1. Suppose a computer centre wants to evaluate the performance of its disk memory system. One measure of performance is the average time between failures of its disk drives. To estimate this value, the center recorded the time between failures for a random sample of 20 disk drive failures. The following sample statistics were computed:

$$\bar{y} = 1762 \text{ hours} \quad , \quad s = 215 \text{ hours.}$$

   Assuming that the times between failures are normally distributed, construct

   (a) a 90%, and

   (b) a 99%,

   confidence interval for the true mean time between failures.

2. Moulded-rubber expansion joints used in heating and air-conditioning systems are manufactured to withstand high pressure. A rubber company has purchased new machinery to produce joints with 5 cm. internal diameters. For a random sample of 50 moulded-rubber expansion joints, the mean internal diameter is $\bar{y} = 4.985$cm. and the standard deviation is $s = 0.03$cm. The diameters can be assumed to be normally distributed $N(\mu, \sigma^2)$. Estimate the mean internal diameter of all expansion joints produced by the new machinery with a 95% confidence interval. Test the null hypothesis

$$H_0 \;:\; \mu = 5$$

   versus the alternative

$$H_1 \;:\; \mu \neq 5$$

   with Type I error $\alpha = 0.05$.

3. A car manufacturer wants to test a new engine to determine whether it meets new air pollution standards. The mean emission, μ, of all engines of this type must be less than 20 parts per million (ppm) of carbon. Ten engines are manufactured for testing purposes, and the mean and standard deviation of the emissions for this sample are determined to be

$$\bar{y} = 17.1 \text{ ppm} \quad , \quad s = 3.0 \text{ ppm.}$$

   Do the data supply sufficient evidence to allow the manufacturer to conclude that this type of engine meets the pollution standards? Assume that the manufacturer is willing to risk a Type I error of size $\alpha = 0.01$. For this test to be valid what assumptions need to be make about the sample?

4. Twenty-four water samples were collected from a lake in order to evaluate concentrations of trace metals. The samples were analysed for the concentration of both lead and aluminium particles.

   (a) The lead concentration measurements had a mean of 9.9nmol/l and a standard deviation of 8.4nmol/l. Calculate a 99% confidence interval for the true mean lead concentration in water samples collected from the lake.

   (b) The aluminium concentration measurements had a mean of 6.7nmol/l and a standard deviation of 10.8nmol/l. Calculate a 99% confidence interval for the true mean aluminium concentration in water samples collected from the lake.

   (c) What assumptions are necessary for the intervals of parts (a) and parts (b) to be valid.

5. Two alloys, A and B, are to be used in the manufacture of steel bars. Suppose a steel producer wants to compare the two alloys on the basis of average load capacity, where the load capacity of a steel bar is defined as the maximum load (weight in tons) it can support without breaking. Steel bars containing alloy A and steel bars containing alloy B were randomly selected and tested for load capacity. Measurements for a particular alloy can be assumed to be normally distributed. The results are summarized below.

Alloy A	Alloy B
$n_1 = 11$	$n_2 = 17$
$\bar{y}_1 = 43.7$ tons	$\bar{y}_2 = 48.5$ tons
$s_1^2 = 24.4$ tons^2	$s_2^2 = 19.9$ tons^2

   Carry out a two-sample t-test in order to test whether the mean load capacity of the two alloys is the same.

1993 Exam Question

1. (a) The specification for steel sheet requires that the thickness should lie between 9.82 mm. and 10.05 mm. If the thickness is normally distributed with mean 10 mm. and variance 0.0081 mm^2 find the probability that the thickness of a randomly selected sheet does not meet the specification.

   (b) The thicknesses of 10 samples of steel sheet from a certain supplier are given below. It is assumed that the thicknesses are normally distributed with unknown mean μ and unknown variance σ^2. Write down the forms of the unbiased estimators of μ and σ^2 and use these to obtain estimates from these data.

9.97	9.78	9.88	9.80	10.06
9.82	9.88	9.99	10.02	10.06

   Test whether the average thickness of sheet from the supplier can be taken to be 10 mm.