Proposed PhD Research Project:
Data-Driven Reduced Models

1. Statement of the Problem

This Project is about multiscale modelling of geophysical turbulence consisting of motions on very different length and time scales but without clear scale separation between them. These motions can be represented in a statistically optimal way by a set of empirical orthogonal functions (i.e., by statistically determined spatial functions) and the corresponding principal components (i.e., their time dependencies). Is it possible to construct a random model that can emulate the above statistics, thus, reproducing spatio-temporal variability of the turbulence with great fidelity? A mathematical formalism — multilayer stochastic model framework — was developed for and applied recently to complicated geophysical multiscale problems (Kondrashov et al. 2015; Kondrashov and Berloff 2015).

In this formalism, on the upper level of the *reduced model*, the vector of macroscopic variables is predicted by solving the equations driven by some hidden (microscopic) variables. In turn, the vector of hidden variables is predicted on the lower "matrioshka"-type level of the reduced model. "Matrioshka" dolls (see Figure) are authentic Russian art-works. You open up a wooden and brightly coloured figure of female beauty and find inside its reduced copy. When you open open the reduced copy, there is even smaller copy found inside, and so on until the smallest figure in the end ("matrioshka" on Figure has 20 dolls in total!). The hidden variables that affect the macroscopic variables are analogous to the smallest "matrioshka". They are predicted by the system of linear equations driven by the next-level hidden variables, which are analogous to the second smallest "matrioshka", and so on. On each level, the hidden variables lose some fraction of their correlations, until the last-level hidden variables, analogous to the largest "matrioshka", degenerate into the white noise. At this point the "matrioshka" sequence of the lower-level model terminates and the reduced model takes its final shape.

Overall, the resulting statistical reduced model is capable of emulating the macroscopic spatio-temporal variability with great statistical accuracy. It is also purely data-driven, which makes it a powerful and versatile tool, ready to be adapted for different applications.

The *first goal of this Project* is to develop and apply the reduced-model formalism to several types of dynamically modelled types of geophysical turbulence. The *second goal of this Project* is to use the resulting reduced models for simulating effects of actual turbulence in non-turbulence-resolving ocean models, such as those routinely used in climate modelling.

2. Milestones of Analysis

The Project is a mind-boggling mixture of Applied Mathematics, Geophysical Fluid Dynamics and Computer Modelling that will involve the following components.

- Statistical multiscale analysis of different types of geophysical turbulence (i.e., anisotropic β-plane turbulence; wind-driven gyres);
- Developing mathematical formalism of reduced models;
- Fitting the reduced models to different types of geophysical turbulence and different types of motion within them;
- Development of non-turbulence-resolving dynamical models with reduced-model parameterizations of the microscopic motions and their effects;
- Dynamical interpretation of the macroscopic and microscopic variable, as well as their interactions in the reduced model.

References

Kondrashov, D., and P. Berloff, 2015: Stochastic modeling of decadal variability in ocean gyres. *Geophys. Res. Lett.*, in press.
Kondrashov, D., M. Chekroun, and M. Ghil, 2015: Data-driven non-Markovian closure models. *Physica D*, in press.