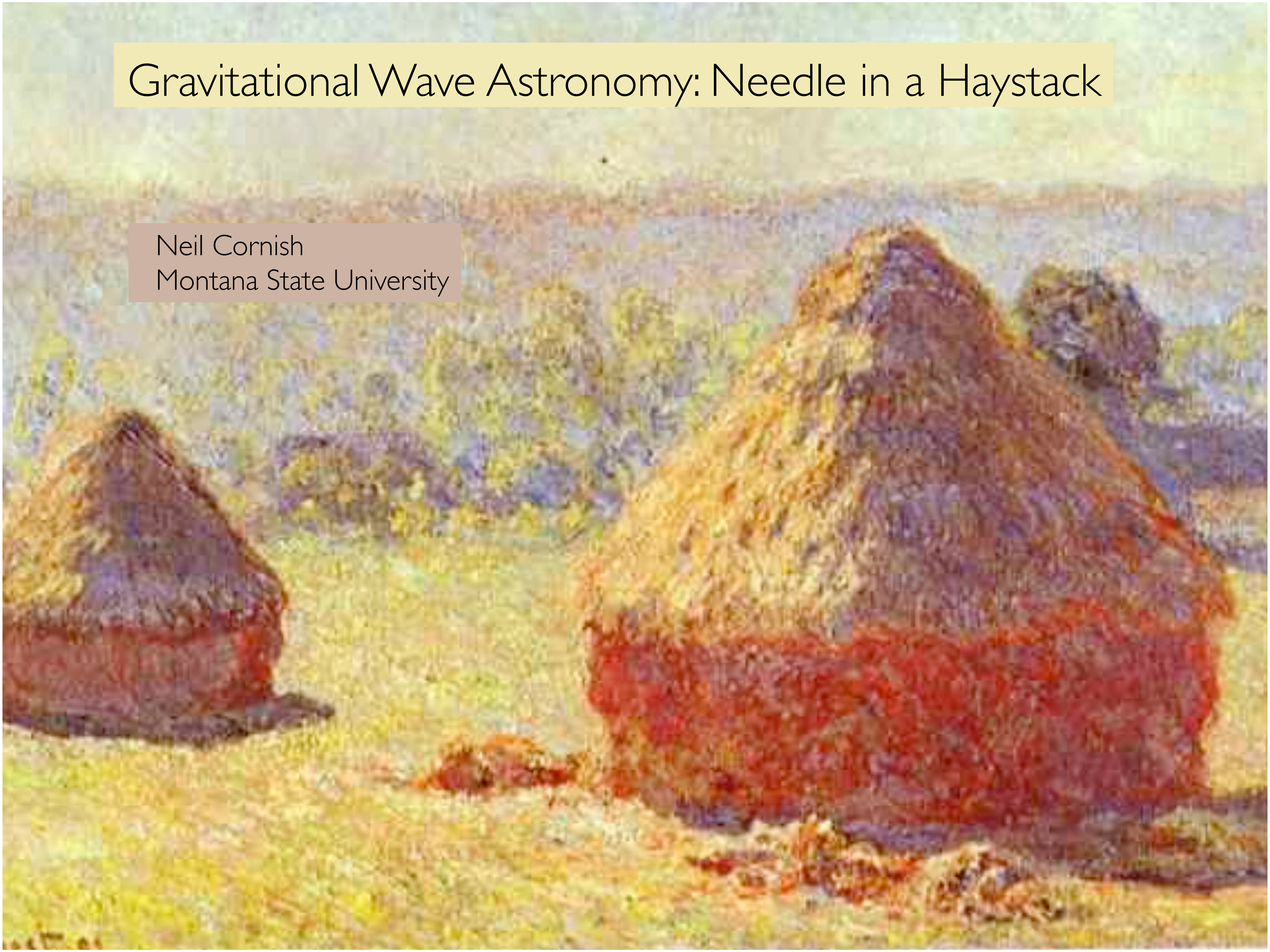
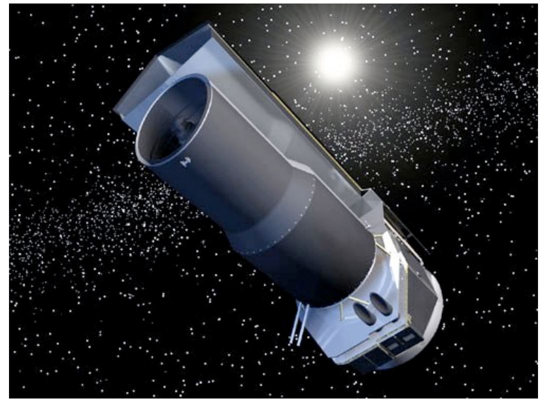
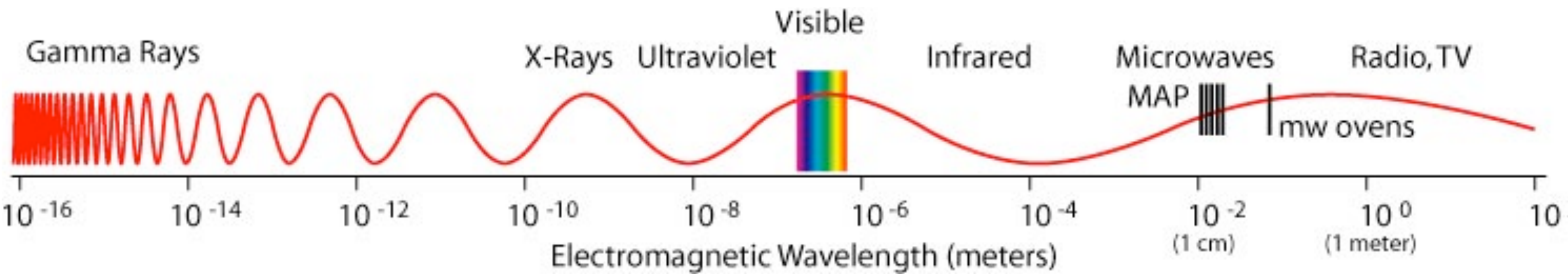
Gravitational Wave Astronomy: Needle in a Haystack

Neil Cornish
Montana State University

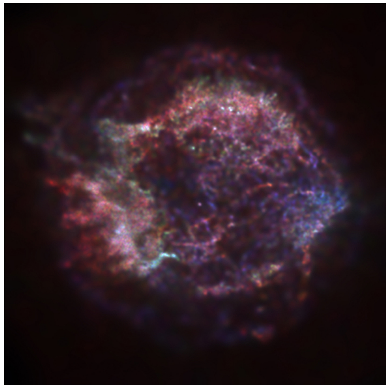


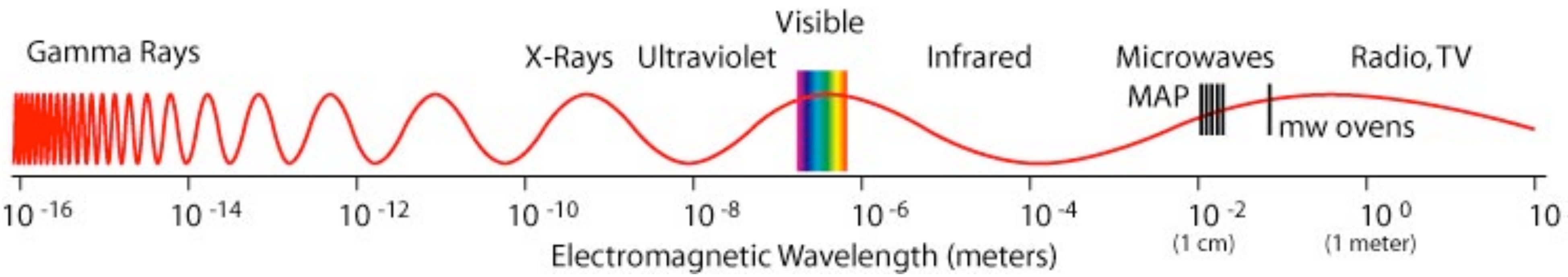
Astronomy Last Century: Multi-Wavelength

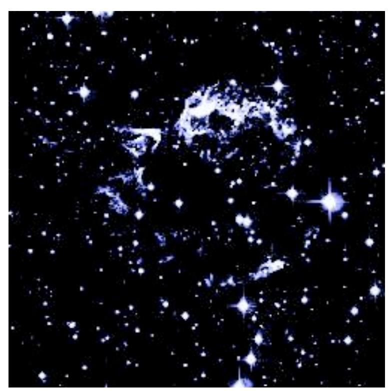


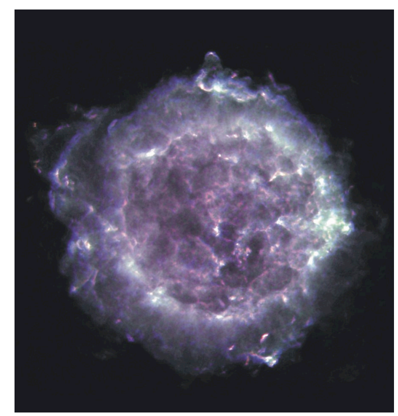


Astronomy Last Century: Multi-Wavelength

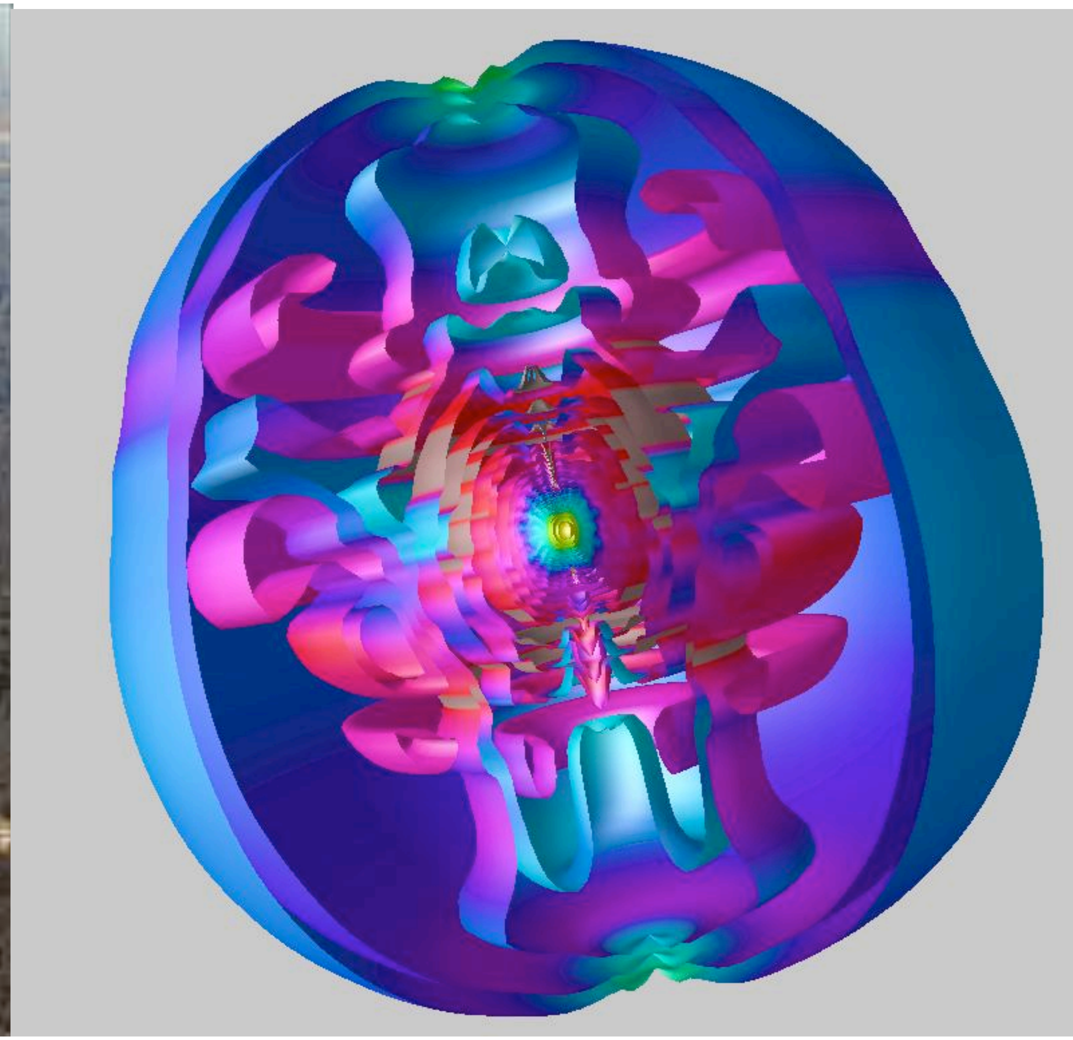




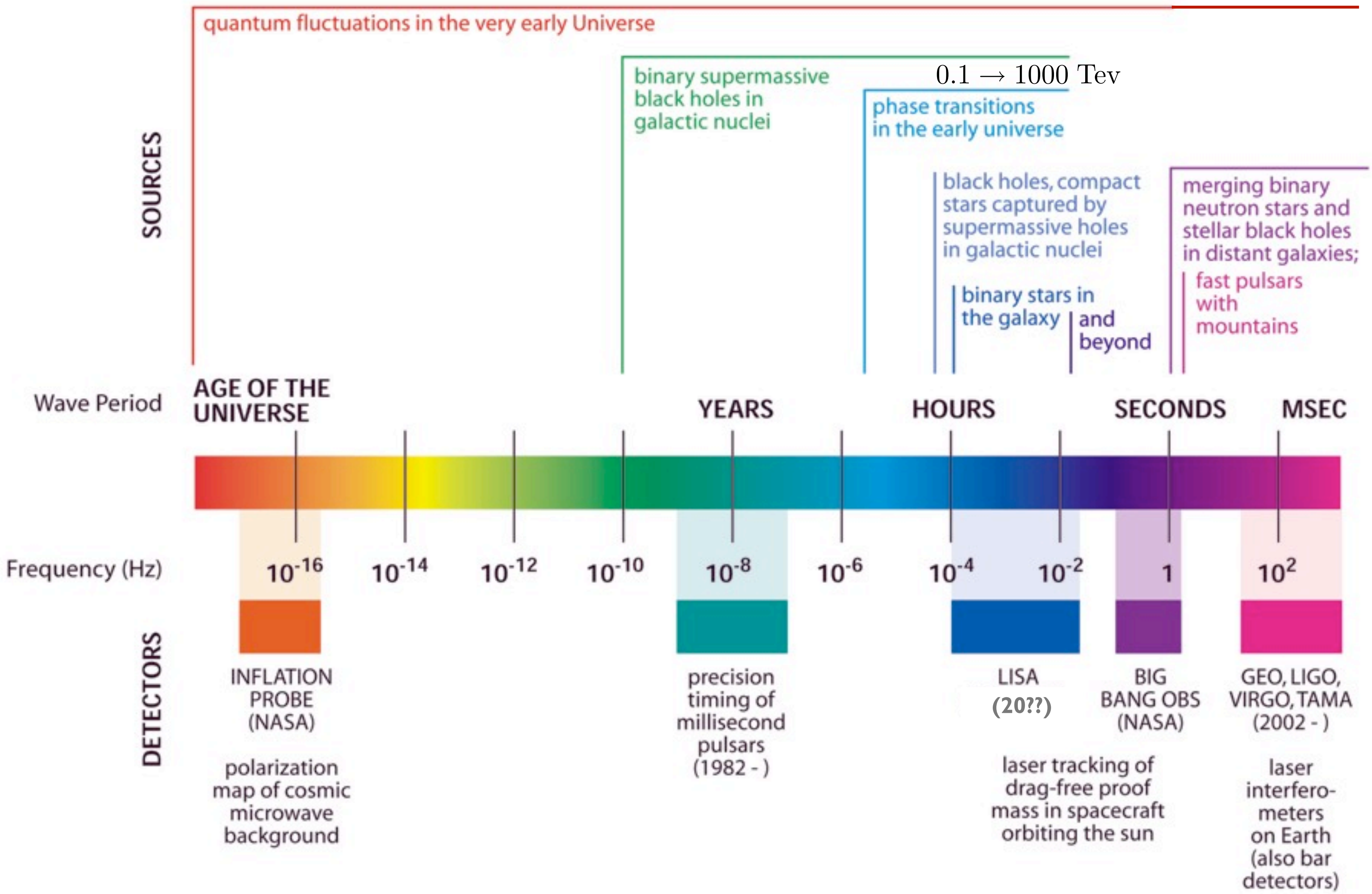


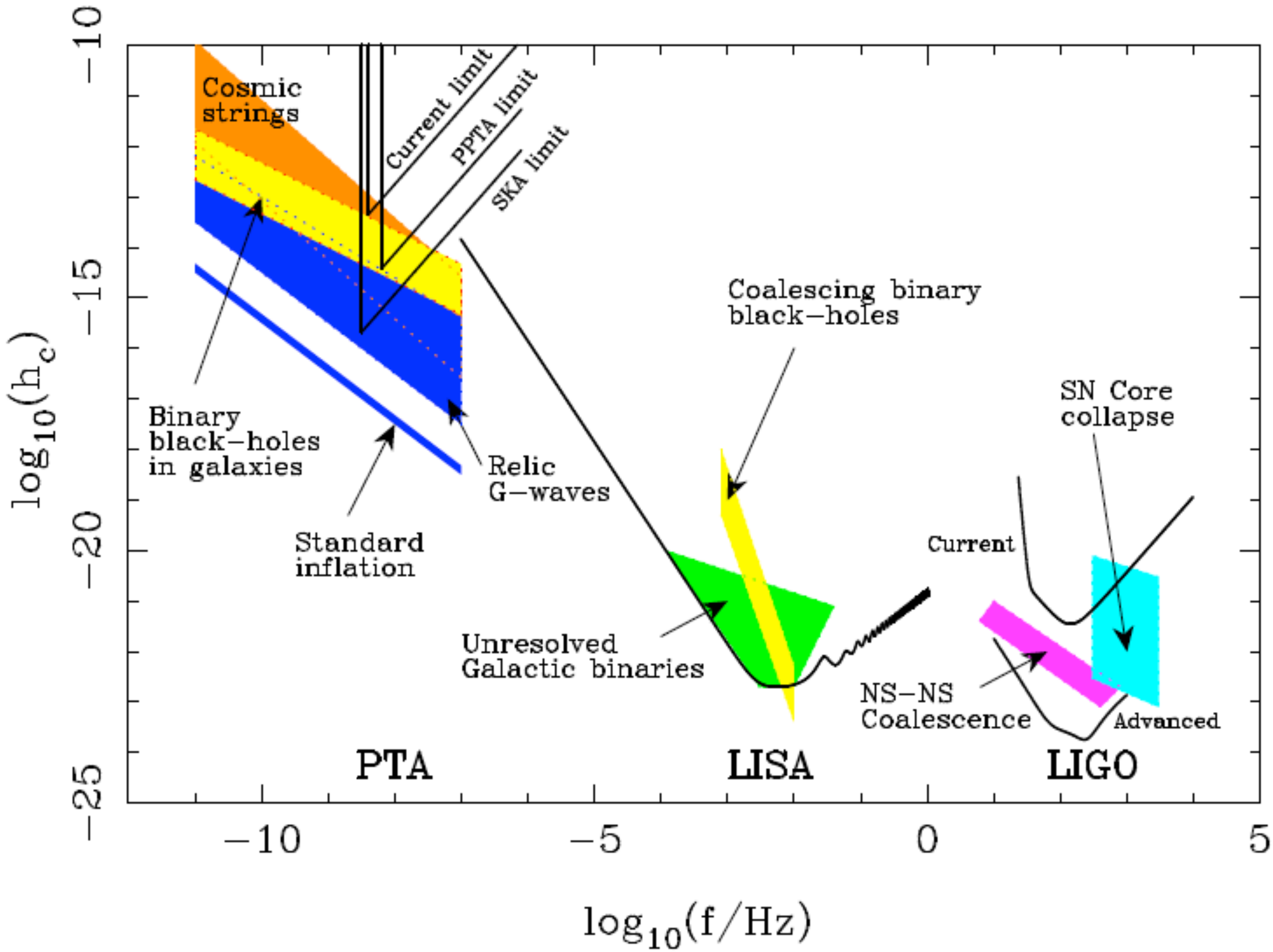


Astronomy this Century: Multi-Spectrum



THE GRAVITATIONAL WAVE SPECTRUM





Cosmic strings
Current limit
PPTA limit
SKA limit
Binary black-holes in galaxies
Relic G-waves
Standard inflation
Coalescing binary black-holes
SN Core collapse
Current
Unresolved Galactic binaries
NS-NS Coalescence
Advanced
PTA
LISA
LIGO
−10
−15
−20
−25
−5
0
5
log10(hc)
log10(f/Hz)

