

Discussion of *Unbiased MCMC methods with couplings* by Jacob, O’ Leary and Atchade

Martin Chak, Nikolas Kantas, Grigorios A. Pavliotis

We would like to congratulate the authors for their major and inspiring contribution. In our discussion we shall sketch our ideas for extending the use of couplings to include variance reduction in addition to bias removal. Our approach is based on extending the framework presented in Nüsken and Pavliotis [2019]. There, couplings of n π -invariant continuous time Markov processes $(Z^i; i = 1, \dots, n)$ were considered for the purpose of variance reduction (w.r.t. asymptotic variance). Sections 3 and 5 of Nüsken and Pavliotis [2019] present non-trivial constructions for such couplings of Markov processes that can be used in sampling contexts. Typically one should expect that these couplings enhance the exploration of the state space, which seems to be in contrast to constructing Markov processes that meet as required for unbiased MCMC. We believe these two opposing behaviours can be combined. Instead of using independent samples of coupled-faithful pairs of chains, one could consider correlating these pairs, $Z^i = (X^i, Y^i)$, such that each of X^i, Y^i have marginally in i the same law and then use an unbiased estimator. Given X^i one could design Y^i so that they eventually meet and the X^i -s could be jointly propagated using correlated noise inputs designed for improved joint asymptotic variance or better exploration of the state space as in Nüsken and Pavliotis [2019]. This can be achieved either in discrete time (or in continuous time) by extending the work of Nüsken and Pavliotis [2019] to discrete time Markov processes and MCMC (or by extending the work in this paper for continuous time Markov processes respectively). Finally we note that in the most simple case moving from \mathcal{X} to work on a joint state space \mathcal{X}^n requires modifying the test function $h(x)$ to $\frac{1}{n} \sum_{i=1}^n h(x^i)$ or a normalised weighted average, so it would be interesting to include variance reduction techniques such as control variates.

References

N Nüsken and GA Pavliotis. Constructing sampling schemes via coupling: Markov semigroups and optimal transport. *SIAM/ASA Journal on Uncertainty Quantification*, 7(1):324–382, 2019.