

Erratum for “Monopole Floer homology and Legendrian knots”

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Proposition 5.6 and Corollary 5.7 of our paper “Monopole Floer homology and Legendrian knots,” which use the invariants ℓ_g of that paper to construct non-loose Legendrian knots in overtwisted contact manifolds, are incorrect. In this erratum we explain the problem with the proof of Proposition 5.6 and why it cannot be true.

[57M27](#); [57R58](#), [57R17](#)

Let $\mathcal{K} \cup \mathcal{S}$ be a two-component Legendrian link in the contact 3-manifold (Y, ξ) such that S is homotopic to a meridian of K in $Y \setminus K$. In Proposition 5.6 of our paper [2], we claim to show that if $tb(\mathcal{S}) \geq 0$ and $\ell_g(\mathcal{K}) \neq 0$, then the invariant $\ell_g(\mathcal{K}_{\mathcal{S}})$ is also nonzero, where $\mathcal{K}_{\mathcal{S}}$ is the image of \mathcal{K} in the contact manifold $(Y_{\mathcal{S}}, \xi_{\mathcal{S}})$ obtained by contact $(+1)$ -surgery on \mathcal{S} , and so by Proposition 4.1 the knot $\mathcal{K}_{\mathcal{S}}$ is non-loose. However, this cannot be the case whenever \mathcal{S} is stabilized because one can find an overtwisted disk in $Y_{\mathcal{S}}$ in a neighborhood of the surgery torus: see e.g. [1, Lemma 3.1]. Thus Proposition 5.6 and Corollary 5.7 are false, and the knots in Figure 7 are loose. To the best of our knowledge, the rest of the paper remains correct.

The error occurs at the end of the proof of Proposition 5.6, where we observe that S is homotopic to a nonseparating curve $c \subset \bar{R} \subset -\bar{Y}$, and we claim that therefore c becomes nullhomotopic when we perform 0-surgery on S . The problem is that c and S cobound an immersed annulus, but since S may intersect the interior of that annulus we cannot guarantee that the annulus will produce an immersed disk bounded by c in the surgered manifold $-\bar{Y}_0$. Then \bar{R} is not necessarily compressible in $-\bar{Y}_0$, and so it does not follow that $KHM((-\bar{Y})_0, K_0) = \widetilde{HM}(-\bar{Y}_0 | \bar{R})$ vanishes as claimed.

We thank Ken Baker for discovering both the incorrectness of these results and the source of the mistake and bringing them to our attention.

References

- [1] P Lisca, A I Stipsicz, *Notes on the contact Ozsváth-Szabó invariants*, Pacific J. Math. 228 (2006) 277–295

- [2] **S Sivek**, *Monopole Floer homology and Legendrian knots*, *Geom. Topol.* 16 (2012) 751–779

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