In these lectures we discuss aspects of hyperbolicity and shadowing, mainly for discrete dynamical systems in finite dimensions. First we elucidate properties of hyperbolicity using the theory of exponential dichotomy for linear difference equations. Then we prove the shadowing theorem and the recently discovered converse. Second we describe the use of shadowing as a tool in computational dynamical systems. This arises from the fact that a computed orbit is a pseudo orbit; shadowing enables us to rigorously verify the existence of a true orbit near a computed orbit, thus validating numerical simulations. We discuss the shadowing by true orbits of finite pseudo orbits, pseudo periodic orbits and pseudo homoclinic orbits of diffeomorphisms; lastly we discuss the shadowing of pseudo homoclinic orbits in autonomous systems of ODEs.

Part 1: Hyperbolicity, Exponential Dichotomy and Shadowing

1. A Sufficient Condition for Hyperbolicity for a Diffeomorphism

A necessary and sufficient condition for hyperbolicity of a compact invariant set of a diffeomorphism in the spirit of Sacker-Sell and Mané is given.

2. Perturbation theory of exponential dichotomies

The classical perturbation theorem for exponential dichotomies is proved. Then we give a finite-time condition for exponential dichotomy; this is a major tool in the proof of the shadowing theorem.

3. Shadowing Theorem

In this section the shadowing theorem is proved. The major tools are the finite-time condition for exponential dichotomy and the BIBO (bounded input - bounded output) property of exponential dichotomy.

4. Converse of Shadowing Theorem

Here we prove the converse of the shadowing theorem due to Pilyugin and Tikhomirov. The major tool is the BIBO criterion for exponential dichotomy.

Part 2: Numerical Shadowing

5. Finite-time shadowing

In this section we give a criterion in terms of a suitable right inverse of a certain operator which guarantees the existence of a true orbit of a discrete dynamical system near a computed orbit with suitably small local errors.

6. Periodic shadowing

Here we give a criterion in terms of the inverse of a certain operator guaranteeing the existence of a true periodic orbit of a discrete system near a computed apparently periodic orbit with suitably small local errors.

7. Homoclinic shadowing

Here we give a criterion in terms of the inverse of a certain matrix (associated with a boundary value problem for a difference equation), which guarantees the existence of a true homoclinic orbit of a discrete system near a computed apparently homoclinic orbit with suitably small local errors.

8. Homoclinic shadowing in ODEs

In this section we give a criterion in terms of the inverse of a certain matrix (associated with a boundary value problem for a difference equation), which guarantees the existence of a true homoclinic orbit of an autonomous ODE near a computed apparently homoclinic orbit with suitably small local errors; the difference here is that the system depends on a parameter and the true orbit is for the system with slightly different parameter value.