Statistical Analysis of Stellar Evolution

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Abstract: Color-Magnitude Diagrams (CMDs) are plots that compare the magnitudes (luminosities) of stars in different wavelengths of light (colors). High non-linear correlations among the mass, color and surface temperature of newly formed stars induce a long narrow curved point cloud in a CMD known as the main sequence. Aging stars form new CMD groups of red giants and white dwarfs. The physical processes that govern this evolution can be described with mathematical models and explored using complex computer models. These calculations are designed to predict the plotted magnitudes as a function of parameters of scientific interest such as stellar age, mass, and metallicity. Here, we describe how we use the computer models as a component of a complex likelihood function in a Bayesian analysis that requires sophisticated computing, corrects for contamination of the data by field stars, accounts for complications caused by unresolved binary-star systems, and aims to compare competing physics-based computer models of stellar evolution.

Stellar Evolution

Stellar Evolution Model: Stars are formed when the dense parts of a molecular clouds collapse into a ball of plasma, see Fig. 3. For a sun-like star, the core eventually ignites in a thermonuclear reaction powered by the fusion of hydrogen into helium. This reaction continues for millions or billions of years depending on the mass and composition of the star. When the hydrogen at the core is depleted, the core begins to collapse and heat up and the star may begin to fuse Helium into heavier elements. At the same time, the diameter of the star increases enormously and its surface temperature cools, resulting in a *red giant* star. During this period the star undergoes mass loss leading to the formation of a short lived *planetary nebula*, see Fig. 3. Eventually a new equilibrium is reached that prevents further collapse of the core. forming a *white dwarf*, see Fig. 1.

Fig. 1: Evolution of a sun-like star.

How a star evolves depends on its initial mass, see Fig. 2. More massive stars are denser, hotter, and burn their fuel faster. Their cores collapse further to from *neutron stars* or *black holes*, often accompanied by a dramatic super nova explosion, see Fig. 3.

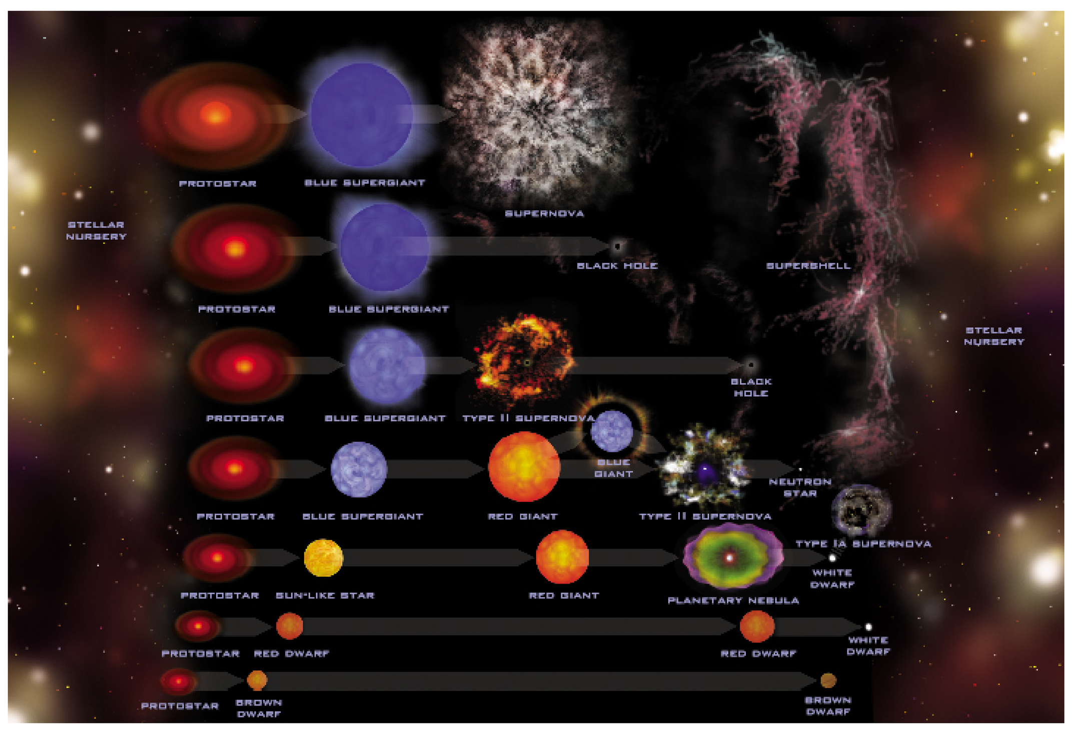

Fig. 2: Stellar evolution depends critically ont initial stellar mass.

