MSE201 1999/2000 Exercises I

1. Let A, B and C be three arbitrary events. Using only the operations of union, intersection and complement, write down expressions for the events that, of A, B, C;

   (a) Only A occurs.
   (b) Both A and B, but not C occurs.
   (c) All three events occur.
   (d) At least one event occurs.
   (e) At least two events occur.
   (f) One and only one event occurs.
   (g) Exactly two events occur.
   (h) No events occur.
   (i) Not more that two events occur.

2. Which of the following is more probable:

   (a) Getting at least one six with 4 throws of a die.

   (b) Getting at least one double six with 24 throws of two dice?

3. A fair sided die is thrown twice.

   (a) Write down the sample space for this experiment.

   (b) Let B be the event that the first number thrown is no larger than 3, and let C be the event that the sum of the two numbers thrown equals 6. Find the probabilities of B and C, and the conditional probabilities of C given B, and of B given C.

4. A box of n light bulbs contains r with broken filaments ($r < n$). Describe a suitable sample space for the number of bulbs tested, if bulbs are tested, one by one, until

   (a) a defective one is found.

   (b) all defectives are found.

5. Tomorrow there will be either rain or snow but not both; the probability of rain is $\frac{2}{5}$ and the probability of snow is $\frac{3}{5}$. If it rains then the probability that I will be late for my lecture is $\frac{1}{5}$, while the corresponding probability in the event of snow is $\frac{3}{5}$. What is the probability that I will be late?

6. A car arriving at a crossroads is equally likely to turn left (L), to turn right (R), or to carry straight on (S).

   (a) What is the probability that it makes a turn? Given that it makes a turn, what is the probability that it turns left?

   (b) Two cars both arrive at the crossroads, and each behaves independently as above. What is the probability that at least one car turns left? Given that at least one car makes a turn, what is the probability that at least one car turns left?

7. A company makes resistors which are nominally 15 ohms. In fact 5% of the output are below 14 ohms and 10% are above 16 ohms. Two resistors are selected at random.

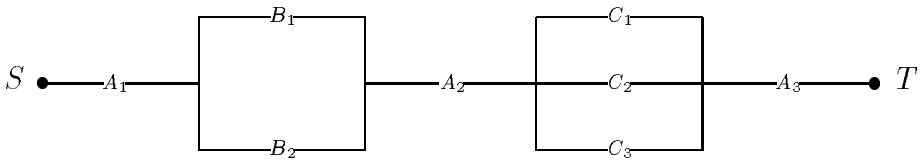
   (a) What is the probability that both are between 14 and 16 ohms?

   (b) What is the probability that at least one is above 16 ohms?

8. A diagnostic test for a disease is 90% accurate, *i.e.*, for an individual with the disease, it is positive with probability 0.9 and negative with probability 0.1, and for an individual without the disease, the reverse. One percent of the whole population have the disease in question.

   Given the the test is positive, what is the probability that the person tested has the disease? Comment.

9. Consider the following network:



   Components labelled A fail with probability 0.1, those labelled B fail with probability 0.2, those labelled C with probability p $(0 < p < 1)$. All the components are independent. The system survives if there is at least one path from S to T along which all components survive.

   (a) If $p = 0.5$, what is the probability that the system survives?

   (b) How large must p be for the probability of survival to be less than 0.5?

10. You are travelling on a train with your sister. Neither of you has a valid ticket and the inspector has caught you both. He is authorized to administer a special punishment for this offence. He holds a box containing nine apparently identical chocolates, but three of these are contaminated with a deadly poison. He makes each of you, in turn, choose and eat a single chocolate.

    (a) If you choose before you sister, what is the probability that you survive?

    (b) If you choose first and survive, what is the probability that you sister survives?

    (c) If you choose first and die, what is the probability that your sister survives?

    (d) Is it in your best interests to persuade your sister to choose first?

    (e) If you choose first, what is the probability that you survive, given that you sister survives?