Differential Geometry: Spring 2024

Plan for course. (The level of detail will depend on how the time goes.)

1. General discussion of various differential geometric structures. The Riemann curvature tensor.
2. The Rauch and Bishop comparison theorems in Riemannian geometry.
3. Symmetric spaces.
4. Riemannian holonomy. The Gibbons-Hawking construction of 4-manifolds with holonomy Sp(1).
5. Positive scalar curvature and minimal hypersurfaces.
6. Spinors and the Dirac operator. Applications to positive scalar curvature.
7. Equivariant de Rham theory and the Gauss-Bonnet formula.
8. The Berline-Getzler-Vergne differential-geometric proof of the index formula for Dirac operators.

Many books treat (1), (2), (3), (4). The reference for (8) is *Heat Kernels and Dirac operators* Berline, Getzler, Vergne Springer 2004, which also covers material for (6),(7). Notes from previous lecture courses on this web page also treat many of these topics.