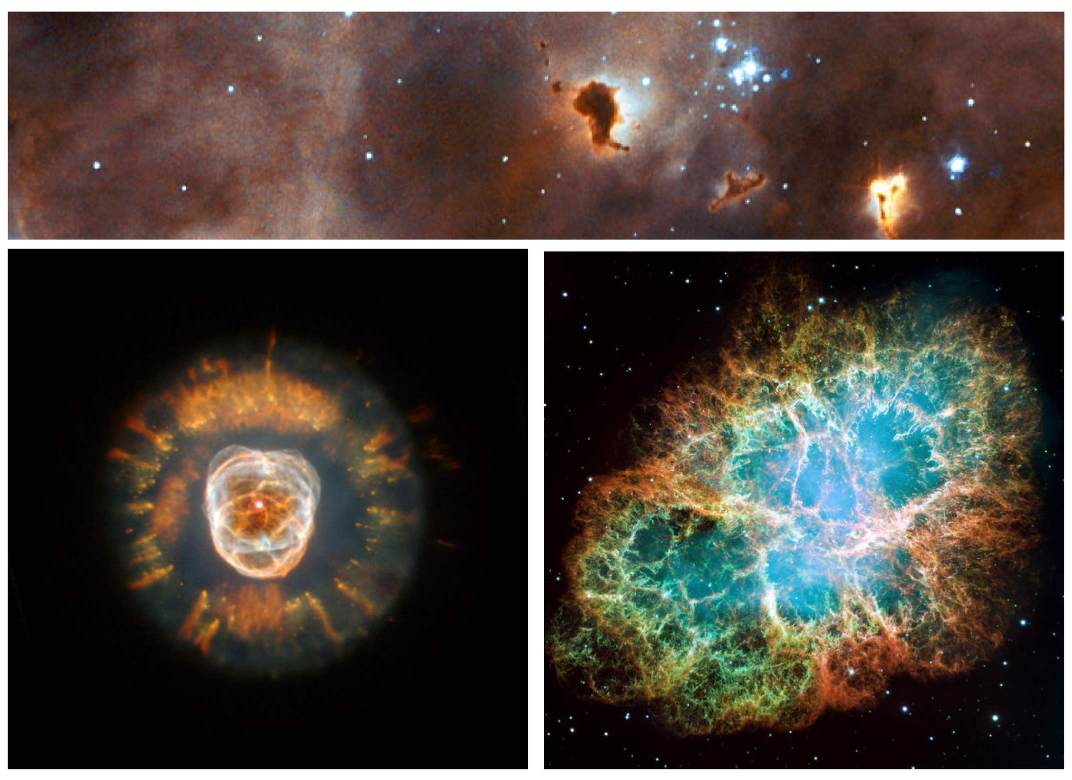

Fig. 3: Photographs of molecular clouds of a stellar nursery (top), a planetary nebula (bottom left), and a super nova (bottom right).

Stellar Parameters: The distribution of the the wavelength of the light emitted by a star (its *spectrum*) is effected by several parameters. More **massive** stars are denser, hotter, bluer, and burn their fuel much more quickly. Composition also effects a spectrum: **Metals** absorb more blue light and **excess helium** at the core reduces the efficiency of the nuclear reaction. As we have seen, the spectrum of the star changes as it **ages**. Finally, some light from a star is **absorbed** by interstellar material and more **distant** stars appear fainter.

