

# Probabilities and Quantum Mechanics

First Year Undergraduate Research Project (M1R)

Imperial Mathematics

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# What do you need to know?

Only some first year maths stuff like matrix math, probability, calculus, differential equations, and complex numbers

You don't need to know anything about quantum mechanics

## What will you do?

1. We'll cover a corner of quantum mechanics. You'll start by learning this.
2. I have four suggested project themes. You choose one. Each theme will contain a few mini problems. You'll work on those. I will help you get started.
3. It will be great if you then can then "direct" your project. Is there an aspect that you find particularly interesting? Pursue it!
4. Make poster and present

# What is quantum mechanics?

Invented in the early 20th century by heroes like Heisenberg, Schrödinger, and Dirac

To understand matter (atoms, molecules, solids, etc) you need to solve the Schrödinger equation, e.g.

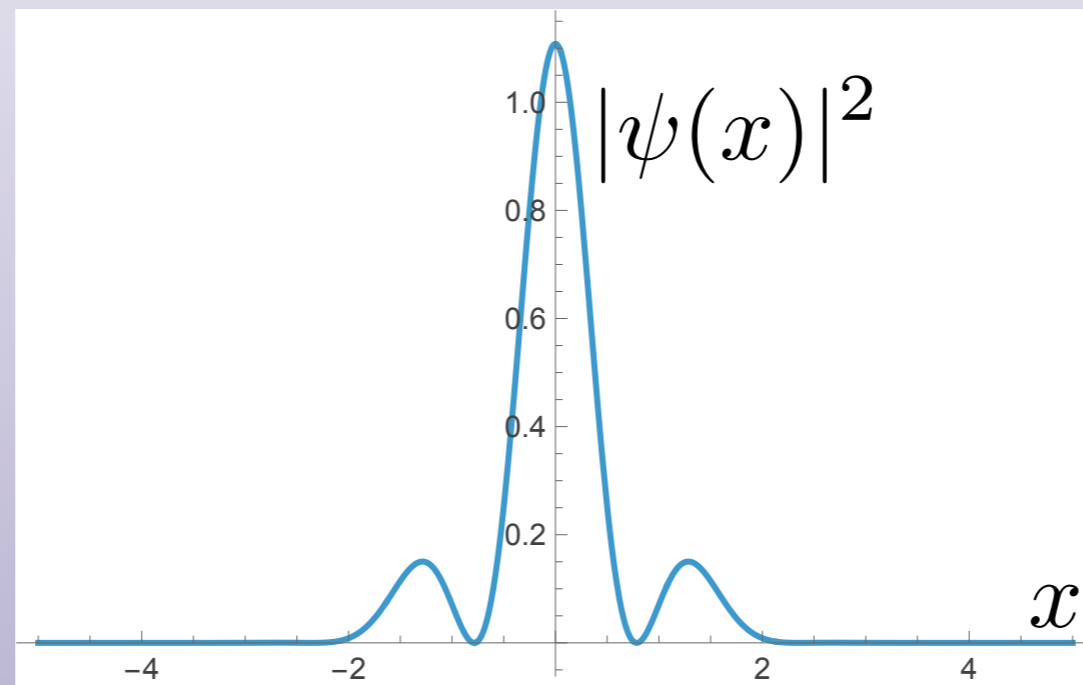
$$\left( -\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) \right) \psi(\mathbf{r}, t) = i\hbar \frac{\partial}{\partial t} \psi(\mathbf{r}, t)$$