Recipe for Making Gravitational Waves

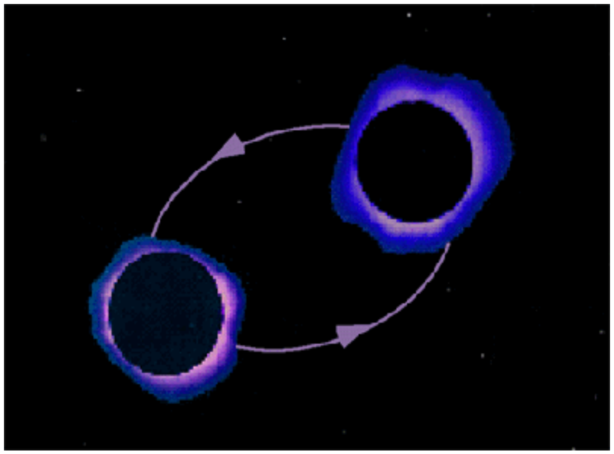
Ingredients:

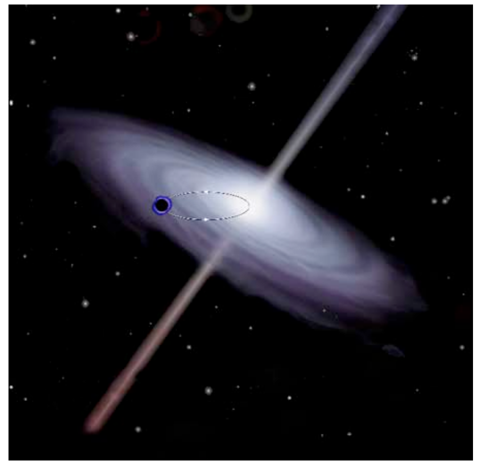
- Large amount of mass/energy (any type will do).

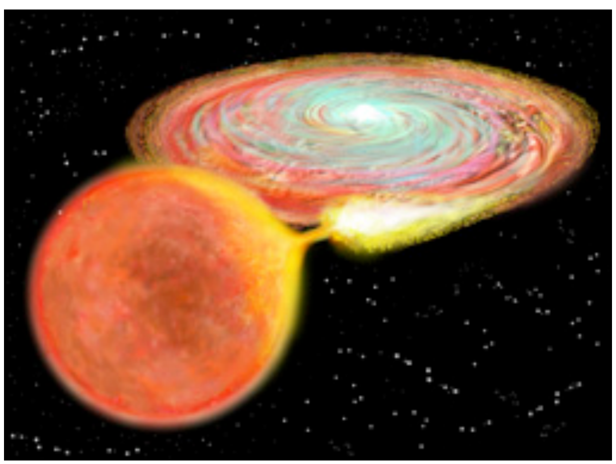
Directions:

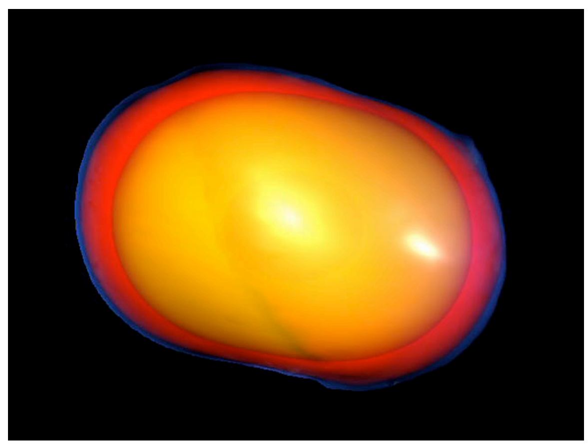
- Squish into a small, lumpy blob. (two blobs work better)
- Shake or stir vigorously.

Gravitational Wave Sources





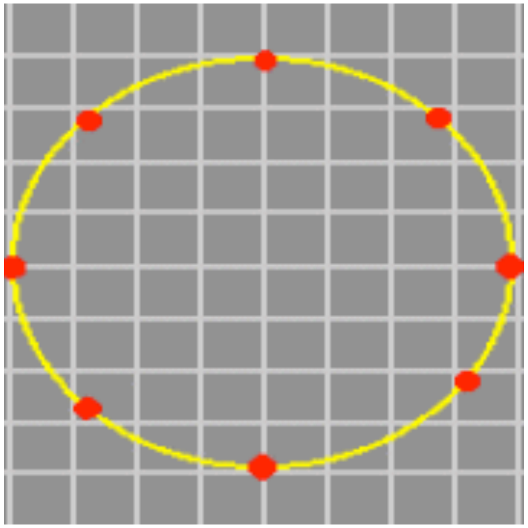




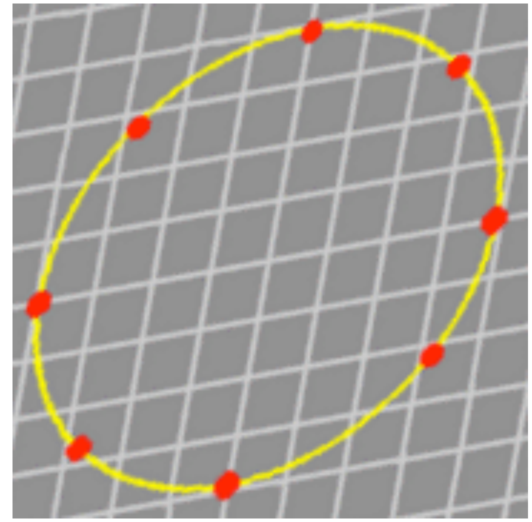
?

Detecting Gravitational Waves

h_+

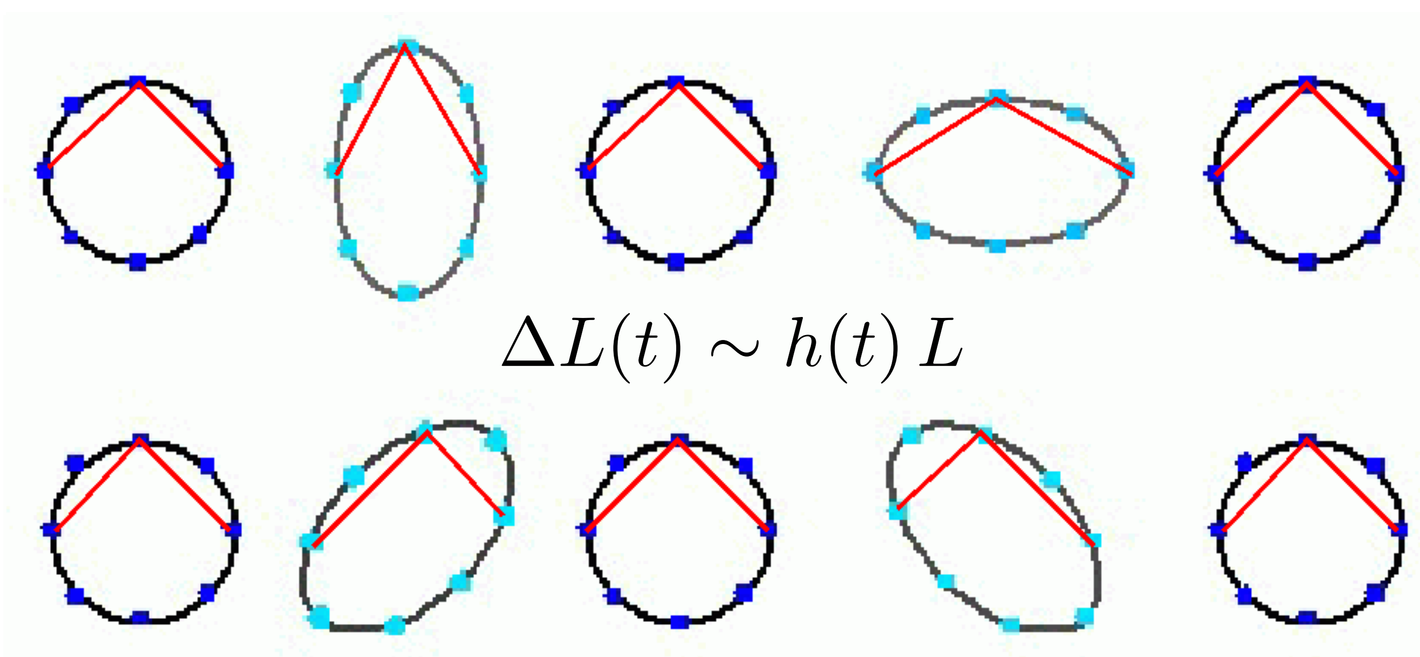


$h_\times$



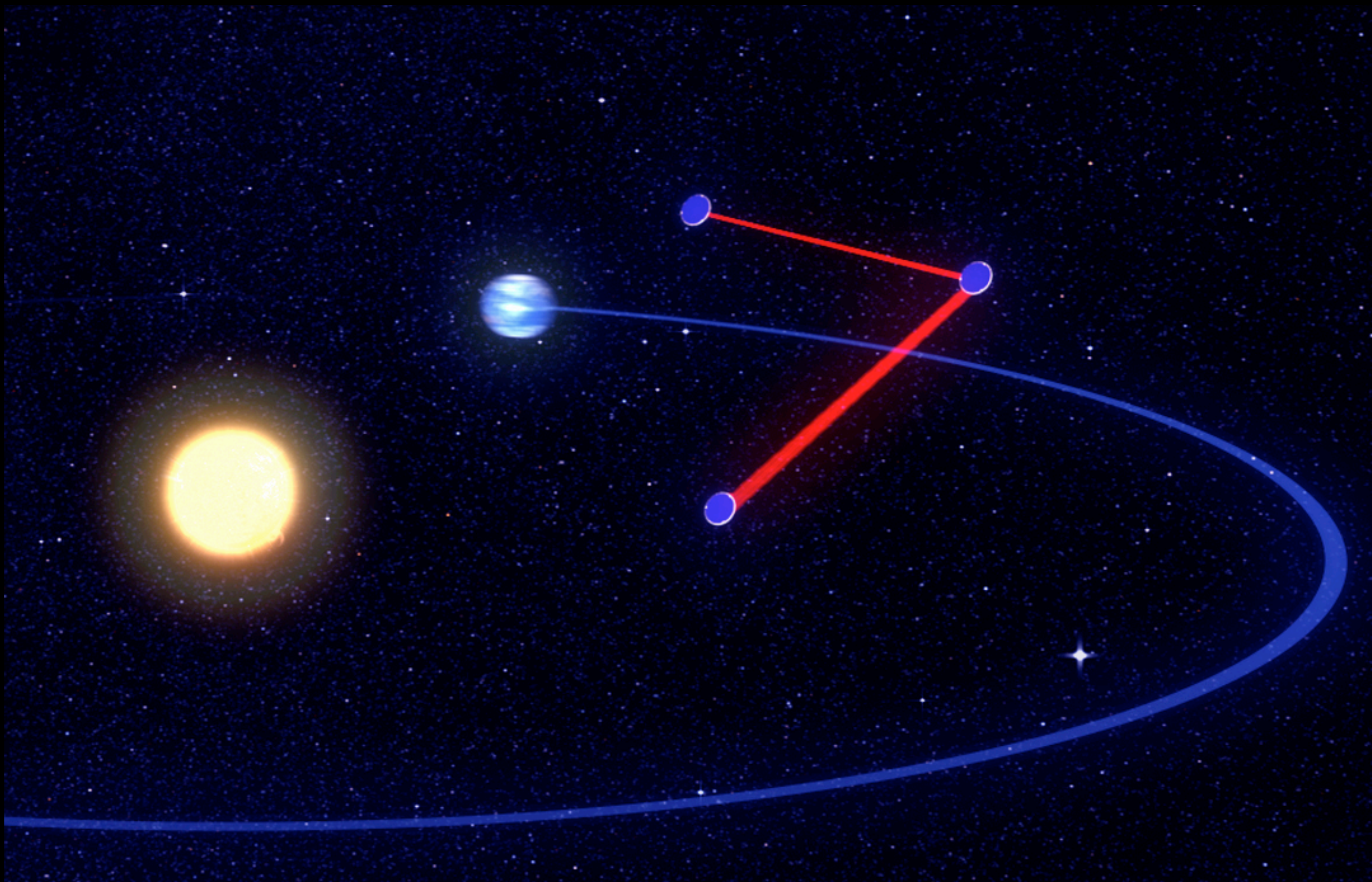
h_+

$h_\times$



$$\Delta L(t) \sim h(t)\, L$$

Laser Interferometer Space Antenna

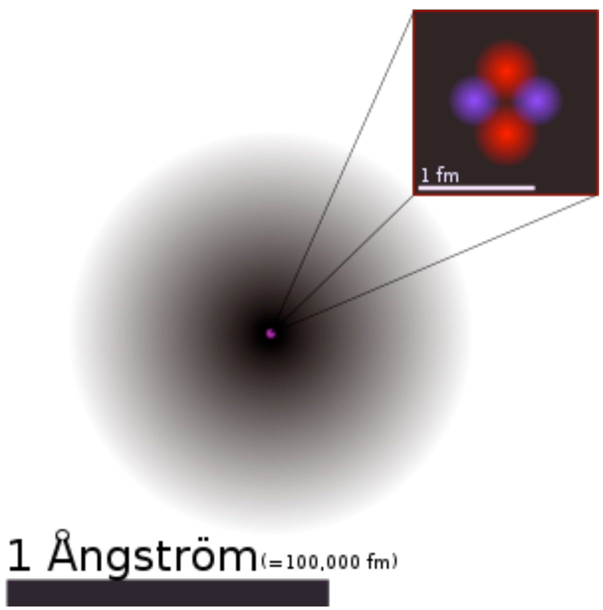


eLISA - http://elisa-ngo.org/

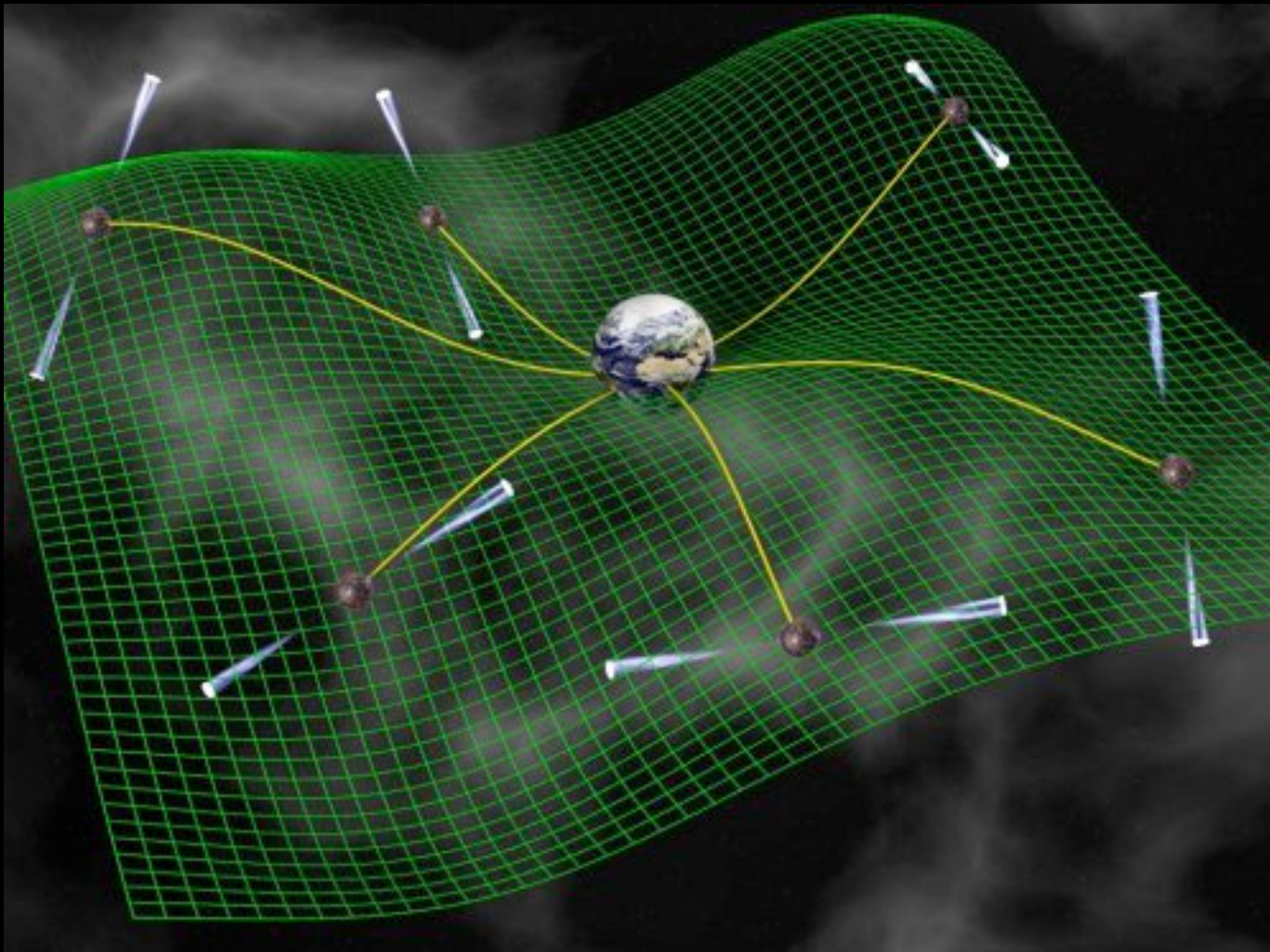
Laser Interferometer Gravitational Observatory

$$h = \frac{\Delta L}{L} \sim 10^{-21}$$

$$L = 4\ \mathrm{km} \;\Rightarrow\; \Delta L = 10^{-3}\ \mathrm{fm}$$



Pulsar Timing Array



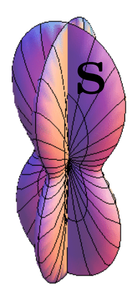
Signal Analysis

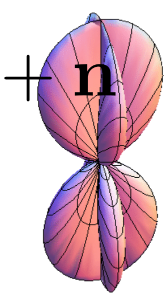
$$s_i(t) = R_i(t,\tau) \star h(\tau) + n_i(t)$$

