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Dispersive and Strichartz estimates for hyperbolic equations with constant coefficients. (English)

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In an article from 1970 [J. Funct. Anal. 5, 218–235 (1970; Zbl 0189.40701)] *R. S. Strichartz* proved that the solution of the Cauchy problem for the homogeneous wave equation satisfies a specific estimate. These type of a priori estimates have been further proved for different equations and today are known under the name of L^q-L^p decay estimates or Strichartz estimates. Using the mentioned estimate, Strichartz could prove the global existence and uniqueness of solutions to the Cauchy problem for nonlinear wave equations with small initial data.

This technique of proving first a priori estimate for the linear equation and using it to prove global existence and uniqueness for nonlinear evolution equations, local existence assumed to be available, is today a largely used procedure.

The purpose of the work is to investigate dispersive and Strichartz estimates for general hyperbolic equations with lower order terms and the influence of lower order terms on the decay properties of solutions.

The analysis is based on the properties of characteristic roots.

Results for different types of characteristic roots, without and with multiplicities, away from the real axis, with non-degeneracies or meeting the real axis, are summarizes in a main theorem. An application to nonlinear problems is also shown.

The subsequent chapters contain the proofs as well as analysis of results. Necessary properties of roots of hyperbolic polynomials are shown and estimates for oscillatory integrals under certain convex assumptions on the level sets of the phase function as well as dispersive estimates for solutions to the Cauchy problem are proved. The case of multiple characteristics is extensively presented.

The last section is devoted to examples and possible applications.

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Keywords : Strichartz estimates; dispersive estimates; general hyperbolic equations; Cauchy problem; decay of solution; L^q-L^p decay estimates; characteristic roots; estimates for oscillatory integrals; multiple characteristics

Classification :

∗35-02 Research monographs (partial differential equations)
35L30 Higher order hyperbolic equations, initial value problems
35L75 Nonlinear hyperbolic PDE of higher (> 2) order
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42B37
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35B40 Asymptotic behavior of solutions of PDE