Courses on QM often focus on solving various versions of this equation

**We won't be solving PDEs like this for the project**

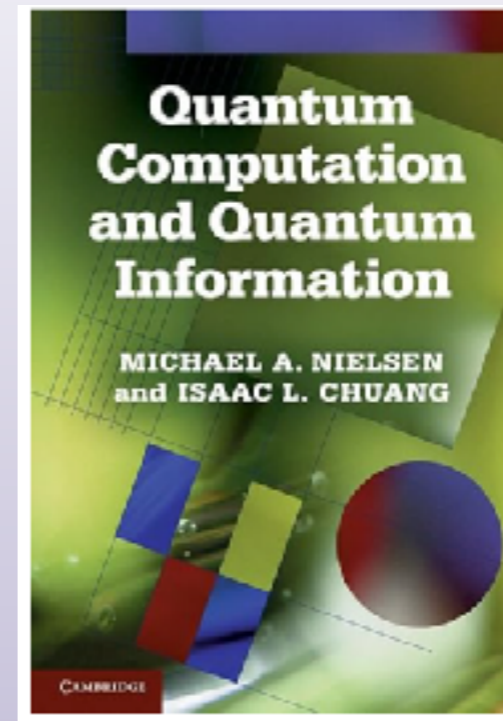
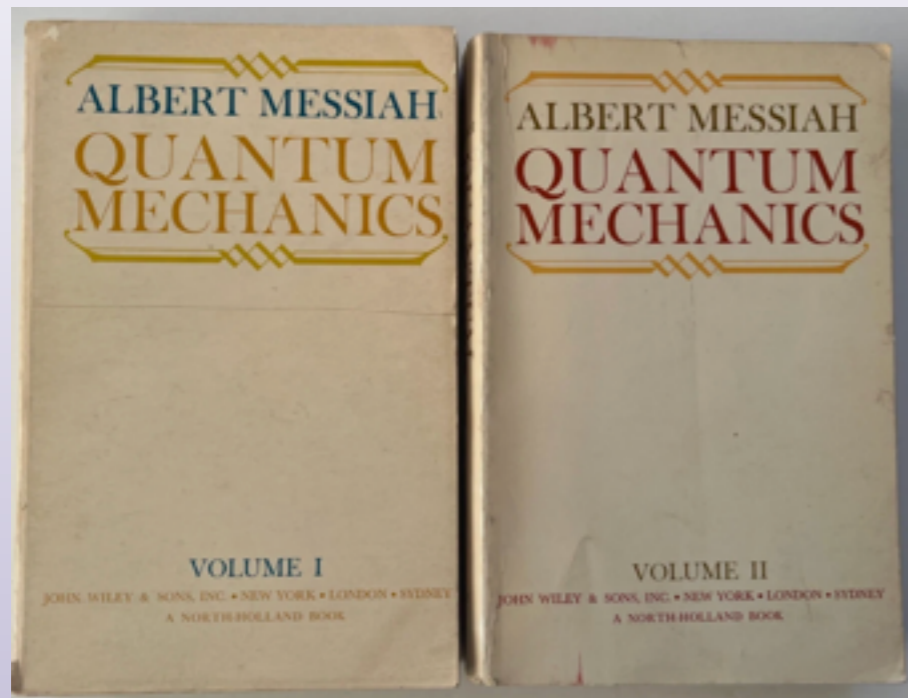
# What is quantum mechanics?

We will think a lot about the wave function  $\psi$  — also called the probability amplitude



$|\psi(x)|^2$  is the **probability density function** for the particle's location

# What is quantum mechanics?



Some approaches focus on solving the Schrödinger equation in various scenarios, with less emphasis on subtle aspects like entanglement. Other approaches put entanglement and measurement front and centre, with Schrödinger as a footnote. We will focus on the latter.

# A quantum coin

Let's look at something that is simple and goes to the essence of quantum

Two classical states:

$$|\text{heads}\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$



$$|\text{tails}\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



Suppose I start with heads:  $|\psi\rangle = |\text{heads}\rangle$

Now look at it.

Probability of heads:  $|\langle \text{heads} | \psi \rangle|^2 = 1$

Probability of tails:  $|\langle \text{tails} | \psi \rangle|^2 = 0$

Kind of dull...

These are Born rules which we'll learn about...

# A quantum coin

Now suppose we let it evolve according to:

$$i \frac{d}{dt} |\psi\rangle = H |\psi\rangle \quad H = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Solve and get:  $|\psi\rangle = \cos(t) |\text{heads}\rangle - i \sin(t) |\text{tails}\rangle$

The unusual thing: it can be both heads and tails. It is called a **superposition** state. Superposition states happen all the time in quantum and have no classical counterpart.

Probability of heads:  $|\langle \text{heads} | \psi \rangle|^2 = \cos^2(t)$

Probability of tails:  $|\langle \text{tails} | \psi \rangle|^2 = \sin^2(t)$

# Wave function collapse

Measurement influences the state

Suppose we look at the state

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\text{heads}\rangle + |\text{tails}\rangle)$$

and see heads.

Then after measurement, the state is:

$$|\psi\rangle = |\text{heads}\rangle$$

# Quantum Computers

The building blocks of classical computers are bits

The building blocks of quantum computers are qubits

Quantum coin is an example of a qubit

Classical bits are either 0 or 1

Qubits, on the other hand, can exist in any superposition:

$$|\psi\rangle = a|0\rangle + b|1\rangle$$

Quantum computers use this extra freedom

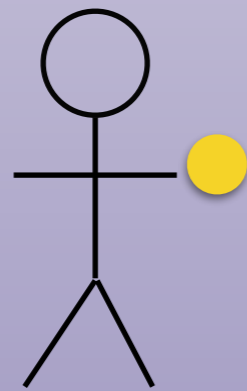
# Entanglement

Let's consider two coins. Alice has one, Bob has the other. Take the coins to be in the **entangled state**:

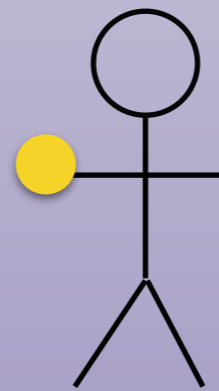
$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\text{heads}\rangle |\text{heads}\rangle + |\text{tails}\rangle |\text{tails}\rangle)$$

Alice's

Bob's



Alice



Bob

# Entanglement

We'll learn how to compute conditional probabilities



Alice looks at her coin first. Then Bob looks at his.