Detector Output   Detector Response   GW Strain   Detector Noise

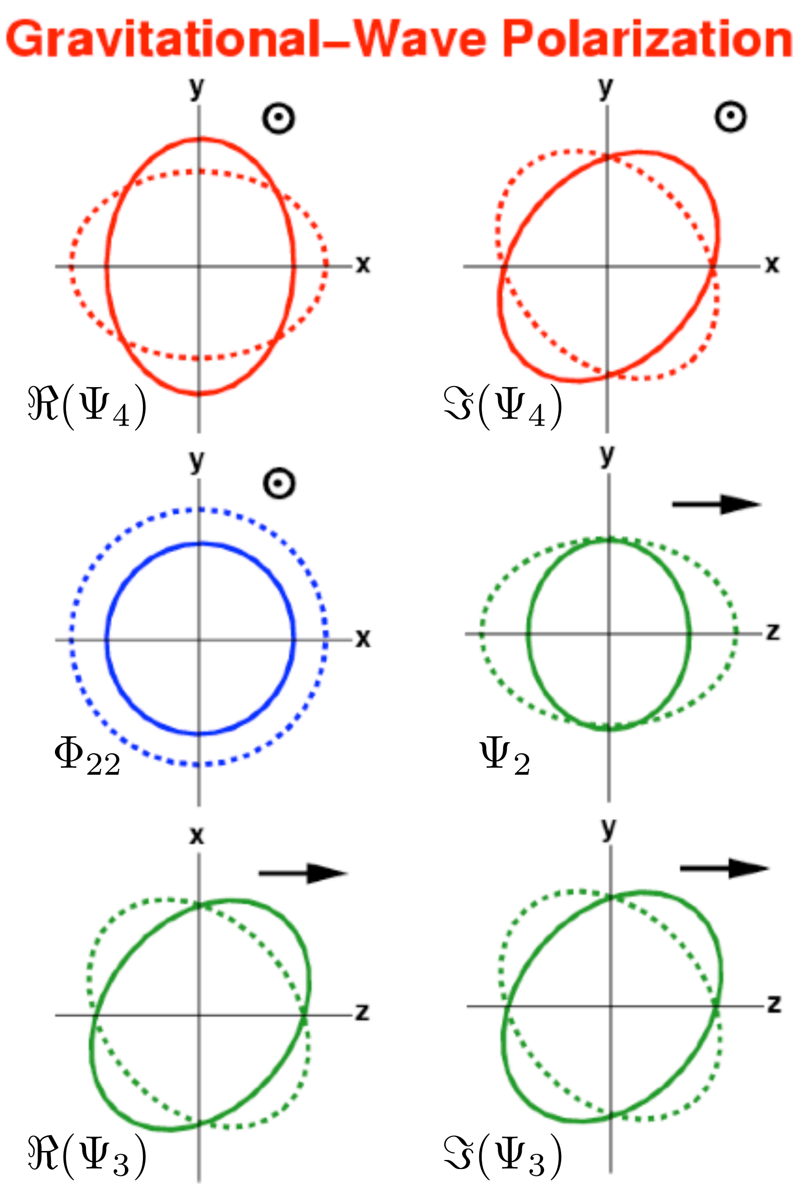
e.g. single LIGO detector $s(t) = F^{+}(t)h_{+}(t-\tau) + F^{\times}(t)h_{\times}(t-\tau) + n(t)$

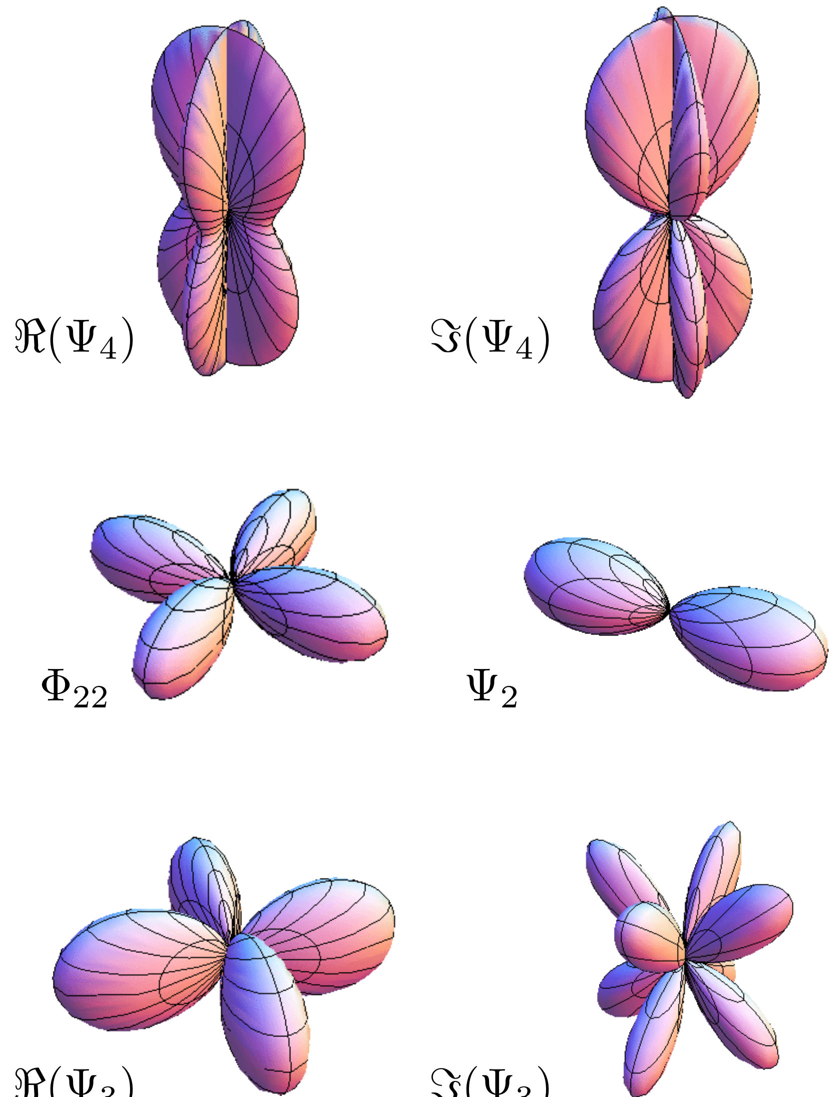
More abstractly: $\mathbf{s} = \mathbf{R} \cdot \mathbf{h} + \mathbf{n}$



