Solutions to Exercises III

1. Variance is unknown so confidence interval is constructed using values from the t-distribution with $n-1$ degrees of freedom. Interval is of the form

$$\left(\bar{y} - \frac{t^{n-1}_{\alpha/2} s}{\sqrt{n}}, \bar{y} + \frac{t^{n-1}_{\alpha/2} s}{\sqrt{n}}\right)$$

where $\bar{y} = 1762, s = 215, n = 20$.

(a) 90% CI gives $t^{19}_{0.05} = 1.729$ giving

$$1762 \pm \frac{1.729 \times 215}{\sqrt{20}} = (1679, 1845) \text{ in hours}$$

(b) 99% CI gives $t^{19}_{0.005} = 2.861$ giving

$$1762 \pm \frac{2.861 \times 215}{\sqrt{20}} = (1624, 1900) \text{ in hours}$$

2. 95% confidence interval, $\bar{y} = 4.985, s = 0.03, n = 50$. $t^{49}_{0.025} \approx 2.009$ so CI is

$$4.985 \pm \frac{2.009 \times 0.03}{\sqrt{50}} = (4.977, 4.994) \text{ inches}$$

For this alternative hypothesis we require a two-sided test. Test statistic is

$$T = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}}$$

and the rejection region is $T > t^{49}_{.025}$ and $T < -t^{49}_{.025}$, in this case therefore we would not reject H_0 if the test statistic is in the interval (-2.009,2.009).

Observed value of the test statistic is

$$\frac{4.985 - 5}{\frac{0.03}{\sqrt{50}}} = -3.54$$

so we reject H_0 at the 5% significance level.

3. We have

$$\begin{aligned} H_0 &: \mu = 20 \\ H_A &: \mu < 20 \end{aligned}$$

where μ is the true mean emission. *NOTE*: one-sided alternative since we are specifically interested in whether the emissions are less than 20 ppm.

$$\text{Test statistic } T = \frac{\bar{y} - 20}{\frac{3}{\sqrt{10}}} = \frac{17.1 - 20}{\frac{3}{\sqrt{10}}} = -3.06$$

Rejection region is (for $\alpha = 0.01$ and $n - 1 = 9$): $T < -t^{9}_{0.01} = -2.821$. So T falls in the rejection region and we reject H_0 at the $\alpha = 0.01$ significance level. Test assumes normality and independence of the sample.

4. In both cases the 99% CI is of the form

$$\bar{y} \pm \frac{t^{23}_{0.005} s}{\sqrt{24}}, \quad t^{23}_{0.005} = 2.807$$

(a) Interval is

$$9.9 \pm \frac{2.807 \times 8.4}{\sqrt{24}} = 9.9 \pm 4.81 = (5.09, 14.71) \ \text{nmol/l}$$

(b) Interval is

$$6.7 \pm \frac{2.807 \times 10.8}{\sqrt{24}} = 6.7 \pm 6.19 = (0.51, 12.89) \ \text{nmol/l}$$

(c) Both intervals assume that samples of each concentration are independent and are normally distributed. Note that this could be dubious for (b) since only positive concentrations are allowed and the standard deviation is greater than the mean.

5. If the normal distributions from which the load capacities for alloys A and B are denoted $N(\mu_1, \sigma^2)$ and $N(\mu_2, \sigma^2)$ then we are interested in testing

$$\begin{array}{lcl} H_0 & : & \mu_1 = \mu_2 \\ H_A & : & \mu_1 \neq \mu_2 \end{array}$$

The test statistic is

$$T = \frac{\bar{X} - \bar{Y}}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

where

$$s^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{10 \times 24.4 + 16 \times 19.9}{11 + 17 - 2} = 21.6$$

so

$$T = \frac{43.7 - 48.5}{\sqrt{21.6}\sqrt{\frac{1}{11} + \frac{1}{17}}} = -2.67$$

The degrees of freedom are $n_1 + n_2 - 2 = 26$. We do not reject H_0 if the observed value of the test statistic is between $-t^{n_1+n_2-2}_{\alpha/2}$ and $t^{n_1+n_2-2}_{\alpha/2}$, where α is the size of the type I error. We have

$$\begin{array}{lclclcl} \alpha & = & 0.10\,, & t^{26}_{0.05} & = & 1.706 \\ \alpha & = & 0.05\,, & t^{26}_{0.025} & = & 2.056 \\ \alpha & = & 0.01\,, & t^{26}_{0.005} & = & 2.779 \end{array}$$

So we would reject H_0 at the 10% and 5% significance levels, but not at the 1% significance level.