Self Organised Criticality Its past and more recent field theoretical insights

Gunnar Pruessner

Department of Mathematics Imperial College London

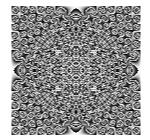
Toulouse, 7 Mar 2012

Outline

- SOC: The early programme
- More models
- Tools in SOC
- Field theory for SOC
- 5 Summary: Any Answers?

Prelude: The physics of fractals
The BTW model
1/f noise — a red herring?
Why SOC?
Experiments

Prelude: The physics of fractals

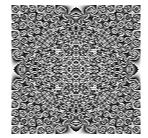


Question: Where does scale invariant behaviour in nature come from?

Answer: Due to a phase transition, self-organised to the critical point.

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Prelude: The physics of fractals

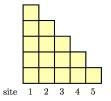


- Anderson, 1972: More is different Correlation, cooperation, emergence
- 1/f noise "everywhere" (van der Ziel, 1950; Dutta and Horn, 1981)
- Kadanoff, 1986: Fractals: Where's the Physics?
- Bak, Tang and Wiesenfeld, 1987: Self-Organized Criticality: An Imperial College Explanation of 1/f Noise

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Prelude: The physics of fractals The BTW model 1/f noise — a red herring? Why SOC?

The BTW Model

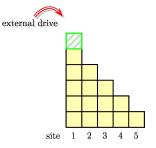


The sandpile model:

- Bak, Tang and Wiesenfeld 1987.
- Simple (randomly driven) cellular automaton → avalanches.
- Intended as an explanation of 1/f noise.
- Generates(?) scale invariant event statistics. (Exact results for correlation functions by Mahieu, Ruelle, Jeng et al.)
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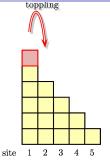
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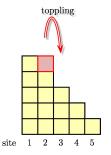
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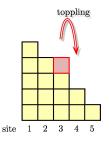


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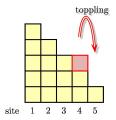
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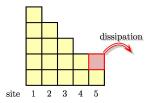
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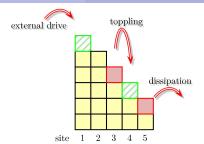


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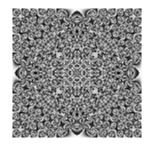
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The physics of fractals.

Prelude: The physics of fractals The BTW model 1/f noise — a red herring? Why SOC? Experiments

The BTW Model



Key ingredients for SOC models:

- Separation of time scales.
- Interaction.
- Thresholds (non-linearity).
- Observables: Avalanche sizes and durations.

1/f noise — a red herring? I

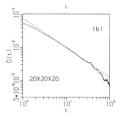


FIG. 3. Distribution of lifetimes corresponding to Fig. 2. (a) For the 50×50 array, the slope $\alpha\approx0.42$, yielding a "1/f" noise spectrum $f^{-1.58}$; (b) $20\times20\times20$ array, $\alpha\approx0.90$, yielding an $f^{-1.1}$ spectrum

From: Bak, Tang, Wiesenfeld, 1987

• Power spectrum $P(f) \propto 1/f$, thus correlation function (via Wiener Khinchin) decays "very slowly".

1/f noise — a red herring? II

Dimensional analysis:

$$\int df \, 1/f^{\alpha} e^{-2\pi i f t} = \dots \propto t^{\alpha - 1} = \text{const}$$

- 1/f noise suggests long time correlations
- Initially, SOC was intended an explanation of 1/f noise.
- Initially the BTW model was thought to display 1/f noise.
- Jensen, Christensen and Fogedby: "Not quite."
- Today: Reduced interest in 1/f.
- Today: Power laws in other observables.

Prelude: The physics of fractals The BTW model 1/f noise — a red herring? Why SOC?

Why is SOC important?

SOC today: Non-trivial scale invariance in avalanching (intermittent) systems as known from ordinary critical phenomena, but without the need of external tuning of a control parameter to a non-trivial value.

Emergence!

- Explanation of emergent,
- ...cooperative,
- … long time and length scale
- ...phenomena,
- ... as signalled by power laws.

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Universality!

- Understanding and classifying natural phenomena
- ... using Micky Mouse Models
- ... on a small scale (in the lab or on the computer).
- (Triggering critical points?)
- But: Where is the evidence for scale invariance in nature (dirty power laws)?

Prelude: The physics of fractals The BTW model 1/f noise — a red herring? Why SOC? Experiments

Experiments:

Granular media, superconductors, rain...



Photograph courtesy of V. Frette, K. Christensen, A. Malthe-Sørenssen, J. Feder, T. Jøssang and P. Meakin.