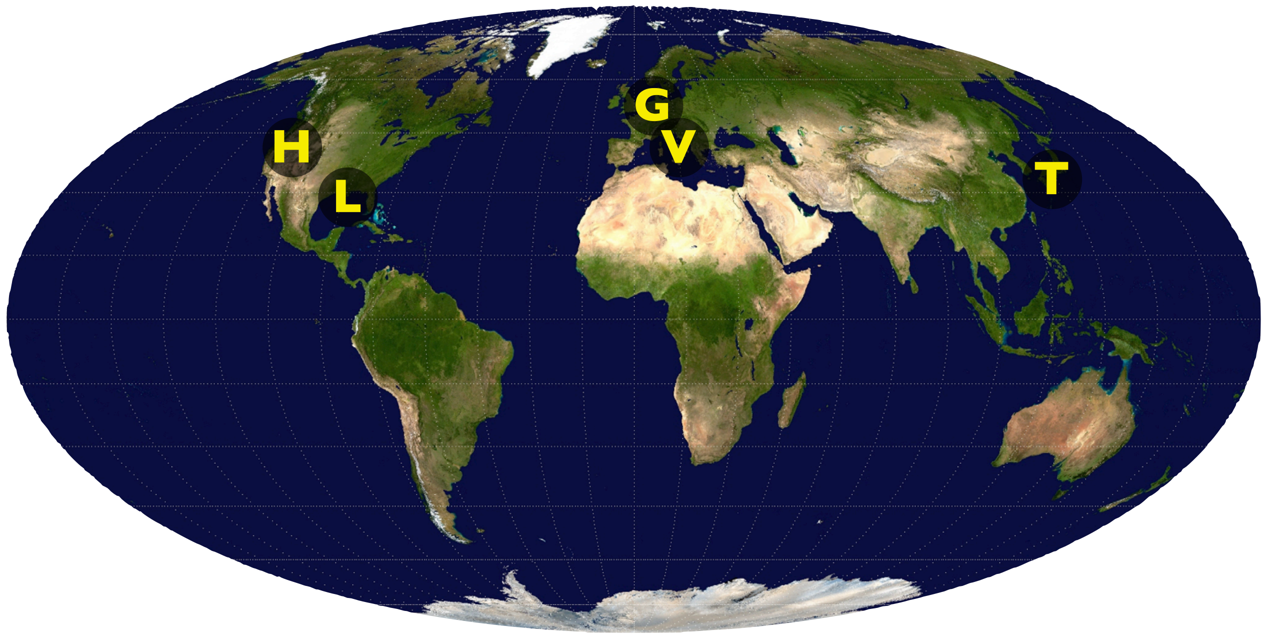


Alternative Theories Predict Additional Polarization States

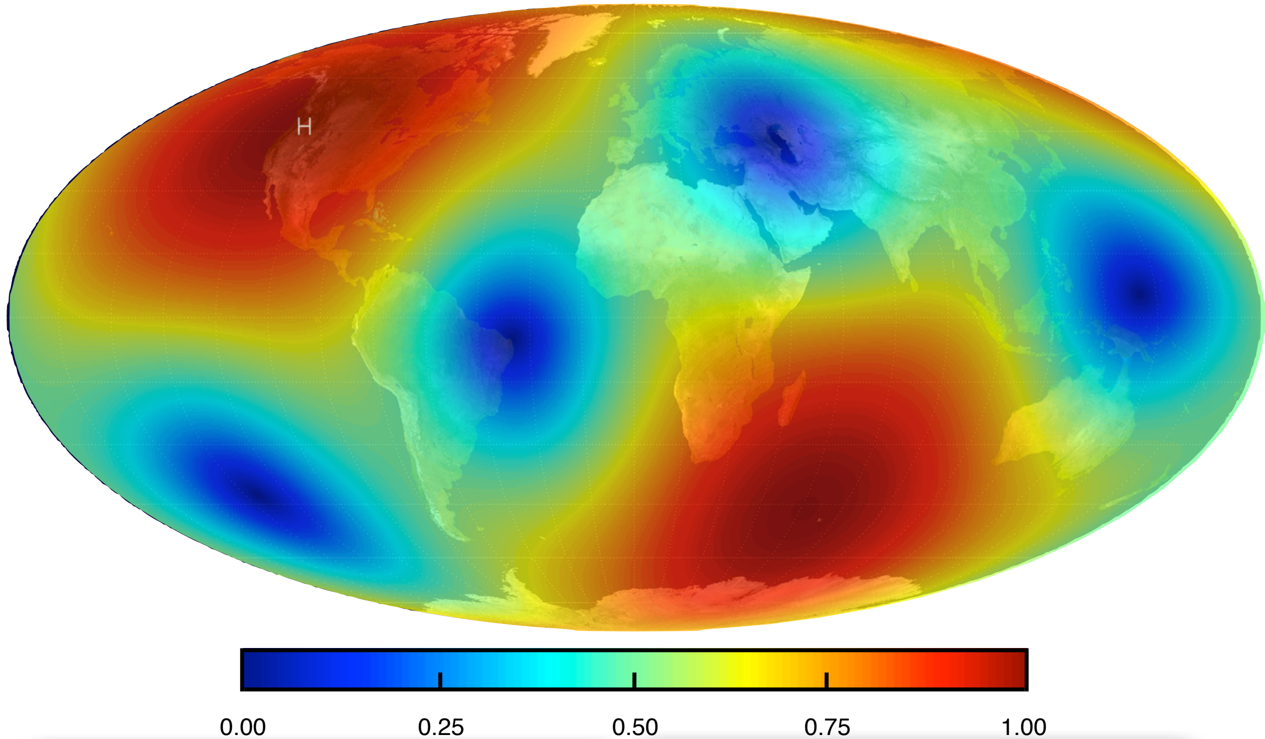


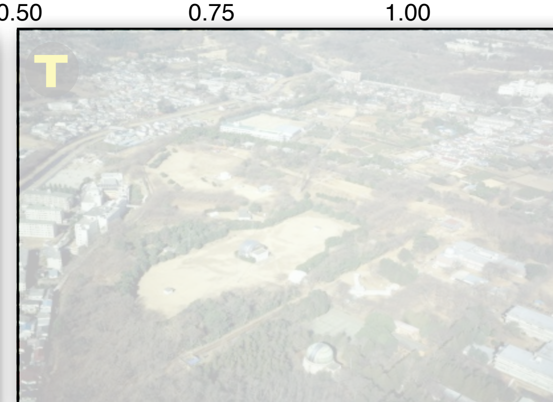


Gravitational Wave Detectors

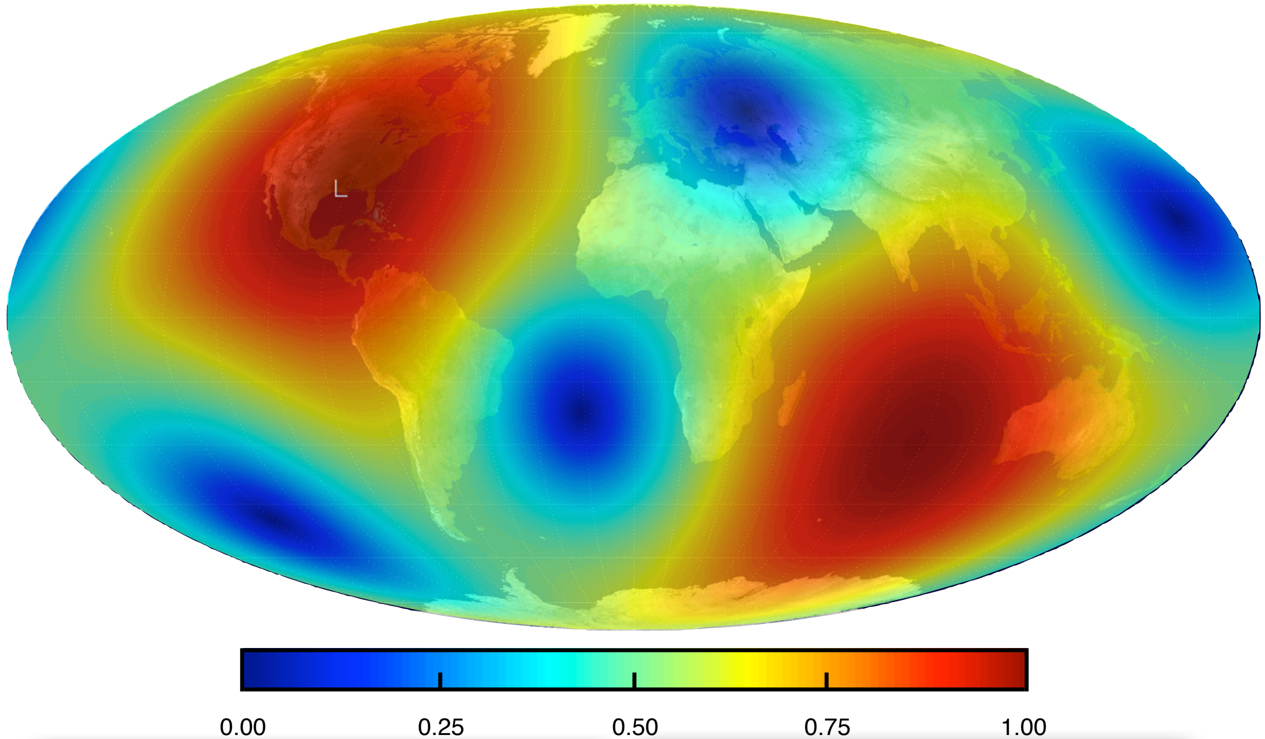


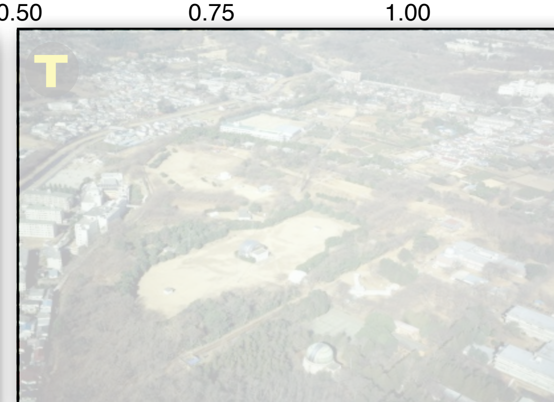
Gravitational Wave Detectors



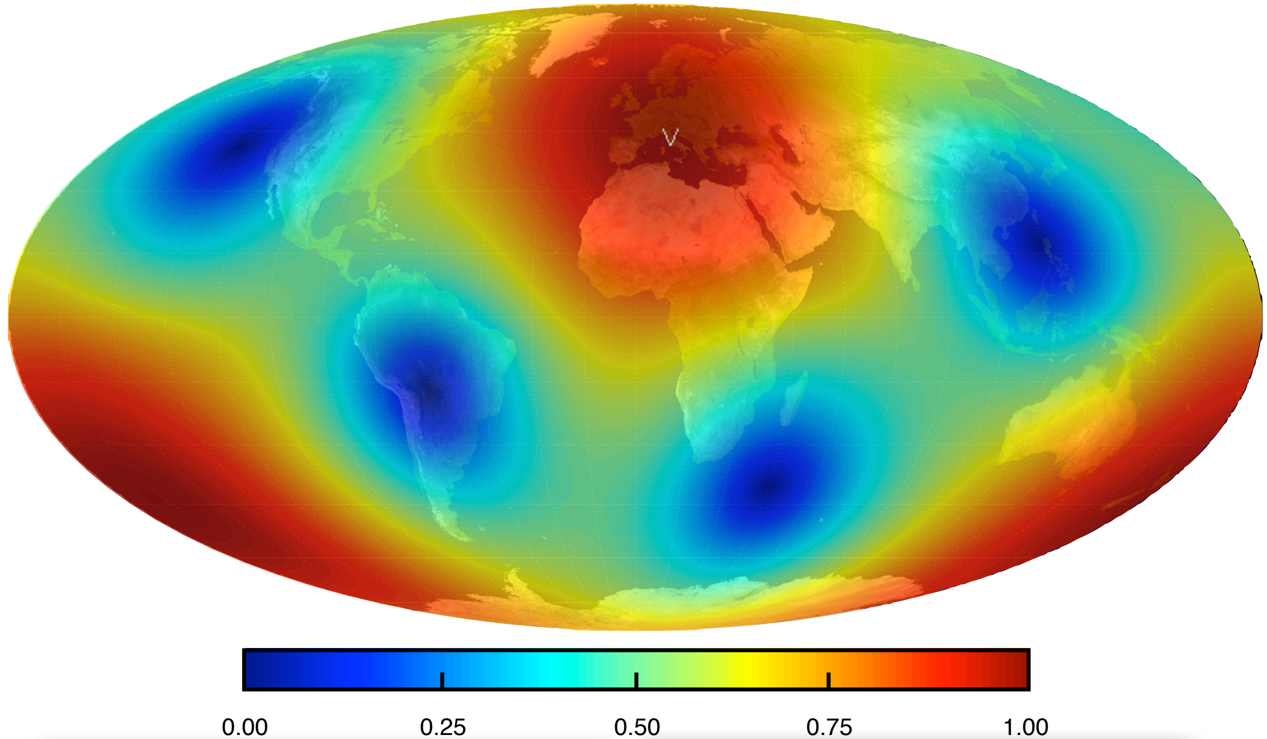


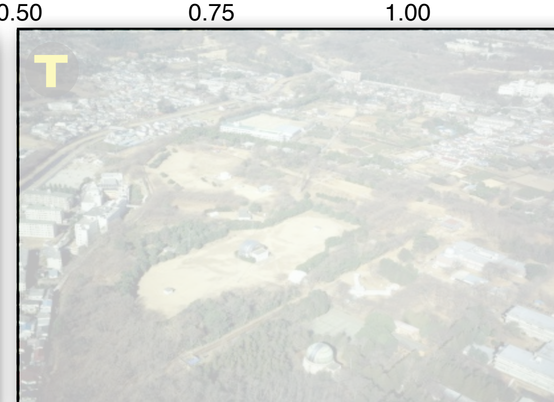
Gravitational Wave Detectors



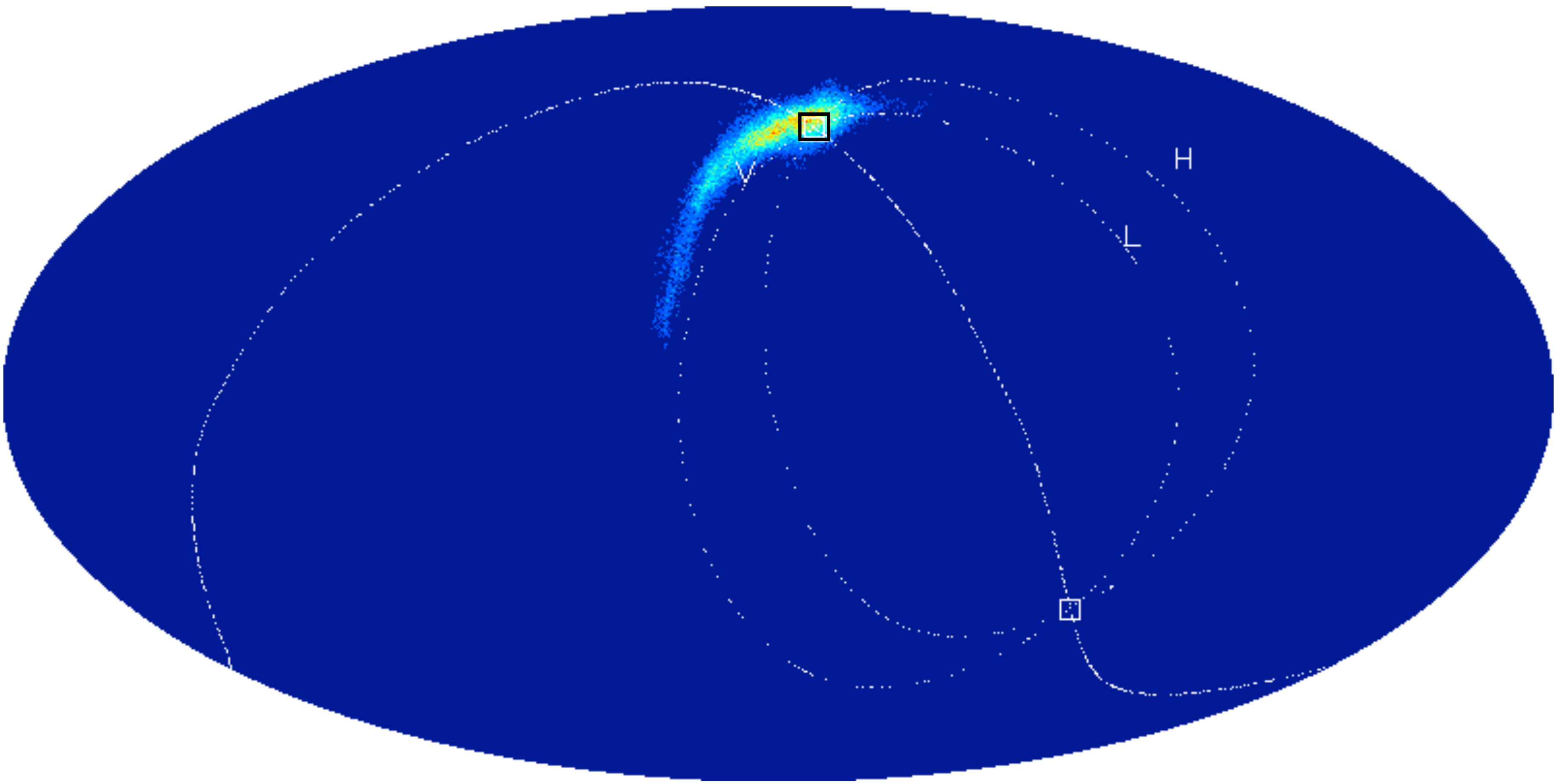


Gravitational Wave Detectors





Triangulation



Signal Analysis

$$\mathbf{s} = \mathbf{R} \cdot \mathbf{h} + \mathbf{n}$$

Main analysis techniques:

(1) Direct Model Inference

(2) Channel Cross Correlation

(3) Inference using Summary Statistics

Model Inference

$$\mathbf{s} = \mathbf{R} \cdot \mathbf{h}_A + \mathbf{n}$$

Form residual: $\mathbf{r} = \mathbf{s} - \mathbf{R} \cdot \mathbf{h}$

Demand that residual is consistent with instrument noise:

$\Rightarrow$ Likelihood $p(\mathbf{s}|\mathbf{h}) = p_n(\mathbf{s} - \mathbf{R} \cdot \mathbf{h})$

Classical Statistics

Neyman Pearson optimal statistic

$$\Lambda = \frac{p(\mathbf{s}|\mathbf{h})}{p(\mathbf{s}|0)}$$

Maximized w.r.t amplitude yields SNR ratio statistic

$$\Rightarrow \quad \rho = \frac{(\mathbf{s}|\mathbf{R}\cdot\mathbf{h})}{(\mathbf{R}\cdot\mathbf{h}|\mathbf{R}\cdot\mathbf{h})^{1/2}}$$

"Weiner matched filtering" - optimal statistic for know signal in stationary, Gaussian noise. Neither of which pertain.

Model Inference

$$\mathbf{s} = \mathbf{R} \cdot \mathbf{h}_A + \mathbf{n}$$

Need models for the instrument response, noise and GW signals

Instrument response ✓

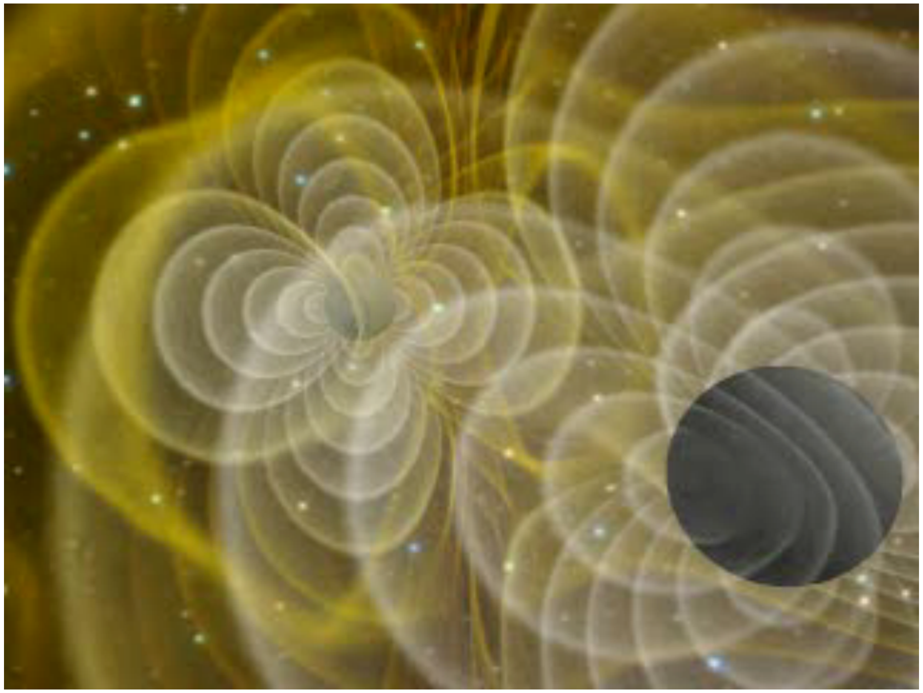
Instrument noise ?

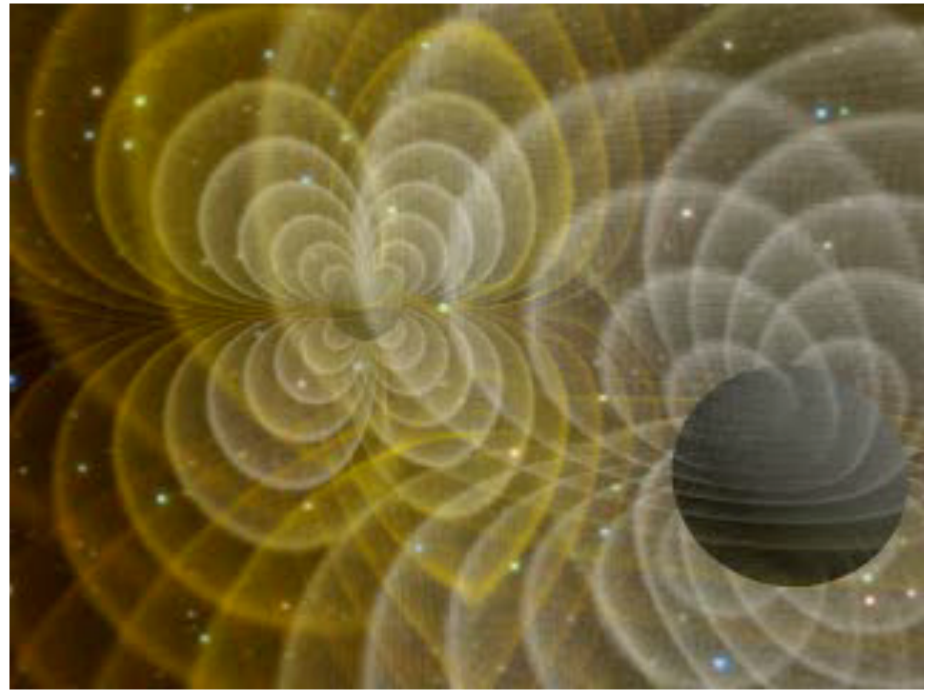
GW signals ?   Some better than others

e.g. Black hole binaries $\mathbf{h}(\vec{\lambda})$

$\vec{\lambda}$ 17-dimensional, includes sky location, merger time, masses, spins, orbital eccentricity etc

Modeling BH Mergers



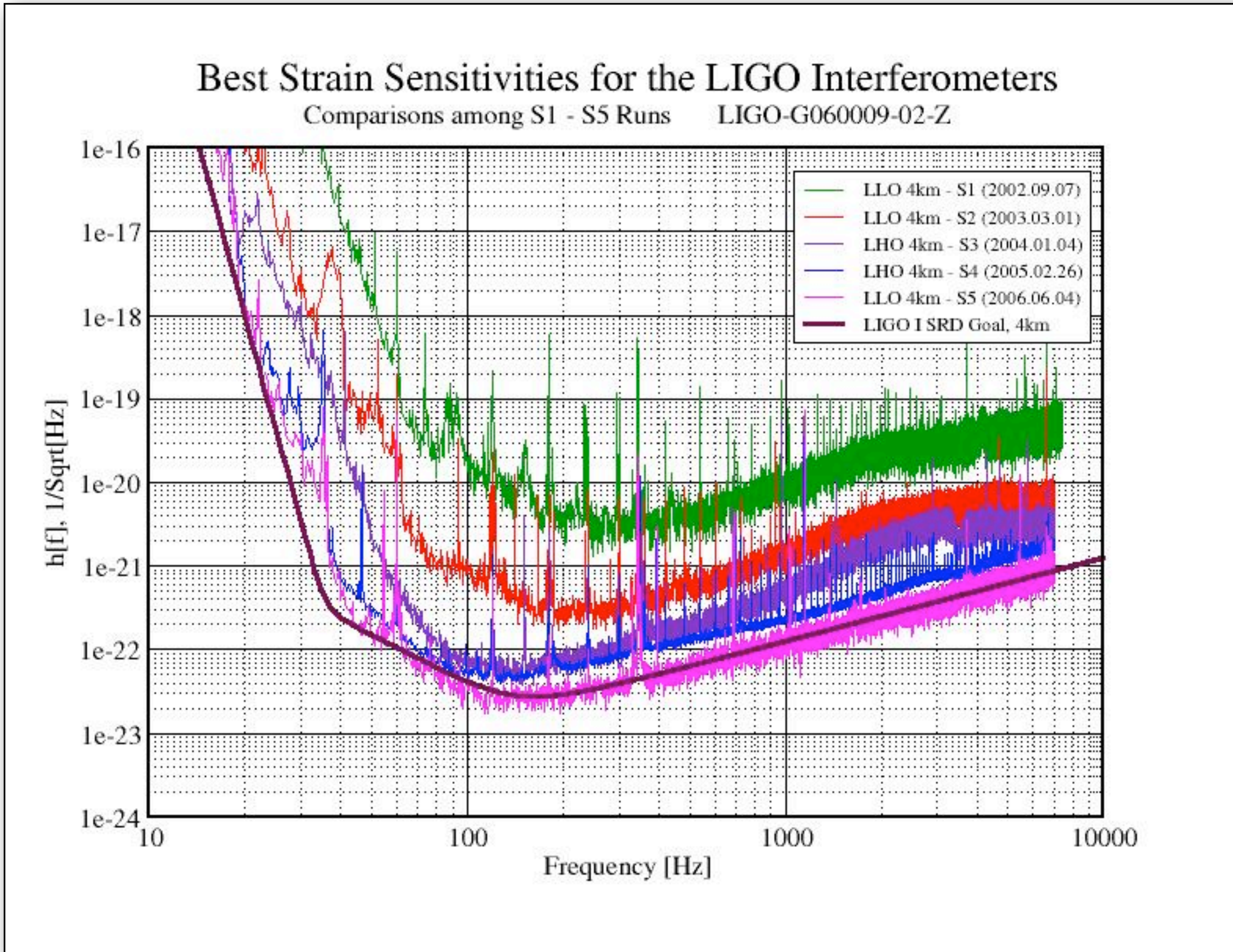


Contours: Curvature components corresponding to "+" polarization of GWs

Contours: Curvature components corresponding to "x" polarization of GWs

Movies courtesy GSFC Numerical Relativity Group

Example LIGO Noise Spectra



Standard (Naive) Noise Model

Stationary, coloured, Gaussian noise

$$E[n(f)] = 0$$

$$E[n(f)n^*(f')] = \frac{T}{2}\delta_{ff'}S_n(f)$$

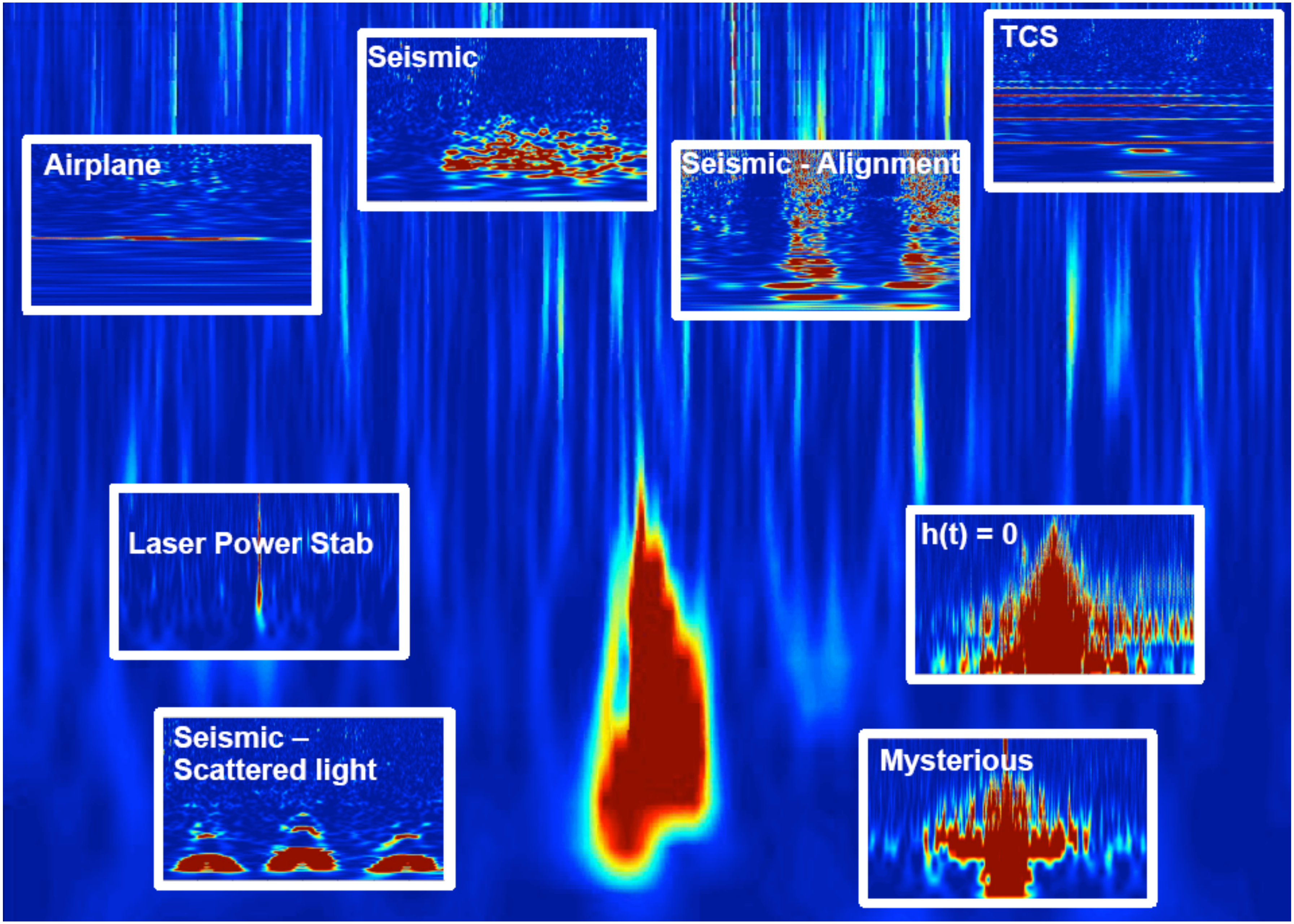
Likelihood

$$\begin{aligned} p(\mathbf{s}|\mathbf{h}) &= \prod_f \frac{1}{2\pi T S_n(f)} \exp\left(-\frac{r(f)r^*(f)}{T S_n(f)}\right) \\ &= C e^{-(r|r)/2} = C e^{-\chi^2/2} \end{aligned}$$

