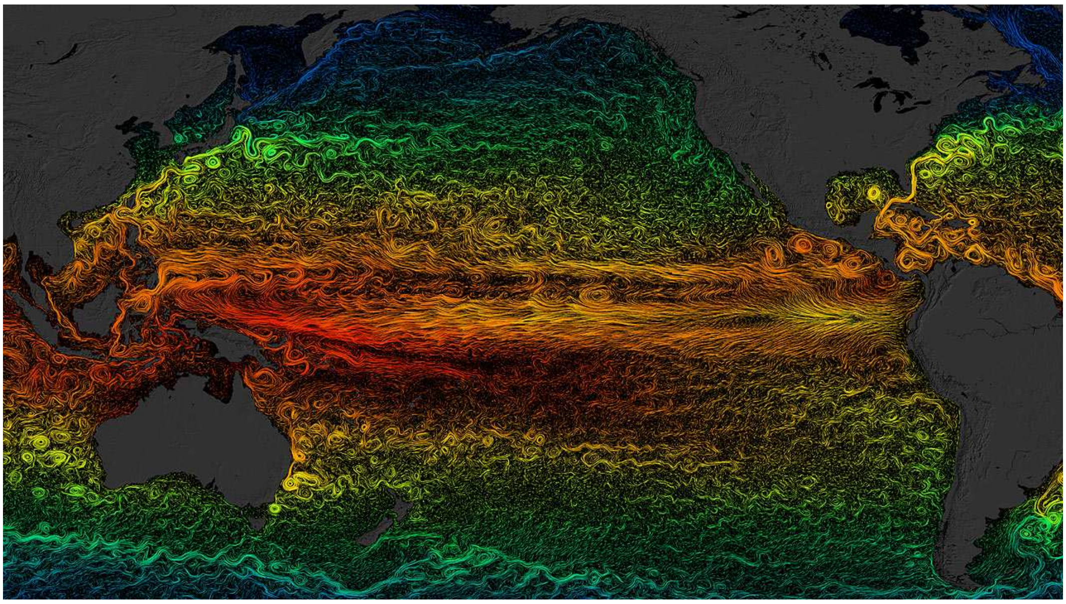
Proposed PhD Research Project:

Rossby Waves in Inhomogeneous Media



1. Statement of the Problem

Rossby waves, also known as planetary waves, are a set of intrinsic transient fluctuations in the atmospheres and oceans that largely owe their properties to the Earth rotation and sphericity. Most of the small wiggles on the Figure can be described as sea-surface manifestations of various families of Rossby waves populating the global ocean. These waves tend to have westward phase velocity, but their group velocity can be in any direction. There are also "barotropic" and the whole set of "baroclinic" Rossby waves distinguished by their vertical structures over the ocean depth. On the fundamental level, Rossby waves are a linear phenomenon, that can be strongly modified by various nonlinear effects (e.g., so-called "eddies" and "coherent vortices"). Oceanic inhomogeneities, such as large-scale currents and density gradients, have even more profound effects: they change locally dispersive and structural properties, as well as stability, energetics and self-interactions of the Rossby waves.

In broader context, this Project deals with a problem of linear wave propagation in inhomogeneous media. For the Rossby waves, this problem remains generally unsolved, despite the fact that the theory of homogeneous Rossby waves is an essential and well-understood part of every textbook on geophysical fluid dynamics. Why is the inhomogeneous problem such a difficult nut to crack down? The main reason for this is technical rather than conceptual. Linear-wave problems are usually solved by Fourier transforms of the governing dynamical equations. This approach results in an algebraic eigenproblem, which relates wave frequency and wavenumbers via dispersion relation, and the Fourier coefficients defining the spatial wave pattern come from the corresponding eigenvectors. If one spatial direction of the linearized wave equation is inhomogeneous (e.g., waves are considered on the jet-like flow with confined velocity profile), the eigenproblem ceases to be algebraic and takes the form of an ODE with boundary conditions. This problem is much more complicated, and when both horizontal directions are inhomogeneous, the problem becomes (in general) even computationally (!) intractable. Linear waves in this case can be understood only approximately, via some local linearizations.

The fully inhomogeneous oceanic Rossby wave problem remained intractable and unpenetrable until Shevchenko and Berloff (unpublished results, 2015) cracked it down on a massively parallel supercomputer, helped by implementation of novel numerical algorithms. As a result of this breakthrough, one can now compute the full spectrum of Rossby waves (i.e., linear normal modes) evolving on a complicated large-scale flow.

The ambitious goal of this Project is full theoretical understanding of linear Rossby waves evolving on oceanic gyres.

2. Milestones of Analysis

The Project will involve computations of the linear normal modes, but most of the efforts will be dedicated to theoretical understanding and various analyses of the outcome, including the following.

- Growth rates, frequencies, spatial patterns, phase and group velocities of the normal modes;
- Non-orthogonal interactions, nonlinear self- and cross-interactions of the normal modes;
- Connection of the normal-mode analyses with initial-value wave propagation problem;
- Dependencies of the Rossby wave properties on the ocean inhomogeneities and other physical parameters.

References: Berloff, P., 2005: On rectification of randomly forced flows. *J. Mar. Res.*, **63**, 497–527.