Data Collection: To fit these parameters, we study a set of course stellar spectra. Using filters attached to earth-based telescopes or the *Hubble Space Telescope*, we measure a star's luminosity in each of several wide wavelength bands. An inverted log transform of the luminosities gives the stars' magnitudes. A typical data set consists of 2 or 3 magnitudes for each of several hundred stars.

Stellar Clusters (Fig. 4) are physical groups of stars formed at the same time out of the same material. These stars have the same metallicity, helium abundance, age, distance, and absorption; only the initial masses vary. This signicantly simplies statistical analysis.

Fig. 4: Three star clusters.

Color Magnitude Diagrams: A color magnitude diagram or CMD plots temperature versus absolute magnitude for a group of stars (Fig. 5, left). These plots are useful to classify stars according to the stages of their lives, but require measurements of temperature and absolute magnitude. When we study star clusters, however, we can use apparent magnitude in place of absolute magnitude, because the stars are all the same distance away. Because temperature is highly correlated with difference in magnitudes in two optical wavelength bands, we can use a difference in place of temperature. The result for the Hyades cluster appers in the right panel of Fig. 5.

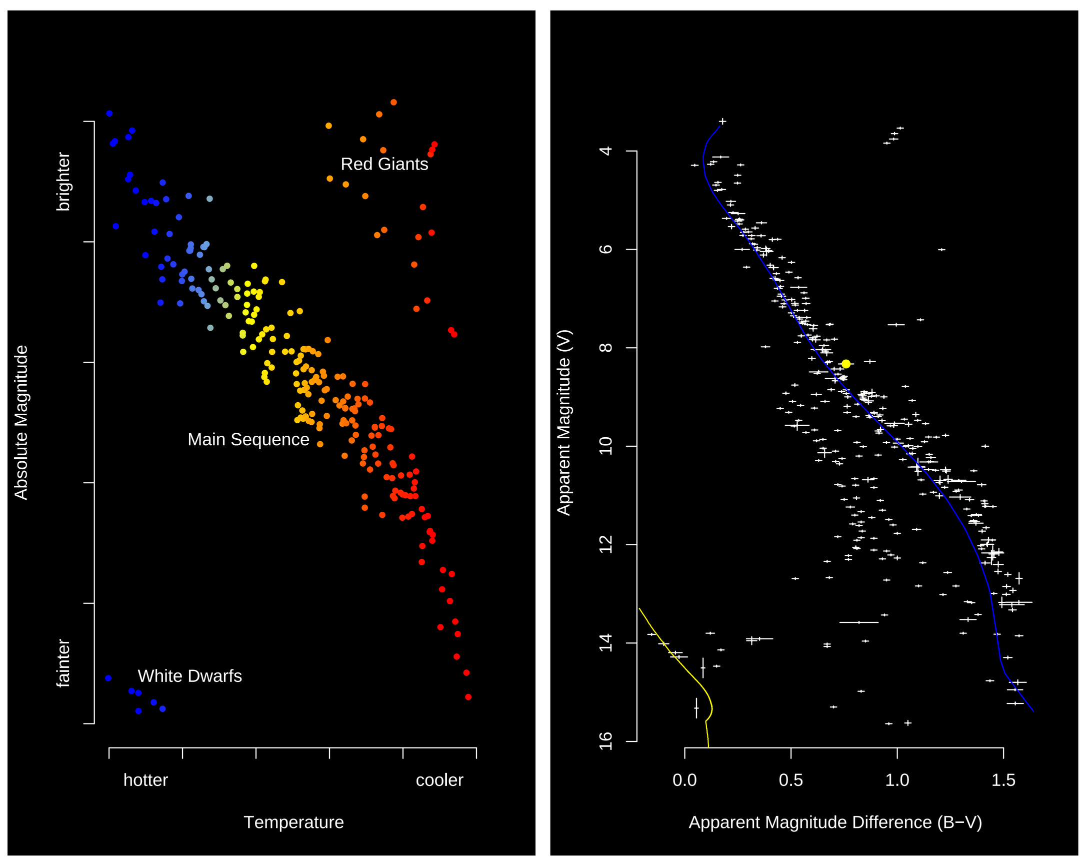

Fig. 5: A schematic CMD (left) and a CMD of the Hyades (right). The fitted main and white dwarf sequence are plotted for the Hyades.