Noise weighted inner product

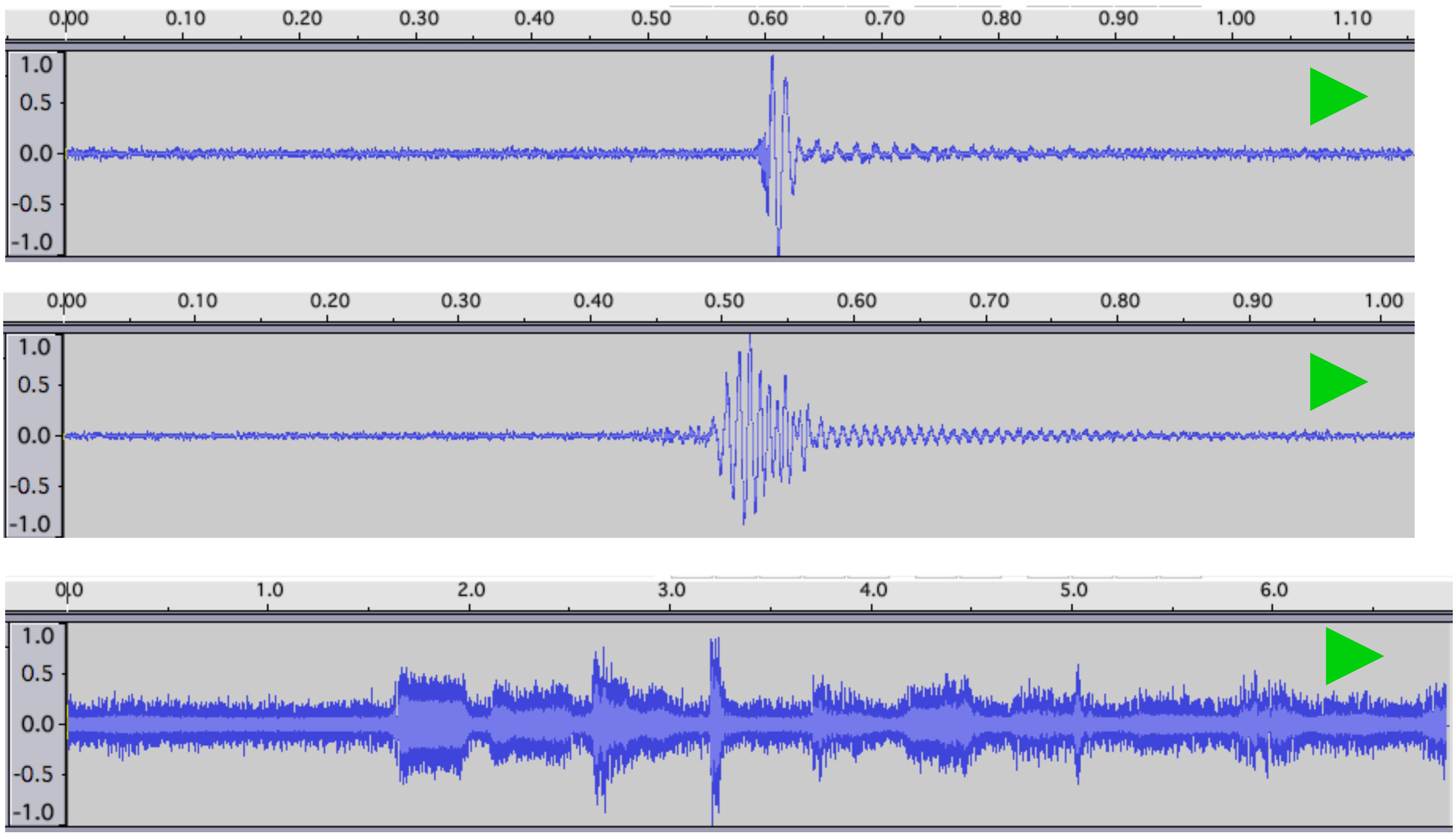
$$(a|b) = \frac{2}{T}\sum_f \frac{a(f)b^*(f) + a^*(f)b(f)}{S_n(f)}$$

Time-Frequency Scalograms of LIGO data



Time domain LIGO data

(Bandpass filtered, Whitened)

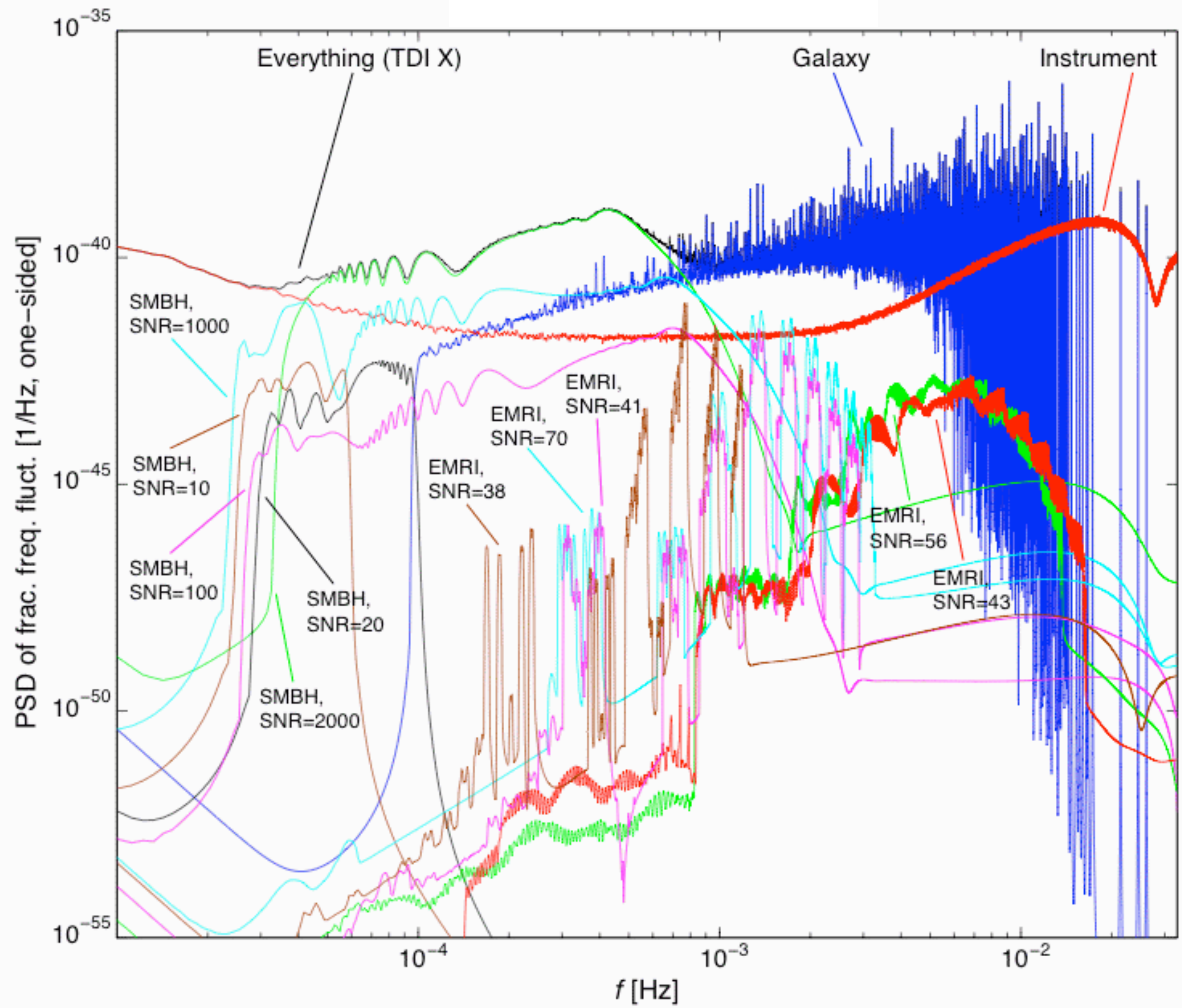


Samples from the Syracuse Audio Study of Glitches

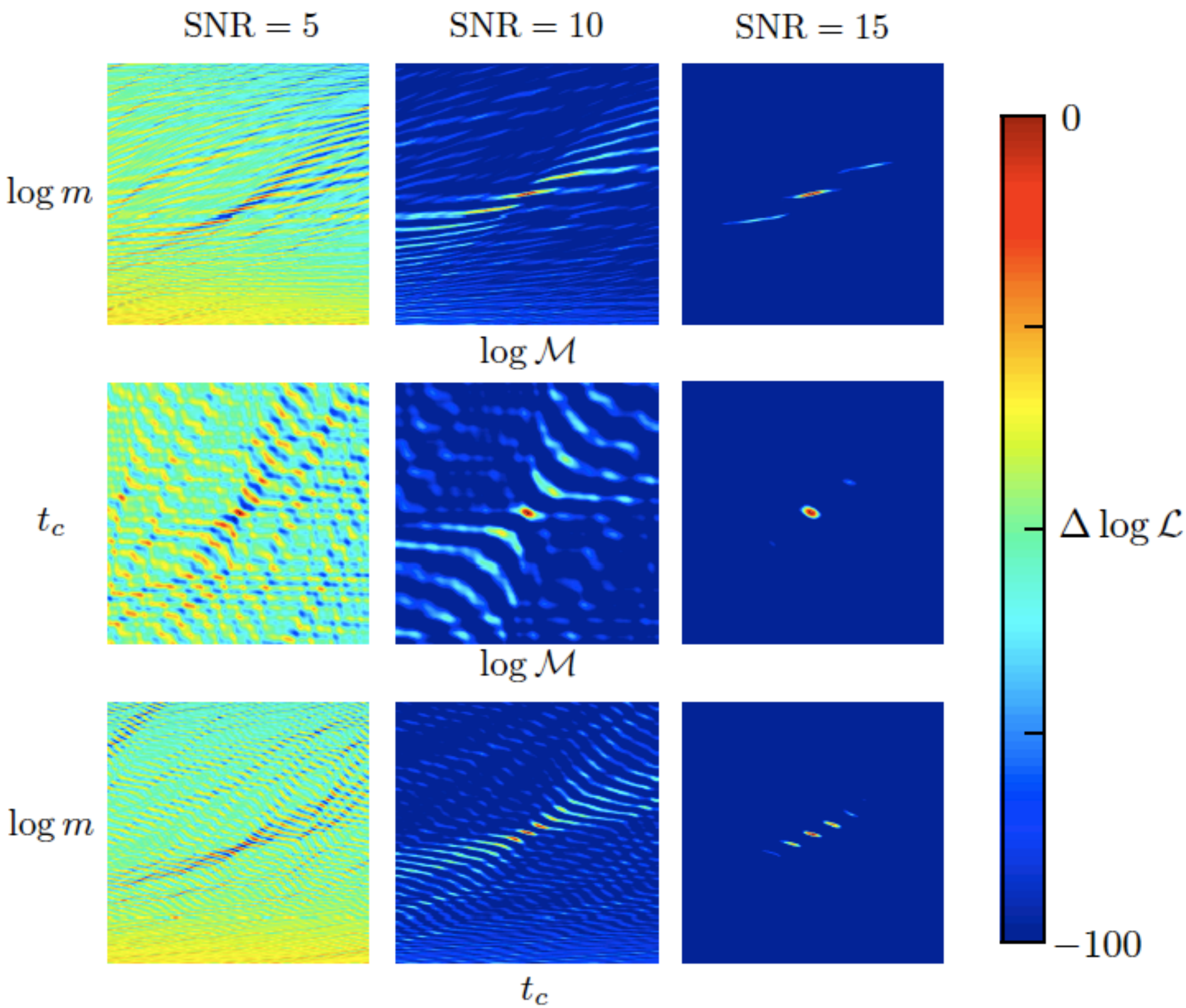
Analysis Challenges

- Rare and weak signals (terrestrial detectors)
- Non-stationary and non-Gaussian noise
- Large model dimension, small posterior to prior volume ratio
- Highly multi-modal posteriors
- Multiple overlapping signals (space based)

Multiple Overlapping Signals - Simulated LISA Data



Multi-modal Posteriors (e.g. Black Holes Inspirals)



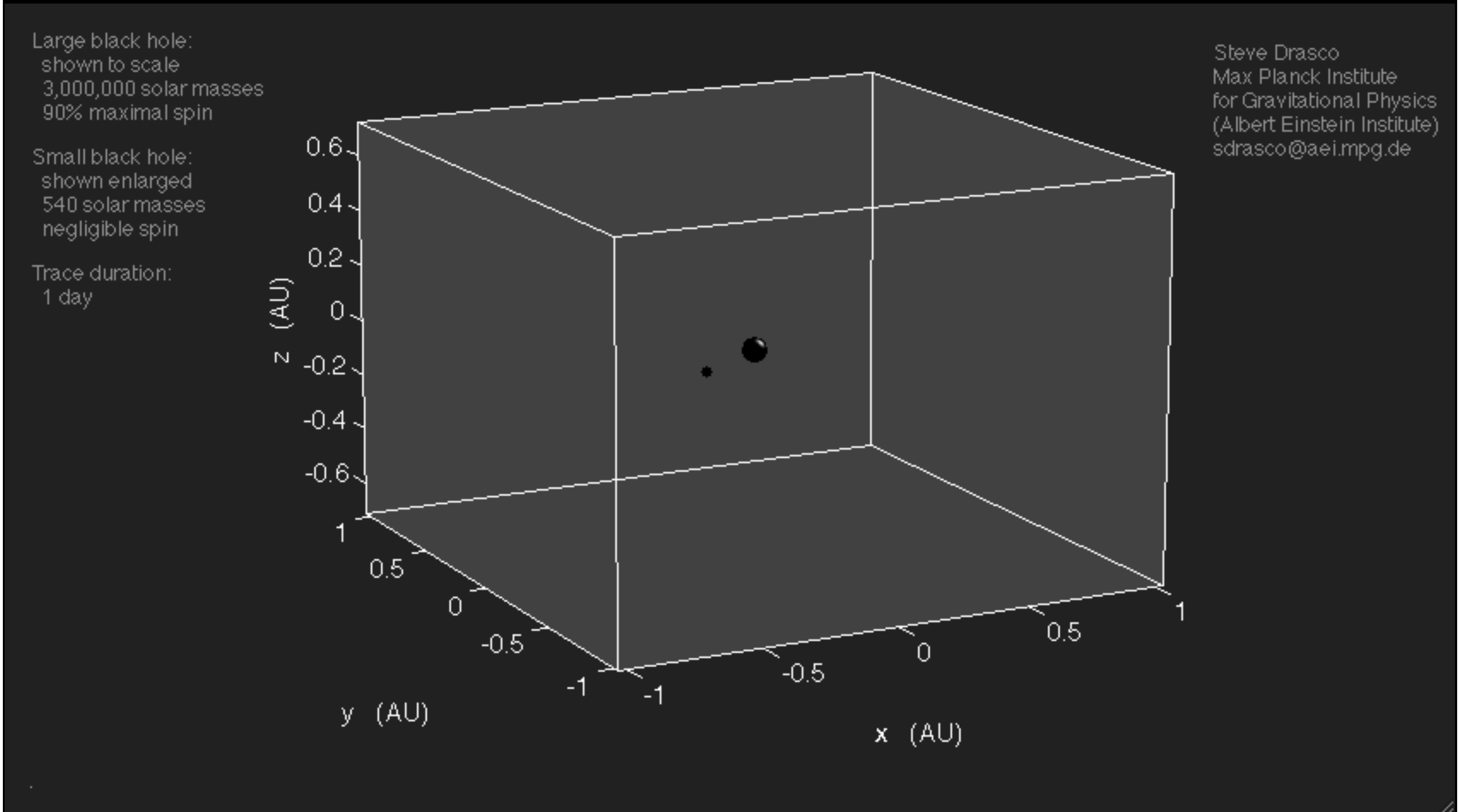
Detecting Weak Signals

Analysis usually conducted in two stages

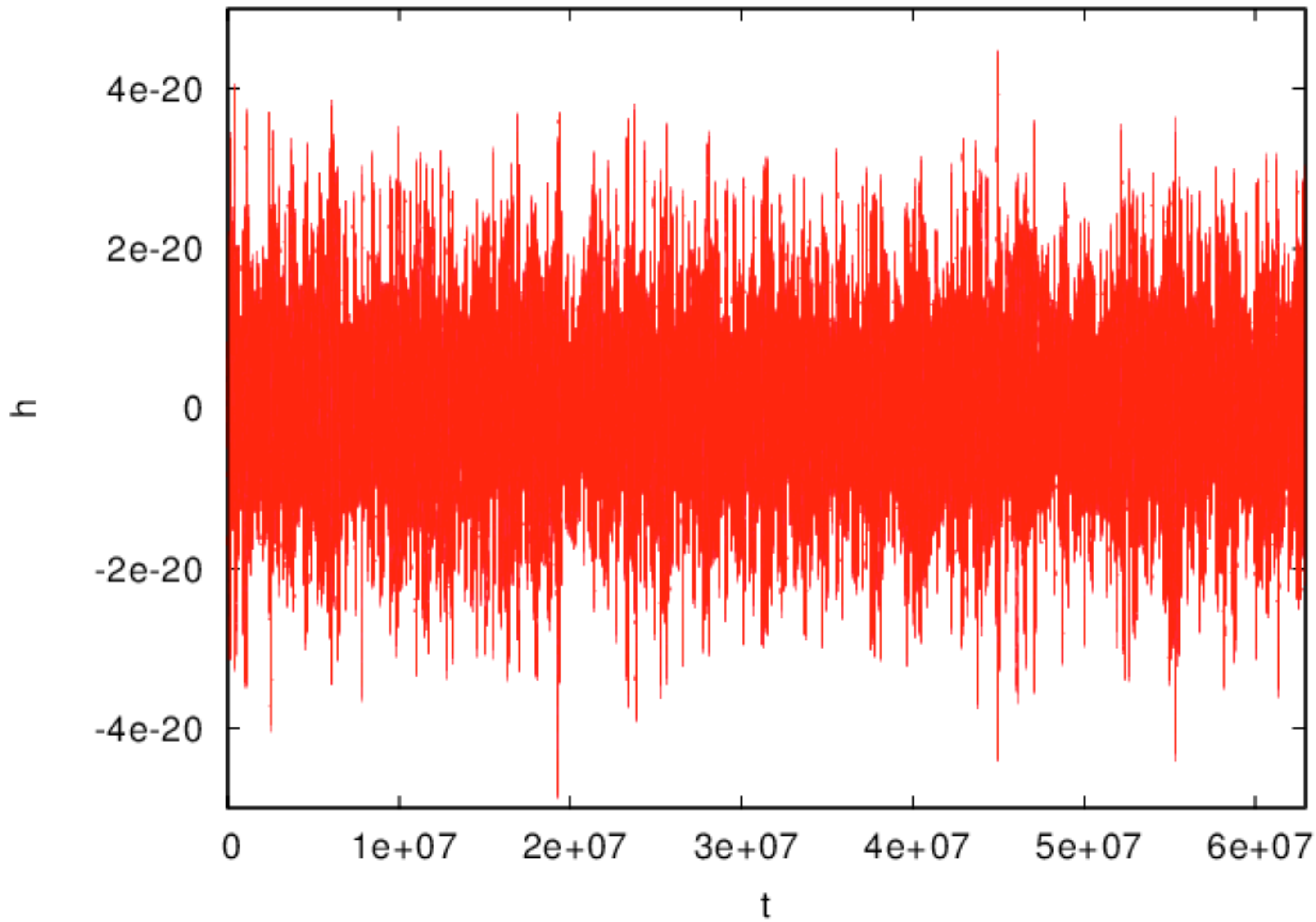
- Search (Optimization)
  - Maximization rather than marginalization
  - Simulated annealing, Genetic algorithms etc
- Characterization (Sampling)
  - Markov Chain Monte Carlo, Parallel tempering, DE etc
  - Nested sampling, MultiNest

