ECSTATIC - Abstracts

Nickolas Castro – Trisection of smooth 4-manifolds with boundary

A trisection is a decomposition of a smooth, compact, oriented 4-manifold into three diffeomorphic 4-dimensional 1-handlebodies whose pairwise intersections are three dimensional handlebodies, and triple intersection is a closed surface. This 4-dimensional analog of Heegaard splittings of 3-manifolds was first defined by Gay and Kirby in the case of empty and connected boundary. We will show that this structure extends to all smooth, compact, oriented 4-manifolds and induces an open book decomposition on each boundary component.

Jocelyne Ishak – Stable Model Categories

A model category is a category C with three distinguished classes of maps verifying certain axioms. One can construct the homotopy category $Ho(C)$ associated to a model category C by taking the homotopy classes of morphisms in C. Then, we can construct an adjoint pair of suspension and loop functors

$$\Sigma : Ho(C) \leftrightarrows Ho(C) : \Omega.$$

C is called stable if Σ and Ω are inverse equivalence of homotopy categories. In this talk, I will give the definition of a model category, and explain how we pass to the associated homotopy category. Moreover, I will explain what a stable model category is, and give some examples of non-stable and stable model categories.

Claudius Zibrowius – Loopy tangles

The starting point of this talk will be a non-glueable categorification of certain tangle invariants generalising the Alexander polynomial of knots and links. After discussing their construction and some properties, we will specialise to 2-stranded tangles and explore what a glueing formula should look like in this case. As an application, we will see why mutation of a $(-2, 3)$-pretzel tangle does not change delta-graded knot Floer homology. This is joint work with Jake Rasmussen.

David Recio Mitter – Topological Complexity and Configuration Spaces

The topological complexity of a space was introduced by Michael Farber in the early 2000's, in the context of topological robotics. It is a natural number which measures the minimal (unavoidable) instability of any motion planning algorithm on a given space. It turns out to be a homotopy invariant and is related to classical concepts such as the Lusternik-Schnirelmann-category and, perhaps surprisingly, the immersion dimension.

This talk will motivate the study of this invariant and show how it can be calculated for the configuration space of n points on the plane. This example is both interesting from a purely mathematical perspective and from the point of view of applications. If time allows we will state some new results.

Patrick Orson – A Khovanov stable homotopy type for colored links

Take a fixed semisimple Lie algebra $\mathfrak{g}$ and a link in the 3-sphere (i.e. a disjoint embedding of circles). If we label the components of the link by irreducible representations of $\mathfrak{g}$ then the Reshetikhin-Turaev construction provides a powerful tool for building invariants of the link. Moreover these invariants have given rise to so-called 'knot homology theories', which are usually defined in a combinatorial way from the link and representations. But if you have an invariant that looks like the homology of something, it is natural to ask: is there a topological space whose homology gives rise to the invariant? If such a space exists, can the topological space yield deeper invariants than just the homology? I will discuss recent results which answer both of these questions affirmatively for certain choices of Lie algebra and representations.

Sarah Whitehouse – TBC

Abstract TBC.

Daniele Celoria – Concordances in 3-manifolds

We will define the equivalence relation of almost-concordance on the set of knots in a closed 3-manifold, and provide some non-trivial examples using an invariant extracted from knot Floer homology. Time permitting, we will discuss some application.

Daniel Graves – An Introduction to functor homology

We can generalize many homology theories using functor homology. These interpretations can be used to obtain more information about the objects under consideration. This expository talk will introduce some basic constructions of functor homology and discuss the generalizations of some classical homology theories.

Christopher Smithers – Topological Machine Learning: Kernels on point clouds

Over the past few years, techniques from algebraic topology have been adapted so as to be usable in helping to understand the "shape" of data, giving rise to the field of topological data analysis. Persistent homology is a tool for finding topological features of a point cloud which persist at multiple scales, giving an indication of the topological properties of the underlying space the points are assumed to approximate. Whilst these persistent features are well understood (as "persistence diagrams" or "barcodes"), there is no comparison between clouds. In fact, by comparing clouds, and looking at differences and similarities in their persistence barcodes, we can hope to learn more about the individual clouds, and classify them more robustly.

