

Logarithmic geometry and stacks in resolution of singularities and moduli

Dan Abramovich, Brown University

LMS lecture series

July 1, 2024

Theme: hidden smoothness

The two underlying topics are about **hidden smoothness**:

1. A variety X with quotient singularities behaves a bit like a smooth variety.

For instance: if X complex projective then $H^*(X, \mathbb{Q})$ has Poincaré duality.

2. A toric variety X is a bit like a smooth variety.

For instance $\Omega_X^1(\log D)$ is locally free, D the toric boundary.

No accidents in mathematics

Accidents like this usually hide deeper meaning

1. quotient singularities hide **smooth** algebraic stacks
2. toric varieties are **logarithmically smooth**

Deeper meaning are combined and used in:

A. **Moduli of stable maps and Gromov–Witten theory.**

A subject with long history around LMS people, many past and present leaders, our lecturers. The lectures go quite far in this direction.

B. **Resolution of singularities**

I'll attempt mainly to explain **why** resolution of singularities in characteristic 0 actually works

Lecture topics

- Introduction to stacks: Abramovich
- Geometry and combinatorics of SNC varieties: Ranganathan
- Moduli spaces: Bousseau
- Gromov–Witten Theory: Carocci
- Logarithmic geometry: Nabijou
- Moduli of logarithmic curves and log GW theory: Argüz
- Resolution of singularities: Abramovich
- Punctured logarithmic invariants: Abramovich
- Research lectures by Nabijou, Arguz, Bousseau, Carocci, Ranganathan

End of introduction

Next: introduction to stacks