- Complicated by large model dimension, small posterior to prior volume ratio, e.g. $\frac{\Delta V_{90}}{V} \sim 10^{40}$
- Can exploit multi-modality of posterior

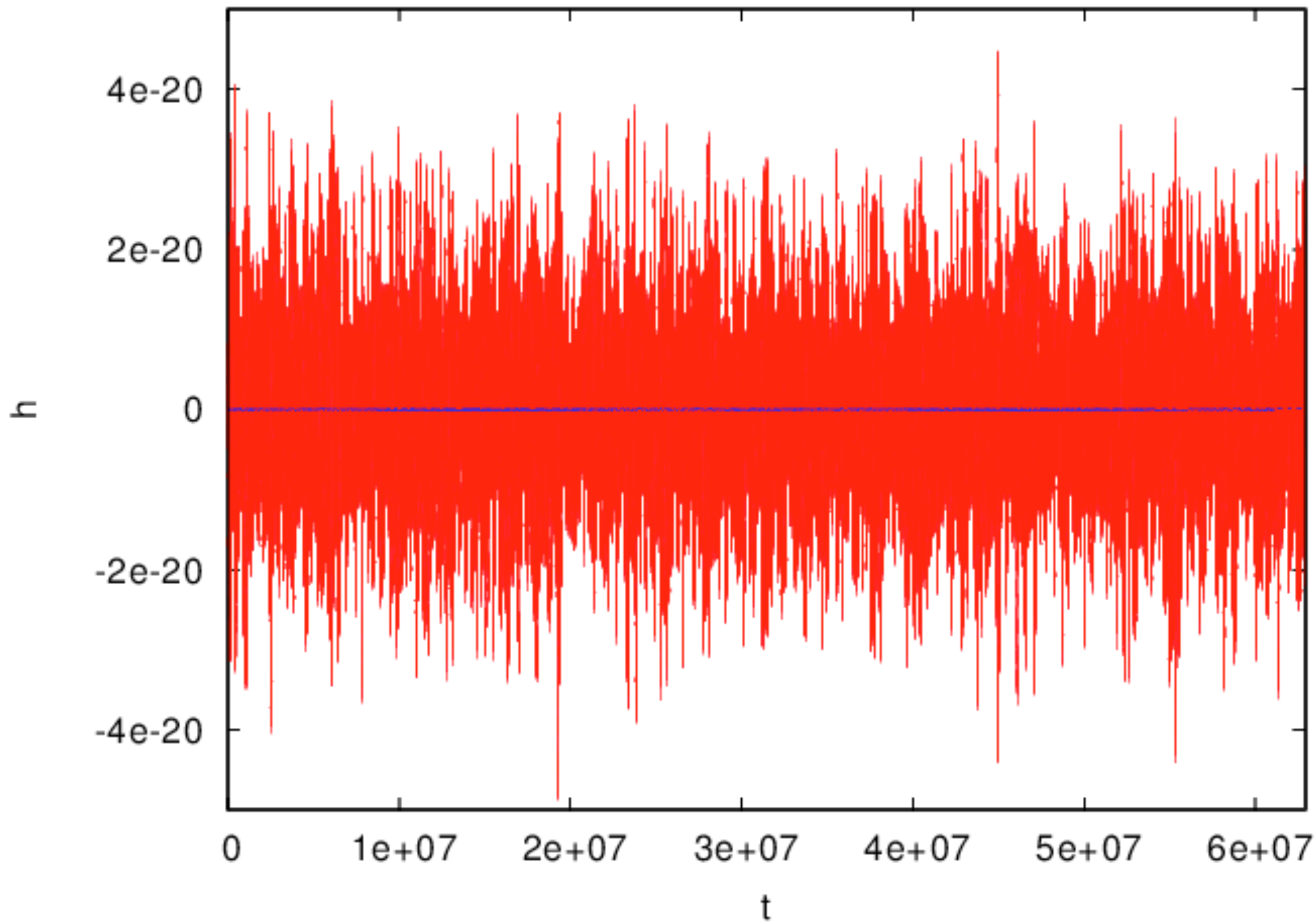
Extreme Mass Ratio Inspirals



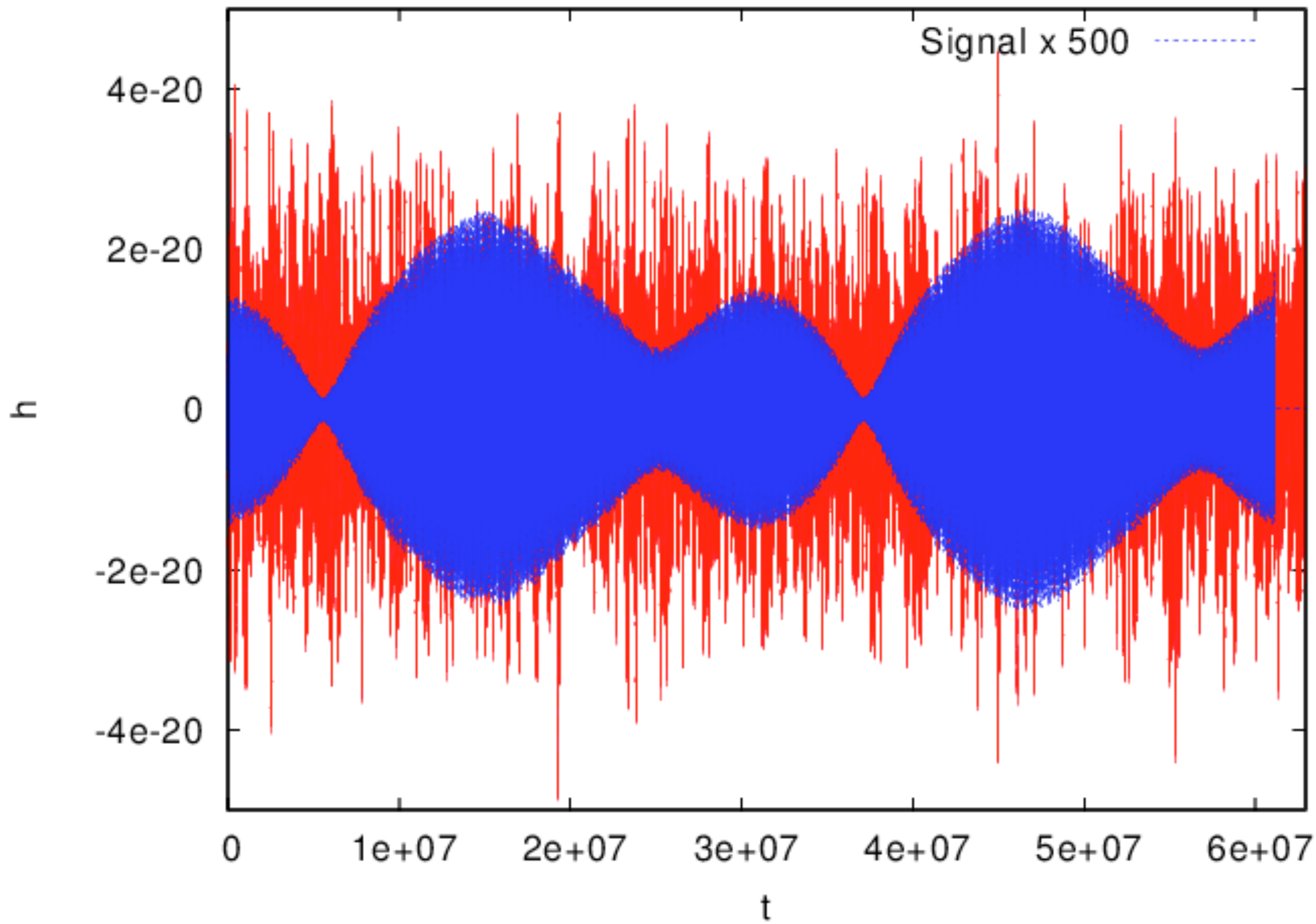
Finding weak signals



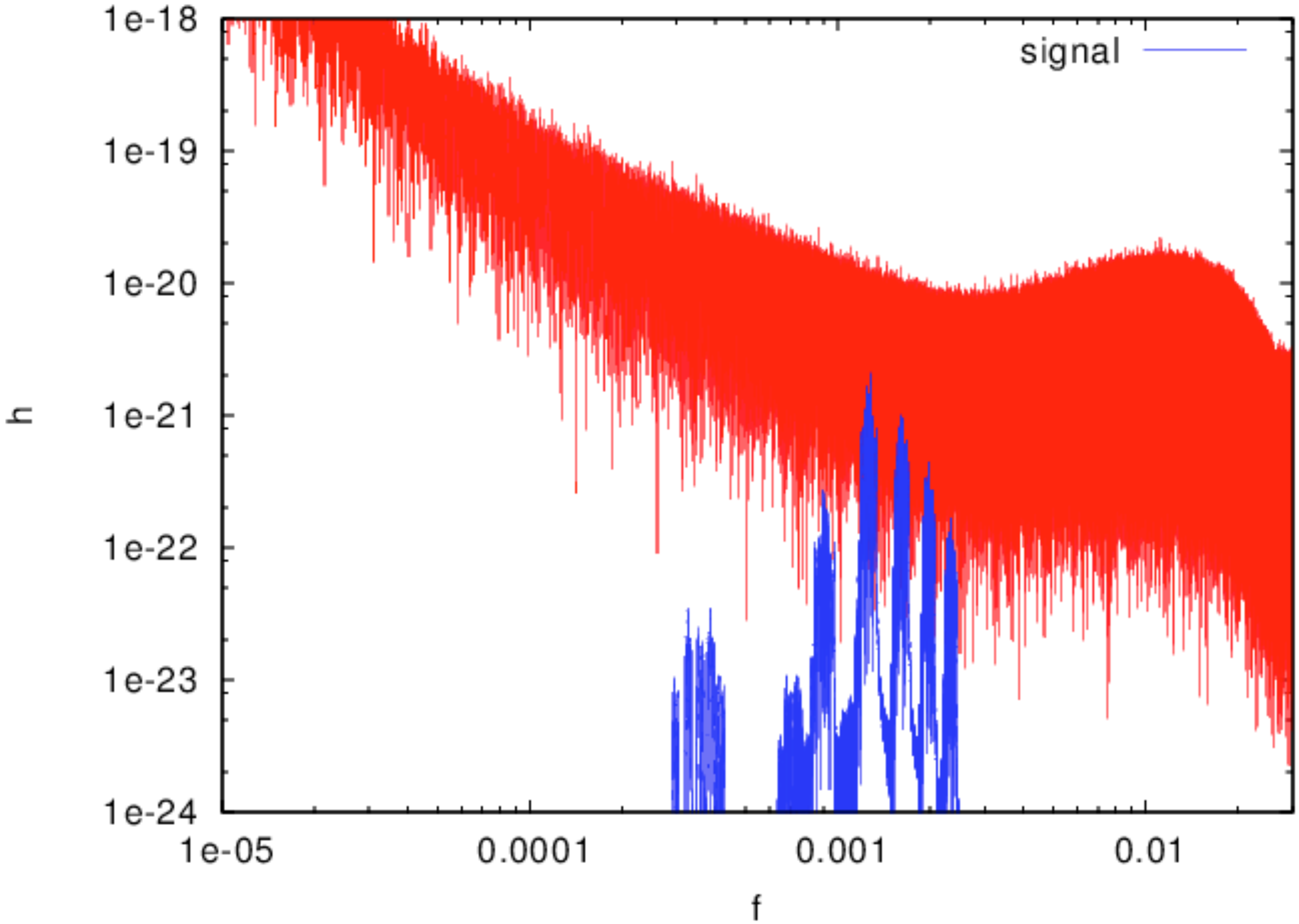
Finding weak signals



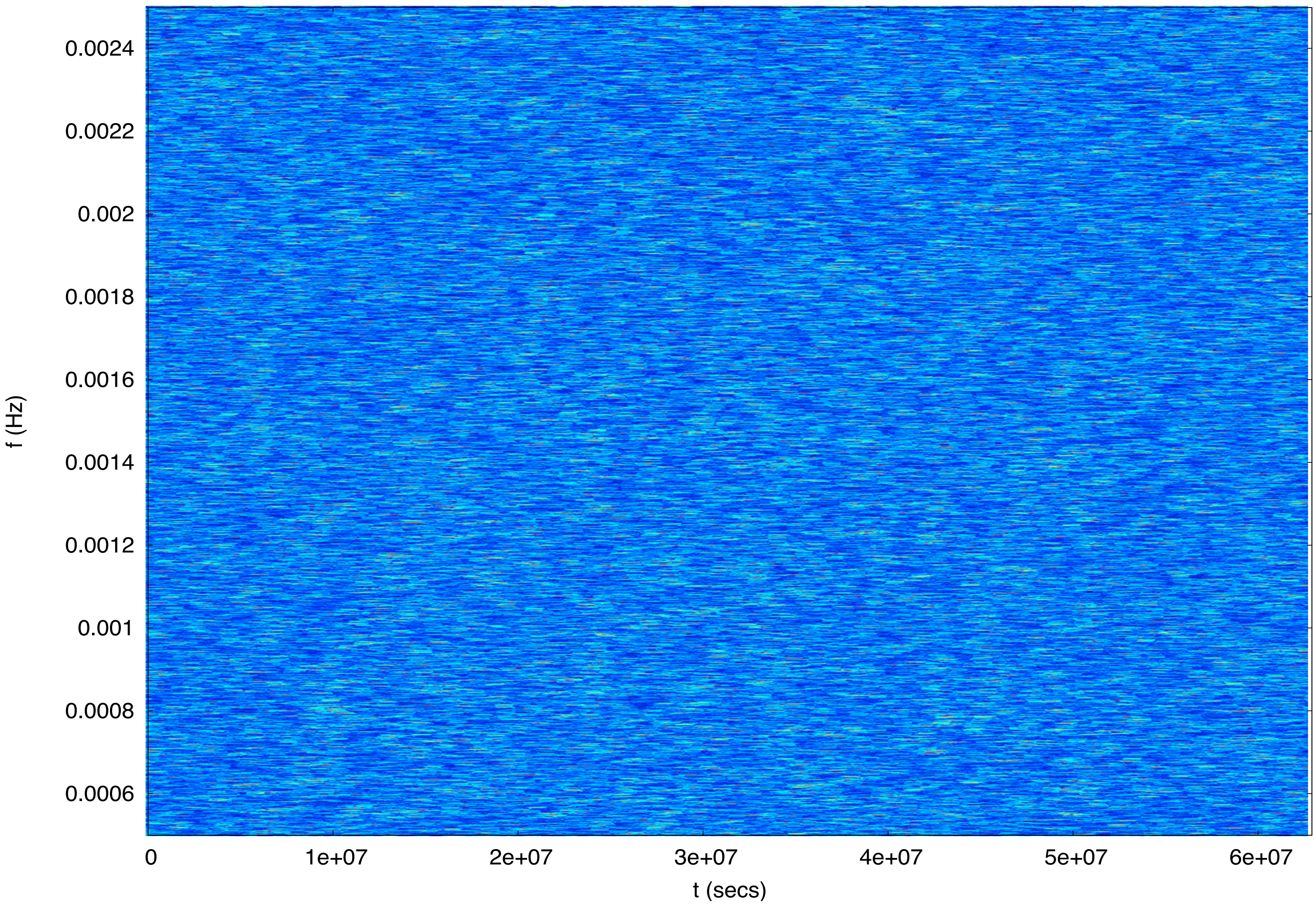
Finding weak Signals

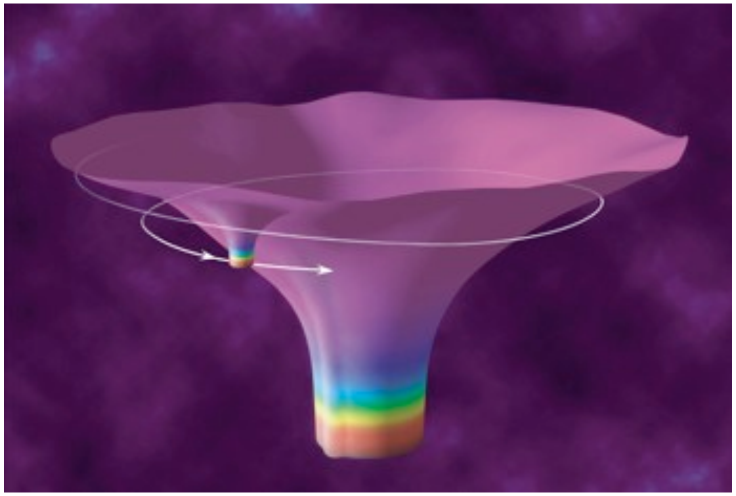


Easier to see in Frequency?