- Large number of experiments and observations:
- Earthquakes suggested by Bak, Tang and Wiesenfeld.
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Non-conservative: The Forest-Fire Models Better Models: The Manna model

Collapse with Oslo Exponents in 1,2,3D

Outline

- SOC: The early programme
- 2 More models
 - Non-conservative: The Forest-Fire Models
 - Better Models: The Manna model
 - Collapse with Oslo
 - Exponents in 1,2,3D
- Tools in SOC
- Field theory for SOC

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More models

- Initial intention for more models: Expand BTW universality class.
- Later: Provide more evidence for SOC as a whole.
- More models...

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Collapse with Oslo Exponents in 1,2,3D

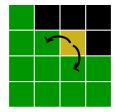
More models

The failure of SOC?

- Zhang Model (1989) [scaling questioned]
- Dhar-Ramaswamy Model (1989) [solved, directed]
- Forest Fire Model (1990, 1992) [no proper scaling]
- Manna Model (1991) [solid!]
- Olami-Feder-Christensen Model (1992) [scaling questioned, $\alpha \approx 0.05$ (localisation), $\alpha = 0.22$ (jump)]
- Bak-Sneppen Model (1993) [scaling questioned]
- Zaitsev Model (1992)
- Sneppen Model (1992)
- Oslo Model (1996) [solid!]
- Directed Models: Exactly solvable (lack of correlations)

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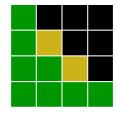
The Bak-Chen-Tang Forest Fire Model



- Originally by Bak, Chen and Tang (1990).
- Intended as a model of turbulence.
- Sites empty, occupied (by tree) or on fire.
- Slow regrowth at rate p.
- Occasional re-lighting.
- Grassberger and Kantz (1991):
 Deterministic pattern, scale given by 1/p.

Non-conservative: The Forest-Fire Models Better Models: The Manna model Collapse with Oslo

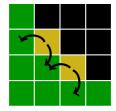
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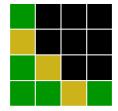


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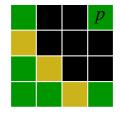
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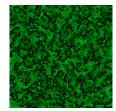
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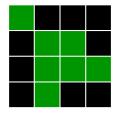


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The Drossel-Schwabl Forest Fire Model



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- Fires instantaneous, explicit lightning mechanism with θ trees grown between two lighntnings attempts.
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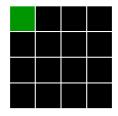
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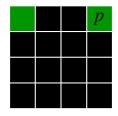


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Exponents in 1.2.3D

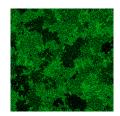
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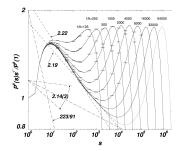
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The Drossel-Schwabl Forest Fire Model

Lack of scaling



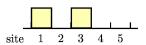
- Finite size not the only scale.
- Scale invariance possible only in the limit of $\theta \to \infty$.
- Lower cutoff moves as well.

Non-conservative: The Forest-Fire Models

Better Models: The Manna model

Collapse with Oslo Exponents in 1,2,3D

Manna Model



Manna Model (1991)

- Critical height model.
- Stochastic.
- Bulk drive.
- Envisaged to be in the same universality class as BTW.
- Robust, solid, universal, reproducible.

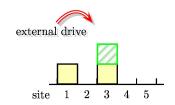
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Defines a universality class.
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 SOC: Past

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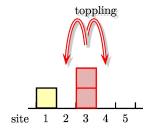
SOC: Past and recent field theory

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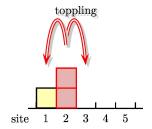
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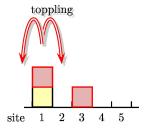
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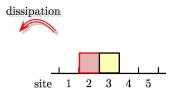
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SOC: The early programme

More models

Tools in SOC

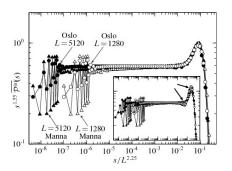
Field theory for SOC

Summary: Any Answers?

Non-conservative: The Forest-Fire Models Better Models: The Manna model

Collapse with Oslo Exponents in 1,2,3D

Collapse with Oslo



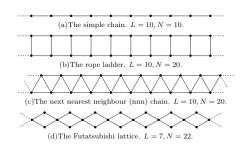
The Manna Model is in the same universality class as the Oslo model.

Non-conservative: The Forest-Fire Models Better Models: The Manna model

Collapse with Oslo Exponents in 1,2,3D

Manna on different lattices

One and two dimensions



From: Huynh, G P, Chew, 2011

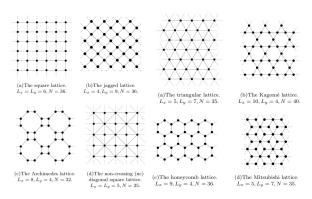
The Manna Model has been investigated numerically in great detail.