In particular, I will discuss current work in using these barcodes as the basis for a number of kernels. Kernels are essentially a measure of similarity between objects, and are used primarily as the input of a machine learning technique known as Support Vector Machine Learning. I will survey some previous progress in developing such kernels, and compare them with a number of proposed alternatives. Our ultimate aim is to be able to automatically detect the topological structures of a large collection of point clouds, grouping those clouds into different classes based on their "shape". For example, can we distinguish between clouds sampling a sphere, and those sampling a torus? What about points on a circle vs points on a square?

András Juhász – Defining and classifying TQFTs via surgery

We describe a framework for defining and classifying TQFTs via surgery. Given a functor from the category of smooth manifolds and diffeomorphisms to finite-dimensional vector spaces, and maps induced by surgery along framed spheres, we give a set of simple axioms that allows one to assemble functorial cobordism maps. This framework is well-suited to defining natural cobordism maps in Heegaard Floer homology. It also allows us to give a short proof of the classical correspondence between (1+1)-dimensional TQFTs and commutative Frobenius algebras. Finally, we use it to classify (2+1)-dimensional TQFTs in terms of a new structure, namely split graded involutive nearly Frobenius algebras endowed with a certain mapping class group representation.

Csaba Nagy – Cobordism groups of branched coverings

We study branched coverings, ie. smooth maps between n-manifolds that have singularities only of type $z^j \times id_{\mathbb{R}^{n-2}}$, where $z^j : \mathbb{R}^2 \to \mathbb{R}^2$ is the complex power function. The cobordism relation can be defined in the natural way between branched coverings of given degree, and the resulting cobordism classes form an abelian group under disjoint union.

For each k we construct a universal k-fold branched covering, from which every k-fold branched covering can be induced. Its base space classifies branched coverings, that is, the nth bordism group of this space is isomorphic to the cobordism group of k-fold branched coverings between n-dimensional manifolds. We calculate the rational homology and bordism groups of the classifying space, which determine the rank of the cobordism group.

Katie Vokes – Bicorn curves and the coarse geometry of curve graphs

Given a surface S, we define the curve graph of S to have as vertex set all isotopy classes of curves on S, with an edge between vertices if they have representative curves which are disjoint. It is used, for example, to study properties of the mapping class group of S. A key property of curve graphs is that they are Gromov hyperbolic, and that the constant of hyperbolicity is in fact independent of the surface. Amongst other methods, one way in which this has been proved is by surgery arguments using "bicorn curves". We aim to introduce this area and to give a sense of how these ideas can be used.

Francesca Iezzi – Graphs of curves, arcs, and spheres, and connections between all these objects.

Given a surface S, the curve graph of S is defined as the graph whose vertices are simple closed curves on S up to isotopy, where two vertices are adjacent if the two corresponding curves can be realised disjointly. This object was defined by Harvey in the 80's, and has been an extremely useful tool in the study of surface mapping class groups.

Similarly one can define the arc graph of a surface with boundary, and the sphere graph of a 3-manifold. In this talk I will introduce all these objects, describe some of their properties and some maps between these objects.

Time permitting, I will describe some joint work with Brian Bowditch, where we prove that, under particular hypothesis, there exists a retraction between the sphere graph of a 3-manifold and the arc graph of a surface.

Alberto Cavallo – Concordance of links in grid homology

We introduce a generalization of the Ozsvath-Szabo τ-invariant to links by studying a filtered version of link grid homology. We prove that this invariant remains unchanged under strong concordance and we show that it produces a lower bound for the slice genus of a link. We also give an application to Legendrian link invariants in the standard contact 3-sphere.

Michael Snape – Invariants of strongly invertible knots

A strongly invertible knot is a knot in the 3-sphere along with an orientation preserving involution of S^3 that reverses the orientation of the knot. Through applying a construction developed by Makoto Sakuma in the 1980's it is possible to associate to each strongly invertible knot a unique knot in the thickened annulus $A \times I$. In this talk I will outline Sakuma's construction and show some applications in light of modern homology theories. In particular, the analogues of Khovanov homology and the Jones polynomial in the annular setting will be discussed.

Rachael Boyd – Calculating the homology of Coxeter groups

Coxeter groups are abstract groups which can be formally described in terms of reflections. They appear in areas of mathematics such as root systems and Lie theory, combinatorics, and geometric group theory. Any Coxeter group can be realised as a group generated by reflections on a contractible complex, called the Davis complex. By studying the stabilisers of cells of the Davis complex we are able to compute the first and second homology groups of an arbitrary Coxeter group, with scope for computations of higher groups.