Easier to see in Time-Frequency?





Mock LISA Data Challenge

N. J. Cornish, Class. Quant. Grav. 28, 094016 (2011)

$\mathrm{SNR} = 21.6$

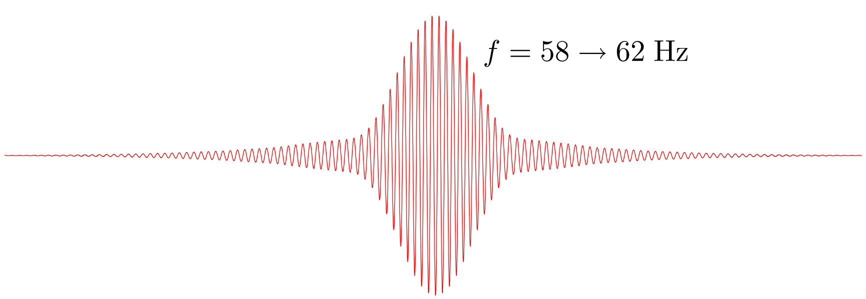
SourceType	ExtremeMassRatioInspiral
EclipticLatitude	0.421812762567
EclipticLongitude	6.12722883835
Polarization	0
PolarAngleOfSpin	1.12926516708
AzimuthalAngleOfSpin	1.02683301361
Spin	0.607839950171
MassOfCompactObject	10.3529155149
MassOfSMBH	9592613.64626
InitialAzimuthalOrbitalFrequency	0.000195874494514
InitialAzimuthalOrbitalPhase	5.21945959065
InitialEccentricity	0.181454715223
InitialTildeGamma	5.87897706737
InitialAlphaAngle	4.97256712568
LambdaAngle	0.809747003385
Distance	1173313438.97

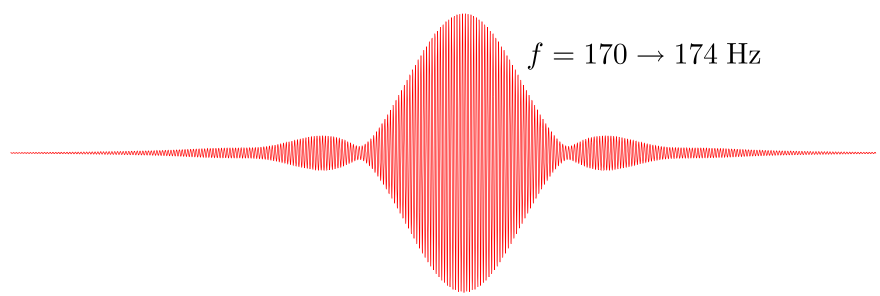
ExtremeMassRatioInspiral
0.428327994937
6.086378539
0
1.06896570232
1.050521566
0.6078238235
10.35303219
9593171.82
0.000195874648
5.270798052
0.181500634
2.707804463
4.908907739
0.809309222072
1022400436.0

Developing a realistic noise model: BayesWave

- Bayesian model selection
  - Three part model $\mathbf{s} = \mathbf{R} \cdot \mathbf{h} + \mathbf{g} + \mathbf{n}$
  - Trans-dimensional Markov Chain Monte Carlo

- Wavelet decomposition
  - Glitch model parameters are wavelet amplitudes
  - Number, amplitude and location of "active" pixels varies





T. B. Littenberg, N. J. Cornish, Phys. Rev. D82, 103007 (2010).

Three part model

Noise model n - defines the likelihood function

$$p(s|a_{ij}) = \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\frac{(s_{ij}-a_{ij})^2}{2\sigma_i^2}}$$

Ideally would also include information from the auxiliary channels

Noise level also an unknown, determined from data

Glitch model g - clustered TF power, incoherent across network

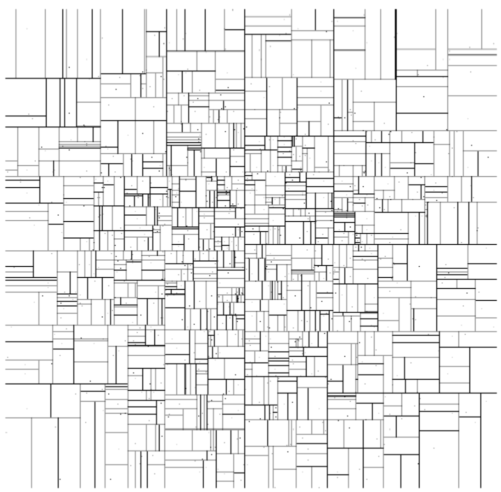
Signal model h - analytic waveforms or clustered TF power, coherent across network

- can incorporate priors such as elliptical polarization

Transitions between signal+noise and noise only models

- Very difficult to compute the evidence for each model individually due to variable and large dimensionality of the glitch model
- Difficult to jump to the signal model since $\Delta V_{90}/V \ll 1$

One solution: Use PDFs from each model as proposal densities

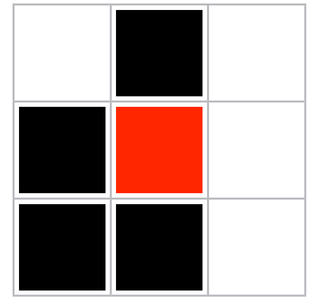


PDFs can be mapped by sparse matrix representations or KD trees

T. B. Littenberg, N. J. Cornish, Phys. Rev. D82, 103007 (2010).

W. M. Farr and I. Mandel, arXiv:1104.0984 [astro-ph.IM] (2011).

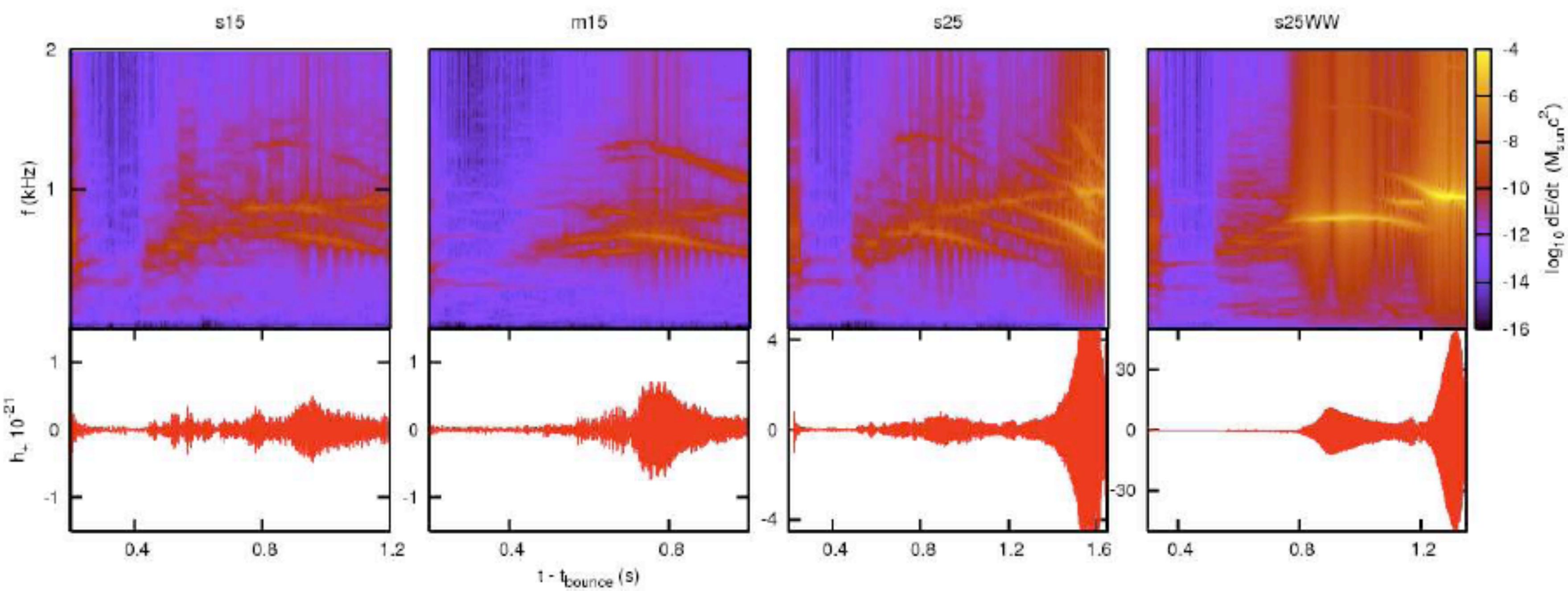
Priors

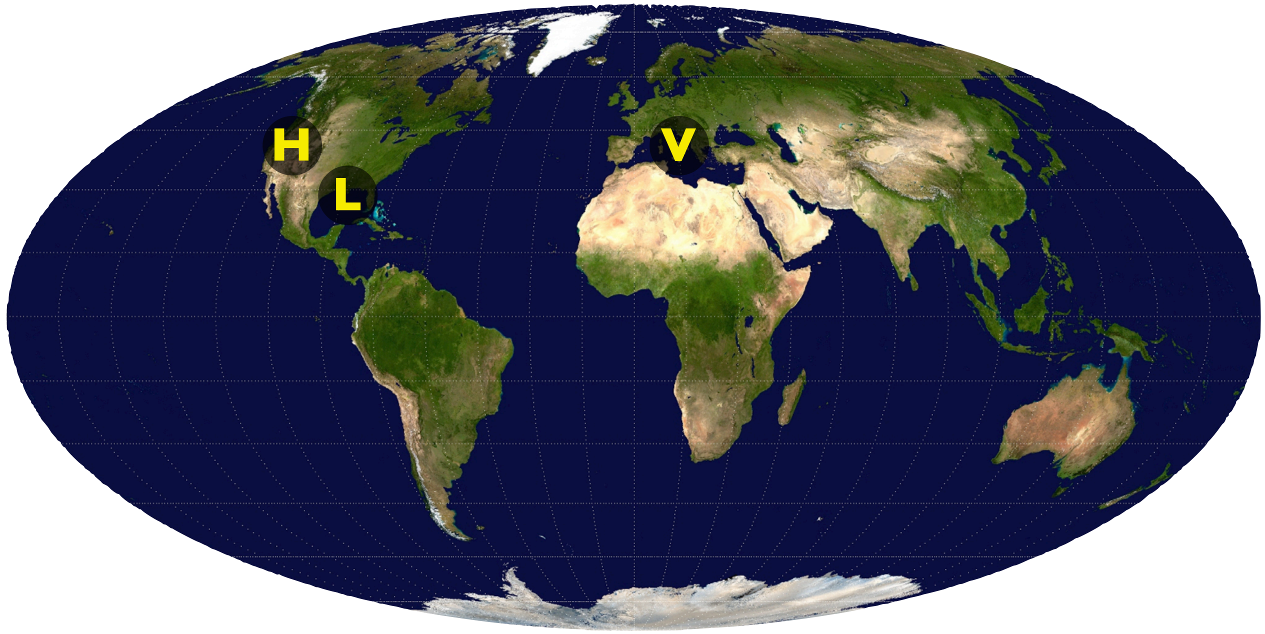


Clustering Prior: $p(ij) \propto \#\text{nearest neighbours}$

Amplitude Prior: $p(a_{ij}) = \frac{1}{\sqrt{2\pi}\kappa\sigma_i} e^{-\frac{a_{ij}^2}{2\kappa\sigma_i^2}}$

Astrophysical model Priors: e.g. time-frequency content of a core collapse SN

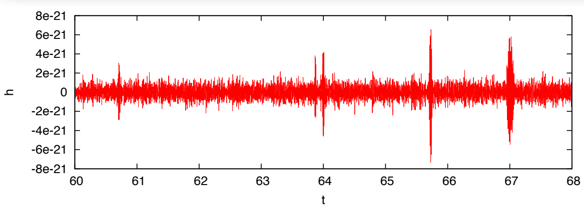


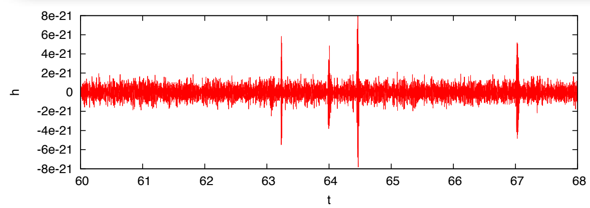

H
L
V

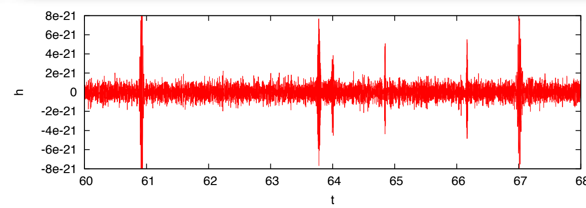

H


L


V


8e-21
6e-21
4e-21
2e-21
0
-2e-21
-4e-21
-6e-21
-8e-21
h
60 61 62 63 64 65 66 67 68
t


8e-21
6e-21
4e-21
2e-21
0
-2e-21
-4e-21
-6e-21
-8e-21
h
60 61 62 63 64 65 66 67 68
t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6e-21
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2e-21
0
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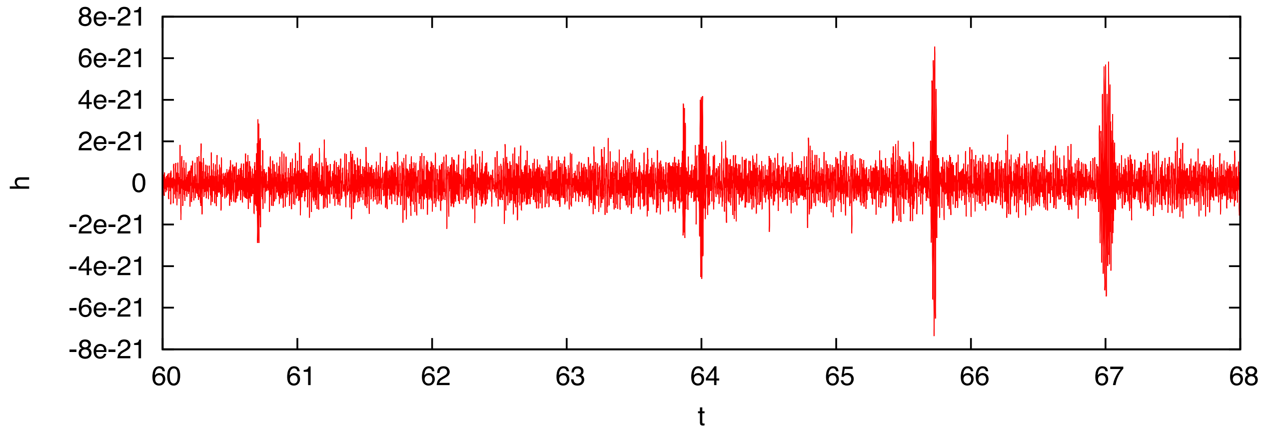
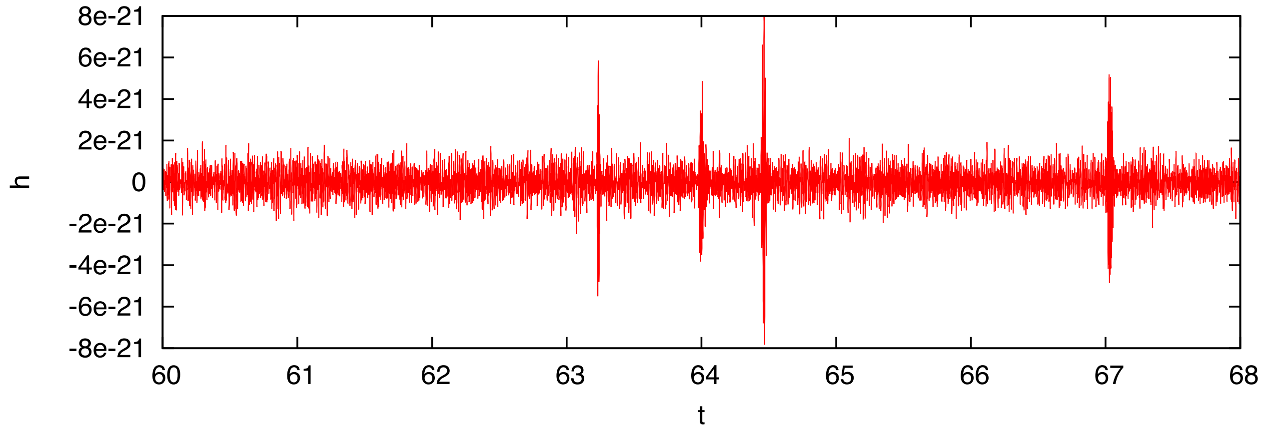
h
60 61 62 63 64 65 66 67 68
t