Non-conservative: The Forest-Fire Models Better Models: The Manna model

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Manna on different lattices

One and two dimensions



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Collapse with Oslo Exponents in 1.2.3D

Manna on different lattices

One and two dimensions

lattice	d	D	τ	z	α	D_a	τ_a	$\mu_1^{(s)}$	$-\Sigma_s$	$-\Sigma_t$	$-\Sigma_a$
simple chain	1	2.27(2)	1.117(8)	1.450(12)	1.19(2)	0.998(4)	1.260(13)	2.000(4)	0.27(2)	0.27(3)	0.259(14)
rope ladder	1	2.24(2)	1.108(9)	1.44(2)	1.18(3)	0.998(7)	1.26(2)	1.989(5)	0.24(2)	0.26(5)	0.26(2)
nnn chain	1	2.33(11)	1.14(4)	1.48(11)	1.22(14)	0.997(15)	1.27(5)	1.991(11)	0.33(11)	0.3(2)	0.27(5)
Futatsubishi	1	2.24(3)	1.105(14)	1.43(3)	1.16(6)	0.999(15)	1.24(5)	2.008(11)	0.24(3)	0.23(9)	0.24(5)
square	2	2.748(13)	1.272(3)	1.52(2)	1.48(2)	1.992(8)	1.380(8)	1.9975(11)	0.748(13)	0.73(4)	0.76(2)
jagged	2	2.764(15)	1.276(4)	1.54(2)	1.49(3)	1.995(7)	1.384(8)	2.0007(12)	0.764(15)	0.76(5)	0.77(2)
Archimedes	2	2.76(2)	1.275(6)	1.54(3)	1.50(3)	1.997(10)	1.382(11)	2.001(2)	0.76(2)	0.78(6)	0.76(3)
nc diagonal square	2	2.750(14)	1.273(4)	1.53(2)	1.49(2)	1.992(7)	1.381(8)	2.0005(12)	0.750(14)	0.75(4)	0.76(2)
triangular	2	2.76(2)	1.275(5)	1.51(2)	1.47(3)	2.003(11)	1.388(12)	1.997(2)	0.76(2)	0.71(6)	0.78(3)
Kagomé	2	2.741(13)	1.270(4)	1.53(2)	1.49(2)	1.993(8)	1.381(9)	1.9994(12)	0.741(13)	0.75(5)	0.76(2)
honeycomb	2	2.73(2)	1.268(6)	1.55(4)	1.51(4)	1.990(13)	1.376(14)	2.000(2)	0.73(2)	0.79(8)	0.75(3)
Mitsubishi	2	2.75(2)	1.273(6)	1.54(3)	1.50(4)	1.999(12)	1.387(12)	1.998(2)	0.75(2)	0.77(7)	0.77(3)

From: Huynh, G P, Chew, 2011

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Non-conservative: The Forest-Fire Models Better Models: The Manna model

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Manna on different lattices

Three dimensions

Lattice	\overline{q}	$\overline{q^{(v)}}$	$\langle z \rangle$	D	τ	z	α	D_a	τ_a	$\mu_1^{(s)}$	$-\Sigma_s$	$-\Sigma_t$	$-\Sigma_a$
SC	6	1	[0.622325(1)]	3.38(2)	1.408(3)	1.779(7)	1.784(9)	3.04(5)	1.45(4)	2.0057(5)	1.38(2)	1.395(16)	1.36(13)
BCC	8	4	[0.600620(2)]	3.36(2)	1.404(4)	1.777(8)	1.78(1)	2.99(2)	1.444(18)	2.0030(5)	1.36(2)	1.390(19)	1.33(6)
BCCN	14	5	[0.581502(1)]	3.38(3)	1.408(4)	1.776(9)	1.783(11)	3.01(3)	1.44(3)	2.0041(6)	1.38(3)	1.39(2)	1.32(7)
FCC	12	4	[0.589187(3)]	3.35(4)	1.402(8)	1.765(16)	1.78(2)	3.1(2)	1.48(14)	2.0035(11)	1.35(4)	1.37(4)	1.5(5)
FCCN	18	5	[0.566307(3)]	3.38(4)	1.408(7)	1.781(14)	1.787(18)	3.00(4)	1.44(3)	2.0051(8)	1.38(4)	1.40(3)	1.32(9)
Overall				3.370(11)	1.407(2)	1.777(4)	1.783(5)	3.003(14)	1.442(12)	2.0042(3)		1.380(13)	

From: Huynh, G P, 2012

The Manna Model has been investigated numerically in great detail.

