EE2 Mathematics

Example Sheet 1: Functions of Multiple Variables

1) Consider $f(p,q) = 2p(q^2 + 2q - 1)$. Find the components of ∇f, i.e. $(\frac{\partial f}{\partial p}, \frac{\partial f}{\partial q})$, at the points i) (-1,0), ii) (1,0), iii) (-1,1) and iv) (1,1). Produce a sketch showing ∇f, the gradient vector, at each of these points.

Pick your answers from:
a) (0,-4) b) (4,8) c) (2,-4) d) (0,4) e) (-2,-4) f) (4,-8) g) (4,4) h) (-2,4)

2) I am an indifferent experimentalist and can be slightly inaccurate in my measurements of time intervals and distances. Recall that a pendulum has period P and length l and these are related by $P = 2\pi(\frac{l}{g})^{\frac{1}{2}}$. What is the percentage error in my estimate of g if
i) I measure P perfectly but make a 0.2% error in estimating l.
ii) I measure l perfectly but make a 0.3% error in estimating P.
It might help you to rearrange the above expression in P.

Pick your answers from:
a) 0.1 b) 0.2 c) 0.4 d) 0.3 e) 0.6 f) 0.8 g) 0.5

3) Consider $f(x,y) = exp(-2xy)$. Find: i) $\frac{\partial f}{\partial x}|_y$ and ii) $\frac{\partial f}{\partial y}|_x$
Check that $\frac{\partial}{\partial y}|_x \frac{\partial f}{\partial x}|_y = \frac{\partial}{\partial x}|_y \frac{\partial f}{\partial y}|_x$. Points (x,y) can be re-expressed in polar co-ordinates (r,θ). Use a *change of variables* (via the chain rule) to obtain expressions for: iii) $\frac{\partial f}{\partial r}|_\theta$ and iv) $\frac{\partial f}{\partial \theta}|_r$. Check that you have obtained the correct answers by first re-expressing $f(x,y)$ as $f(r,\theta)$.

Pick your answers from:
a) $-2xy\exp(-2xy)$ b) $-2x\exp(-2xy)$ c) $-2y\exp(-2xy)$ d) $-2\exp(-2xy)$ e) $-2\exp(-2xy)/r$
f) $2(y^2 - x^2)\exp(-2xy)$ g) $-xy\exp(-2xy)/r$ h) $-4xy\exp(-2xy)/r$

4) Consider the equation $xyz + x^3 + y^4 + z^6 = 0$. What is the value of the product below?

$$\frac{\partial x}{\partial y}|_z . \frac{\partial y}{\partial z}|_x . \frac{\partial z}{\partial x}|_y \tag{1}$$

Pick your answer from:
a) 1 b) 0 c) -1 d) ∞
[harder] Show that this holds in general for $f(x,y,z) = 0$. It might help to consider $x = x(y,z)$, $y = y(x,z)$ and the total differentials dx and dy.

5) Expressions for $f(x,y)$ can be re-expressed in polar co-ordinates as $f(r,\theta)$. i) Does the following expression have the correct dimensions? ii) And is it correct?

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \frac{\partial^2 f}{\partial r^2} + \frac{1}{r}\frac{\partial f}{\partial r} + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} \tag{2}$$

Pick your answers from:

a) yes! b) no.

6) For:

- $(x^2 + y^2 + 1)^{-1}$ find i) all stationary points and ii) their characters.
- $\sin x \sin y$ with $(0 < x < \pi, 0 < y < \pi)$ find iii) all stationary points and iv) their characters.
- $x^4 + y^4$ find v) all stationary points and vi) their characters. [tricky to do properly]

Calculating the Hessian in each case might be useful.

Pick your answers from:
a) $(1, 1)$ b) $(1, 0)$ c) $(0, 1)$ d) $(0, 0)$ e) $(\pi/4, \pi/2)$ f) $(\pi/2, \pi/2)$ g) max h) min i) saddle