

M2PM2 Progress Test 1

1. For each of the following statements, decide whether it is true or false, and justify your answer.

- (a) A_9 has elements of every order from 1 up to 9.
- (b) A_{10} has elements of every order from 1 up to 10.
- (c) Up to isomorphism, there is only one abelian group of order 12.
- (d) The two groups S_4 and $C_2 \times A_4$ are isomorphic to each other.

Solution

(a) False: A_9 has no element of order 8 (since an 8-cycle is an odd permutation). **2 marks**

(b) True: cycle-shapes of elements of orders $1, \dots, 10$ are

$e, (2, 2), (3), (4, 2), (5), (6, 2), (7), (8, 2), (9), (5, 2)$. **3marks**

(c) False: C_{12} and $C_6 \times C_2$ are abelian, and are not isomorphic as $C_6 \times C_2$ has no element of order 12, hence is not cyclic. **2 marks**

(d) False: $C_2 \times A_4$ has an element of order 6 (e.g. $(-1, (123))$), but S_4 doesn't. **3 marks**

(No marks unless some reasoning is given.)