Probability that Bob's coin is heads provided Alice saw heads: 100%

Probability that Bob's coin is tails provided Alice saw tails: 100%

Alice's measurement outcome influenced Bob's coin!

Entanglement is important for quantum computers too.

# Summary (so far)

- \* The wave function is the fundamental object in quantum. Probabilities follow from it.
- \* Superposition
- \* Wave function collapse
- \* Entanglement

# Research Theme 1: Quantum Games

Games (board games, etc) often involve probability and random numbers



Can cool stuff happen when we put in [quantum](#)?

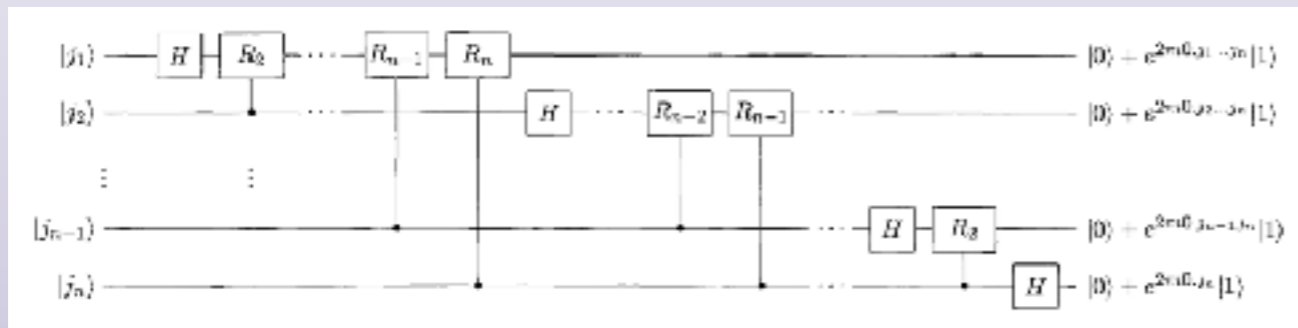
What if we allow for dice superposition or even entanglement?

We will start with the **Quantum Penny Flip Game** (D. Meyer) and consider variations and extensions.

Tools: Linear algebra, statistics, creativity(!)

# Research Theme 2: Engineering quantum gates

To build a universal quantum computer, you need to have a certain collection of primitive gates at your disposal.



(Nielsen and Chuang, Cambridge Press)

Quantum Fourier Transform

This theme will explore how to achieve these gates from **physical** Hamiltonians like:

$$H = \epsilon Z + \lambda X \cos(\omega t)$$

Which combo of parameters will lead to the gates we need?

Tools: Linear algebra, differential equations, numerical methods

# Research Theme 3: Entanglement

Is this state entangled? Meaning will Alice's measurement influence Bob's state?

$$|\psi\rangle = (-0.320706 - 0.0241544i)|00\rangle + (0.0560778 + 0.200581i)|01\rangle \\ + (0.610594 + 0.434295i)|10\rangle + (0.390946 - 0.372708i)|11\rangle$$

It is not obvious.

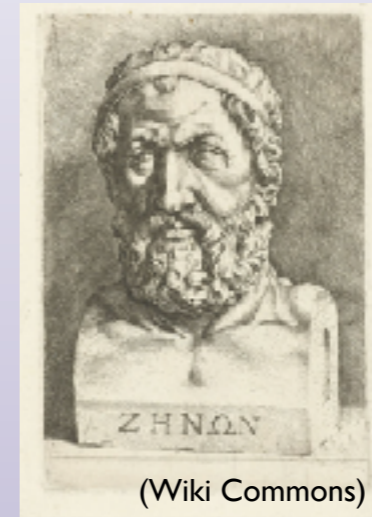
How do we quantify entanglement? Is there a geometrical way to do it? What about *qutrits*?

Tools: Linear algebra (slightly more advanced), geometry

# Research Theme 4: Quantum Zeno

The ancient Greek philosopher's paradox where you can never reach your destination seems to be resolved since

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots = 1$$



(Wiki Commons)

However, suppose during the journey, we perform several “where am I?” quantum measurements? Do Zeno's concerns persist?

Tools: density matrices (we will learn), numerical methods