5. The signal strength of a wireless router from a laptop is classified into three categories: *excellent, good* and *weak*, depending on the distance, X, of the laptop from the router,

Classification	Distance
Excellent	$X < 10$m.
Good	10m. $< X < 30$m.
Weak	$X > 30$m.

The distance, X, from the router follows an exponential distribution with parameter $\lambda = 0.1$,

$$f(x) = \begin{cases} \lambda \exp(-\lambda x) & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

(i) Find an expression for $P(X < x)$.

(ii) Determine the probabilities that the signal strength will be classified as excellent, good and weak.

The observed probabilities of successfully downloading a file if the signal strength is classified as excellent, good or weak are 1, 0.9 and 0.1 respectively.
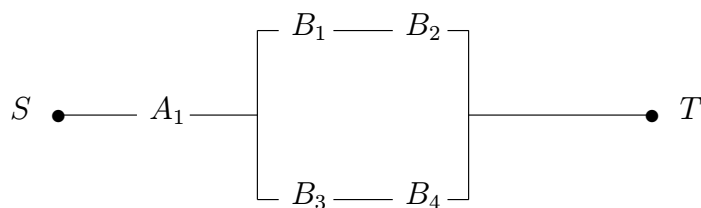
(iii) Find the unconditional probability that a file is downloaded successfully.

(iv) If a file is downloaded successfully, determine the probability that the signal strength was classified as excellent.

Assuming that files are downloaded sequentially and independently,

(v) determine the maximum number of files that can be downloaded, such that the probability that they are all downloaded successfully is greater than 0.5.

(vi) find an expression for the probability that the first unsuccessful download occurs after n download attempts.

6. The lifetimes, T_A and T_B of components of type A and B, in hours, follow normal distributions with variances 4 and 9 respectively.

   (i) The lifetimes of a sample of size $n_A = 16$ components of type A have a sample mean of $\bar{x}_A = 26$, and the liftimes of a sample of size $n_B = 16$ components of type B have a sample mean of $\bar{x}_B = 30$. Calculate 95% confidence intervals for the mean lifetimes of components A and B.

   (ii) Find expressions for the reliability and hazard function of a component with lifetime $T \sim N(\mu, \sigma^2)$.

   (iii) Assuming that the mean lifetimes of components A and B are 26 and 30 hours respectively, find the reliabilities of a component of type A and B at one day.

   (iv) Comment on a potential problem of modelling lifetimes using a normal distribution.

The following network is constructed using one component, A_1, of type A and four components, B_1, B_2, B_3 and B_4, of type B, all operating independently. The network functions if there is a path of functioning components between S and T.

$$S \bullet \rule{1cm}{0.4pt} A_1 \rule{1cm}{0.4pt} \begin{array}{c} B_1 \rule{0.8cm}{0.4pt} B_2 \\ \\ B_3 \rule{0.8cm}{0.4pt} B_4 \end{array} \rule{1cm}{0.4pt} \bullet T$$

   (v) Determine the reliability of the network at one day.