Tools in SOC Link to growth phenomena Field theories for Manna and Oslo The Absorbing State Mechanism

Outline

- SOC: The early programme
- 2 More models
- Tools in SOC
 - Tools in SOC
 - Link to growth phenomena
 - Field theories for Manna and Oslo
 - The Absorbing State Mechanism
- Field theory for SOC

Tools in SOC Link to growth phenomena Field theories for Manna and Oslo The Absorbing State Mechanism

Tools in SOC

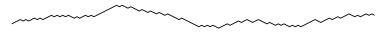
- (Extensive) numerics (BTW, FFM, BS, Manna, Oslo).
- Analytical tools:
 - Exact solutions (so far: directed models only).
 - Mappings to known (understood?) phenomena.
 - Growth processes and field theories.

Tools in SOC Link to growth phenomena Field theories for Manna and Oslo The Absorbing State Mechanism

Link to growth phenomena

Generic scale invariance

Stochastic evolution of sandpile surface.



$$\partial_t \phi(\mathbf{r}, t) = (\mathbf{v}_{\parallel} \partial_{\parallel}^2 + \mathbf{v}_{\perp} \partial_{\perp}^2) \phi + \eta(\mathbf{r}, t)$$

- Generic scale invariance (Hwa and Kardar, 1989, and Grinstein, Lee and Sachdev 1990)
- No mass term $-\epsilon \phi$ on the right \longrightarrow conservative dynamics (finiteness generates ϵ).
- Anisotropy (boundaries?) required in the presence of conserved noise.
- Non-trivial exponents in the presence of non-linearities and non-conserved noise.

Effect of a mass term

Mass term

$$\partial_t \phi = \nu \nabla^2 \phi - \epsilon \phi + \ldots + \eta$$

represents disspation

$$\partial_t \int_V \mathrm{d}^d x \, \phi = \text{surface terms} - \epsilon \int_V \mathrm{d}^d x \, \phi$$

and correlation length

$$\Phi = \dots e^{-|x|\sqrt{\epsilon/\nu}}$$

But: How can a renormalised $\epsilon=0$ be maintained without trivialising the phenomenon?

Tools in SOC Link to growth phenomena Field theories for Manna and Oslo The Absorbing State Mechanism

Field theories for Manna and Oslo

Number of charges interpreted as an interface.



- Manna model has a (weird!) Langevin equation.
- Oslo model implements quenched Edwards Wilkinson equation → interfaces!
- Field theories for both still unclear.
- Mechanism of self-organisation still unclear.
- Link to known universality classes.
- Link to directed percolation?

The Absorbing State Mechanism

Dickman, Vespignani, Zapperi 1998

- SOC model: activity ρ_a leads to dissipation
- dissipation reduces particle density ζ
- density is reduced until system is inactive
 - → absorbing phase
- external drive increases particle density
 - → back to active phase

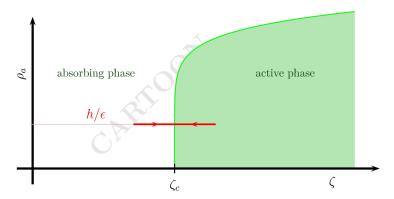
An SOC model can be seen as an AS model that drives itself into the inactive phase by dissipation ϵ and is pushed back into the active phase by external drive h.

$$\dot{\zeta} = h - \varepsilon \rho_a \xrightarrow{\text{stationarity}} \rho_a = h/\varepsilon$$

Imperial College London

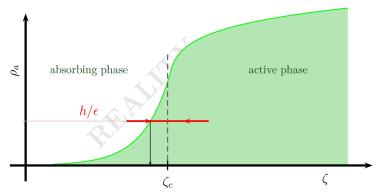
Toulouse, 03/2012

The Absorbing State Mechanism



Idea: SOC drives
$$h/\epsilon=\rho_a$$
 to 0 as $L\to\infty$
Leading orders: $h(L)=h_0L^{-\omega}$ and $\epsilon(L)=\epsilon_0L^{-\kappa}$

The Absorbing State Mechanism



Problem: SOC exponents would be affected by the way how driving and dissipation are implemented \longrightarrow no universality.

Fey, Levine and Wilson suggest that critical point is not reached.

The Manna Model Vertices tree level The SOC mechanism

Outline

- Field theory for SOC
 - The Manna Model
 - Simplifications, bare propagators
 - Vertices, tree level
 - The SOC mechanism

- Does SOC exist in computer models? Yes. Manna and Oslo models are robust and universal.
- Does SOC exist in nature or experiments? Probably:
 Superconductors, granular media, earthquakes, precipitation
- Is SOC ubiquitous? Apparently not.
- Is SOC understood? Yes, it looks good!
- Is it worth understanding? Certainly: Understanding of long-range correlations in nature and criticality without tuning.

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