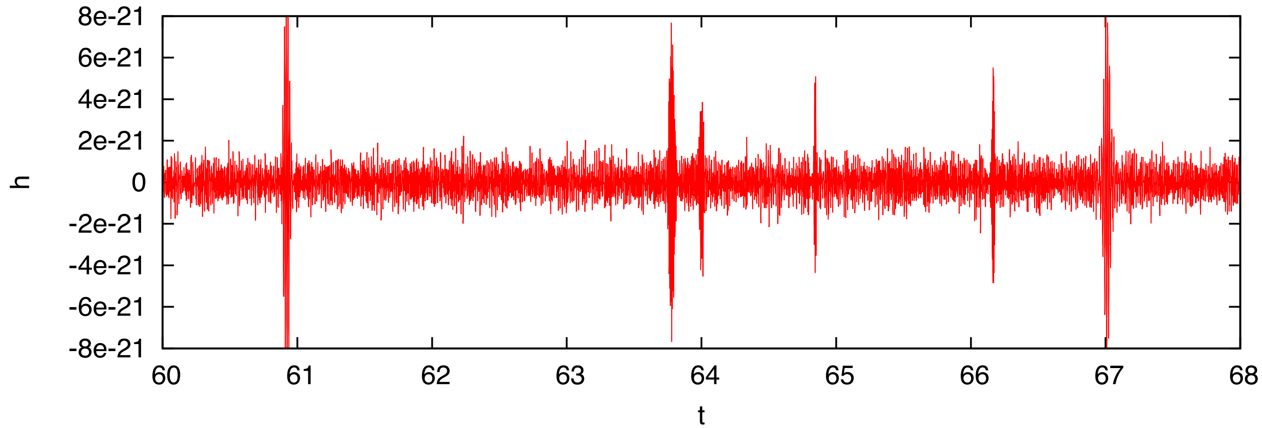
H



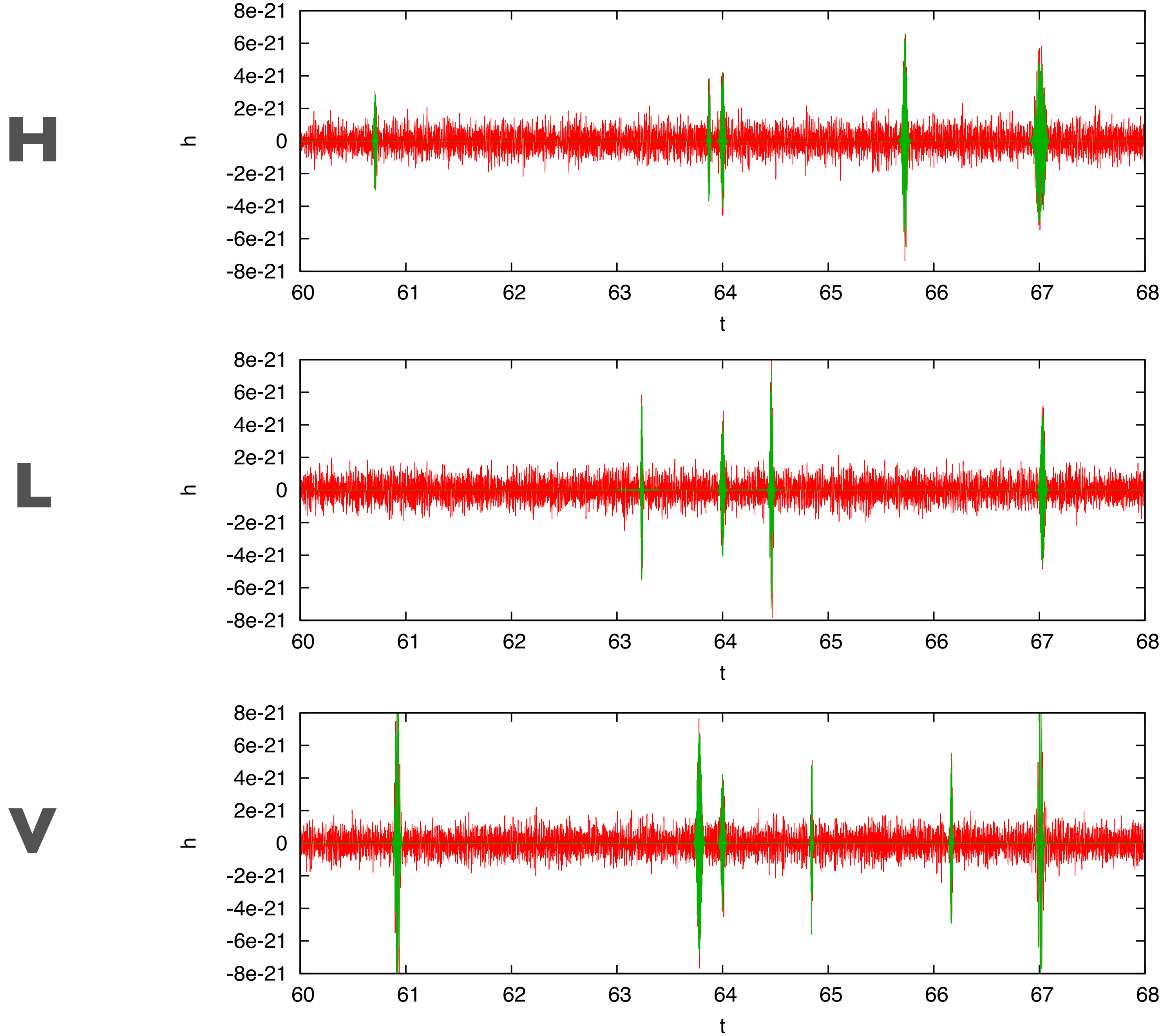
L



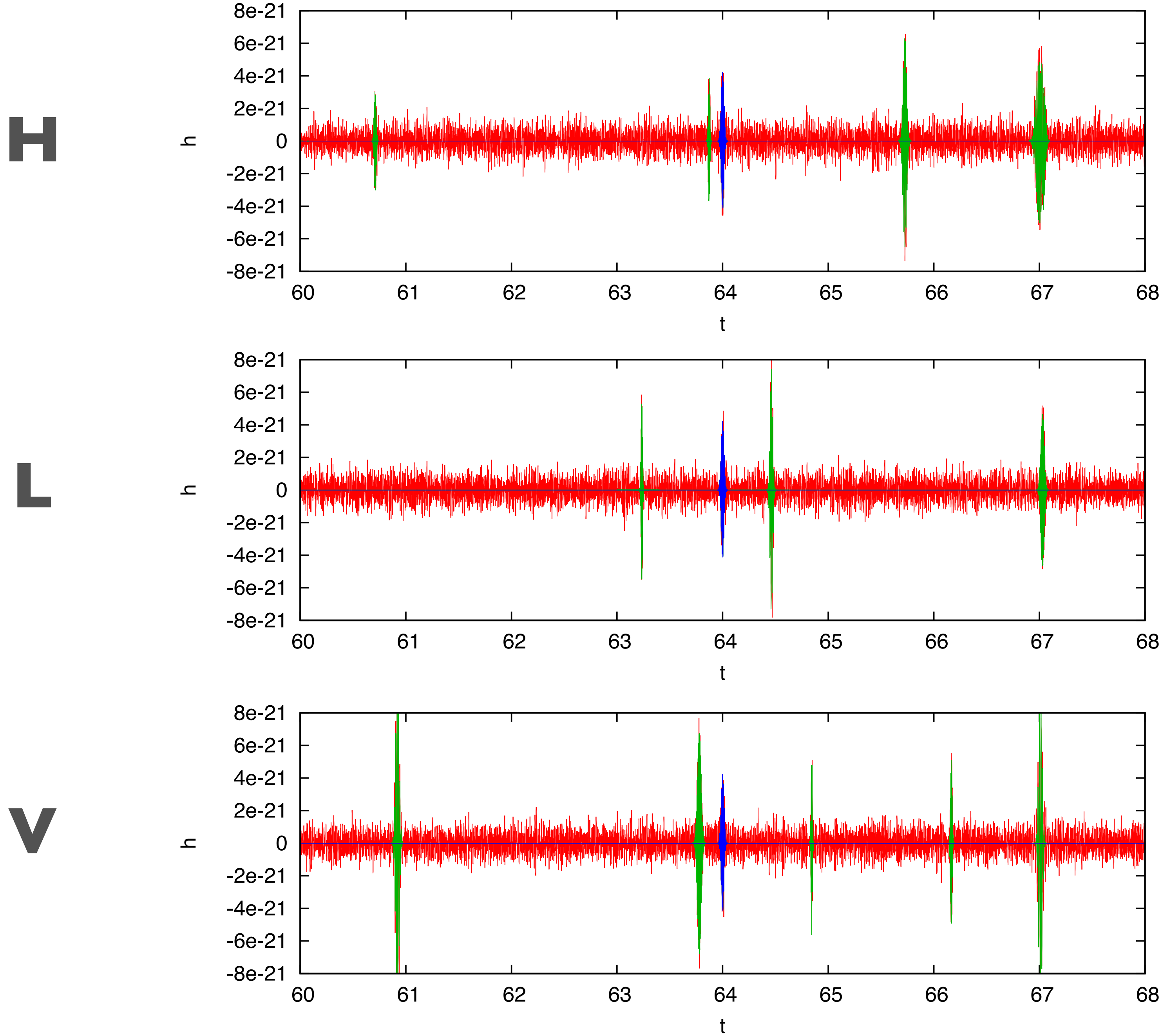
V



Simulated data, band-passed 60-300 Hz

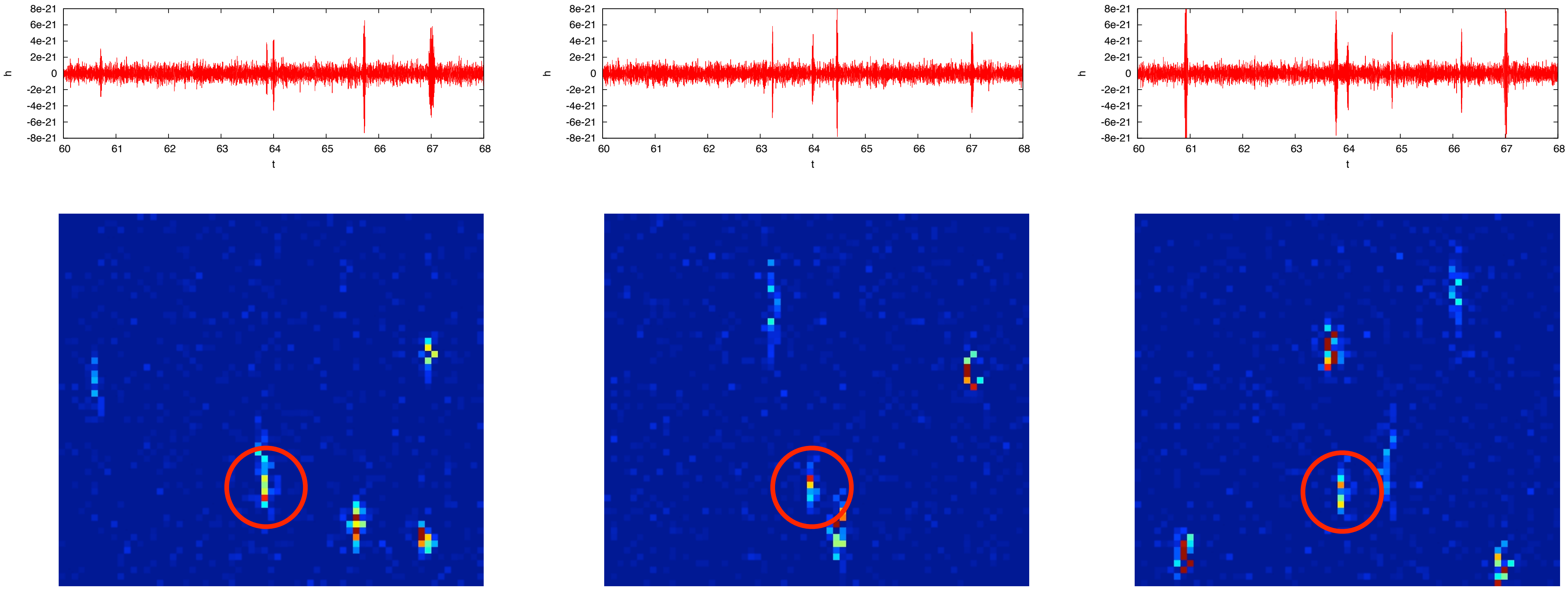


Simulated data, band-passed 60-300 Hz



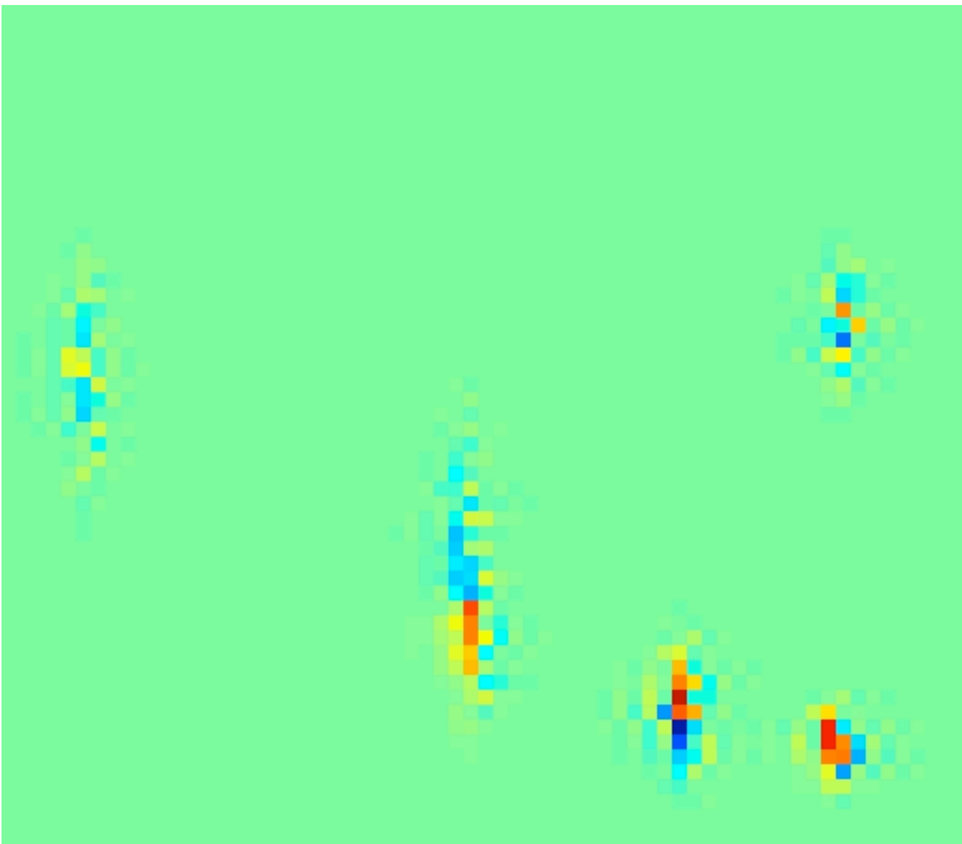
Simulated data, band-passed 60-300 Hz

Coherent Network Analysis



GW signals line up in time *and* frequency
and morphology

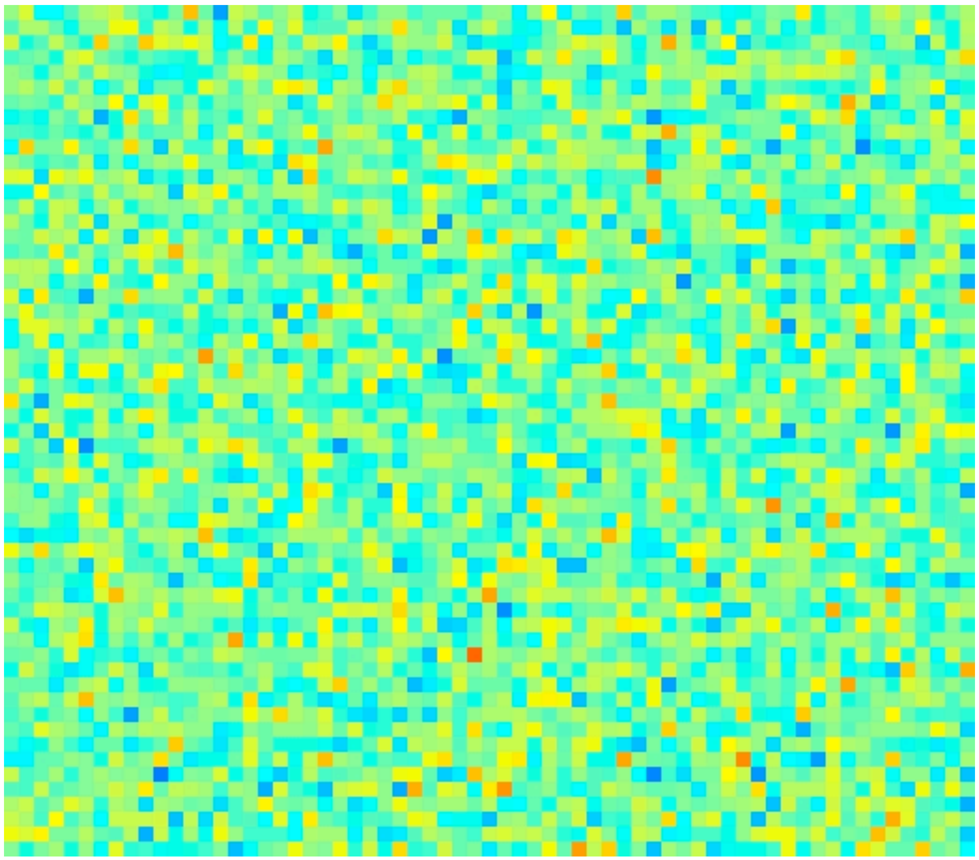
Glitch & Signal Recovery



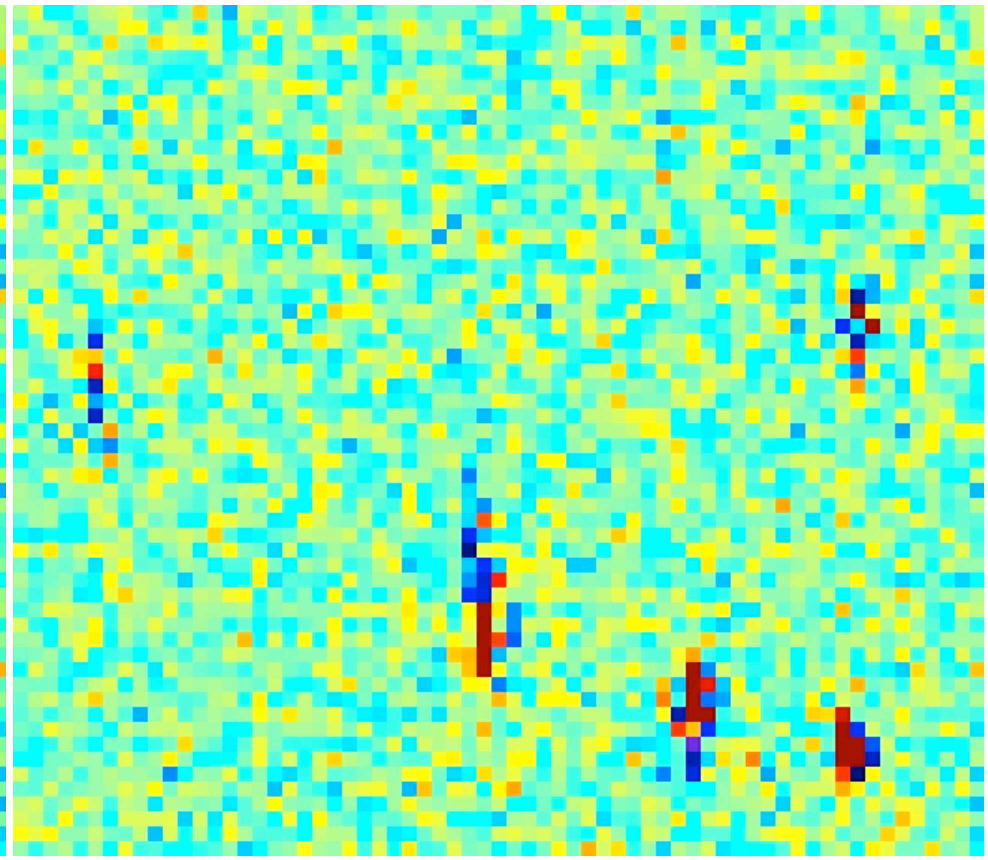
Injected

Recovery

Noise Recovery



Injected



Recovery

Help Wanted

- First direct detection of gravitational waves within 5 years
- Terabytes of data, weak signals, complex waveforms, glitchy detectors

...and a bunch of theoretical physicist trying to do the analysis