Proposed PhD Research Project:

Reconciling the Eulerian and Lagrangian Models for Turbulent Transport

Supervisor: Prof. Pavel Berloff (Department of Mathematics)

Project Description

Big Picture. This Project is for a student who wants to develop skills in geophysical fluid dynamics, turbulent transport processes, stochastic PDEs and modelling, analytical skills, scientific computing, statistical analyses and data processing. Eddying ocean currents constantly transport and stir huge amounts of water and its material properties. These processes have profound effects on the general ocean circulation and global climate system, therefore, their description, understanding and predictive modelling are among the main topics in climate research. This Project aims to transform the fundamentals of how we think about and deal with turbulent material transport.

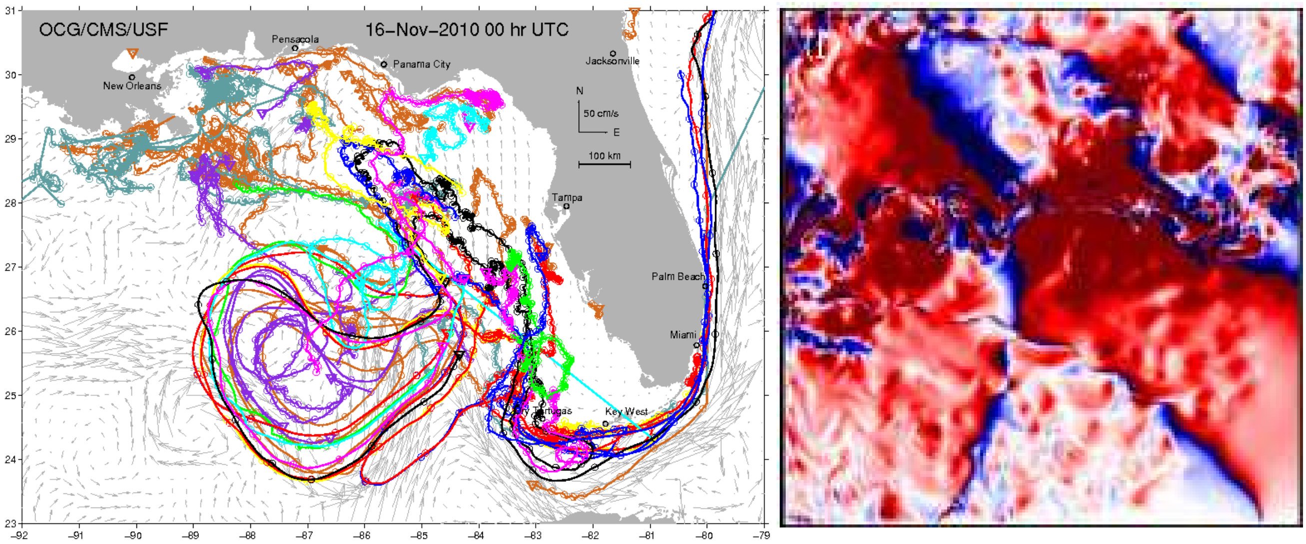

Figure 1: Illustration of the material transport properties in geophysical fluids. Left panel shows differently coloured Lagrangian trajectories of the observed oceanic drifters in the Gulf of Mexico and in the Gulfstream current exiting it. One can detect in these trajectories multiscale motions, ranging from the large-scale geostrophic currents to the variety of small-scale fluctuations induced by submesoscale motions and inertial waves. Important transport properties can be extracted from trajectory analyses, but they are nonlocal in space and time and describe only positive-diffusion characteristics captured via Lagrangian particle dispersion. Right panel shows an instantaneous snapshot of the spatial map for a transport tensor component, which was found for a modelled geophysical turbulence with ensemble of multiple passive tracers. The evident spatial complexity of the tensor, and even more so its negative diffusivities and advective effects, can not be accurately captured by statistical analyses of the Lagrangian trajectories, because the corresponding mathematical methodology has not been developed.

Main Objective. An efficient way of detecting transport processes and properties is by releasing Lagrangian floats and drifters, and by tracking their trajectories (see Fig. 1 for the superposition of float trajectories observed in the ocean) and measuring diffusive part of the transport from single-particle dispersion (Rypina et al. 2012). This Lagrangian description of transport is fundamentally nonlocal in space and time and misses such effects as negative diffusivity and eddy-induced advection of tracer concentration. An alternative Eulerian diagnostics, which is local in space and time, is provided by considering passive-tracer concentrations and extracting the corresponding turbulent fluxes. Through the classical flux-gradient relation these fluxes, as well as their components, can be translated into spatio-temporal maps of the corresponding Eulerian transport tensors (Fig. 1), which can be further reduced to the anisotropic turbulent diffusion and eddy-induced tracer advection (Haigh et al. 2021a,b). The main theoretical disconnect at this point is that the real observations are mostly Lagrangian, whereas the turbulent transport coefficients employed in ocean circulation models are to be the Eulerian ones. The state-of-the-art is such that not only the Lagrangian analyses omit a lot of important information, but also the remaining information is inaccurately translated into the Eulerian parameters due to the nonlocality issue. This problem is fundamental and requires fundamental-science breakthrough, which is offered by this Project.

Work Plan. The underlying transformative idea is both fundamental and highly practical: to close the theoretical gap between the existing Lagrangian and Eulerian approaches for characterizing turbulent transport. This will be achieved by connecting several research tracks. First, the benchmark turbulent solutions computed for geophysical turbulence will provide the reference fields for releasing ensembles of passive tracers, analyzing their solutions and eventually mapping out the Eulerian transport tensors with full details. Second, we will upgrade the existing families of Lagrangian stochastic transport models (Berloff and McWilliams 2002, 2003) by considering co-existing populations of Lagrangian particles with different characteristics and also with the prescribed tracer concentrations. The goal is to be able to model effects of the Eulerian transport in the Lagrangian way, including negative diffusion and eddy-induced tracer concentration advection. Third, the new stochastic modelling methodology will be used in the inverse way for estimating the Eulerian properties from the Lagrangian particles.

In addition to developing conceptual models and methods for cross-fitting the Lagrangian and Eulerian transport characteristics, we will explore and cross-fit these processes for material tracer transports in eddy-resolving benchmark solutions of the ocean circulation. The outcome will provide both theory and methodology for improving parameterizations of turbulence in climate-type ocean models. The Project provides unique opportunity to get training in fluid mechanics, geophysical fluid dynamics, turbulence, stochastic and diffusion-advection transport modelling, statistical Lagrangian-particle analyses, numerical methods and scientific computing. Interactions with research partners in the USA (University of Miami and Woods Hole Oceanographic Institution) will be encouraged.

References

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