Computer Models for Stellar Evolution: Computer models are used to predict observed magnitudes given the stellar parameters. This involves iteratively solving a set of coupled differential equations which results in a prediction of how a star of a particular mass and radial abundance profile would appear in terms of its luminosity and color. Stars are evolved by updating the mass and abundance prole to account for the newly produced elements. Finally interstellar absorption and distance can be used to convert absolute magnitudes into apparent magnitudes.

There are separate implementations of the model for Main Sequence / Red Giant and White Dwarf evolution and competing implementations for the Main Sequence. An empirical model is used to link the models via an initial mass / white dwarf mass relationship. Evaluating the full model can take from seconds to more than an hour, depending on the evolutionary state of the star.

Statistical Model

The computer evolution model predicts the magnitudes of star i, $\boldsymbol{G}(M_i,\Theta) = \mathrm{E}(\boldsymbol{X}_i)$, as a function of mass, M_i, and the cluster parameters, Θ. Under a normal error assumption the likelihood is

$$L_0(\boldsymbol{M},\Theta|\boldsymbol{X}) = \prod_{i=1}^{N}\left(\prod_{j=1}^{n}\left[\frac{1}{\sqrt{2\pi\sigma_{ij}^2}}\exp\left(-\frac{(x_{ij}-G_j(M_{i1},\Theta))^2}{2\sigma_{ij}^2}\right)\right]\right),$$

where j indexes the observed magnitudes of star i. The actual likelihood must also account for binary stars and field star contamination.

Binary Stars: Between 1/3 and 1/2 of stars are actually unresolved binary or multi-star systems. The observed luminosities are the sums of those of the component stars, resulting in an offset on the CMD that is informative for the individual masses,

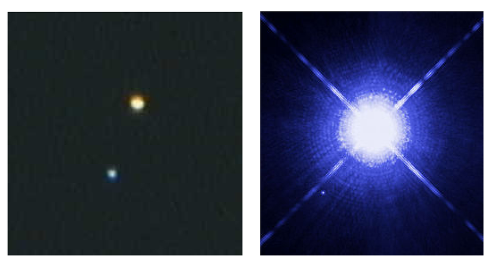

Fig. 6: Resolved binary stars.

$$-2.5\log_{10}\left[10^{-\boldsymbol{G}(M_{i1},\Theta)/2.5} + 10^{-\boldsymbol{G}(M_{i2},\Theta)/2.5}\right].$$

Field Stars: Although field stars appear in the field of view, they are not part of cluster and their magnitudes do not follow the pattern on the CMD. More distant stars are dimmer and below the main sequence, see right of Fig. 5. We use a uniform distribution, $\boldsymbol{U}(\boldsymbol{X}_i)$, for their magnitudes and formulate the likelihood as a mixture,

$$L(\boldsymbol{M},\Theta,\boldsymbol{Z}|\boldsymbol{X}) = \prod_{i=1}^{N}\prod_{j=1}^{n}\left[\frac{Z_i}{\sqrt{2\pi\sigma_{ij}^2}}\exp\left(\frac{-1}{2\sigma_{ij}^2}\left\{x_{ij}+2.5\log_{10}\left[10^{\frac{-G_j(M_{i1},\Theta)}{2.5}}+10^{\frac{-G_j(M_{i2},\Theta)}{2.5}}\right]\right\}^2\right)+(1-Z_i)\boldsymbol{U}(\boldsymbol{X}_i)\right]$$

where Z_i is an indicator for cluster membership for star i.

Prior Distribution: We use both informative and non-informative prior distributions. An informative truncated Gaussian is used on log mass, representing the population distribution of stellar masses. The ratio of the smaller and larger mass is assumed uniform. For well studied clusters such as the Hyades, there are informative star-by-star priors for the cluster membership. A mildly informative population-based prior is used for the cluster age. Finally, the remaining cluster parameters are considered on a case-by-case basis.

Statistical Computation

Basic Strategy: Our Metropolis within Gibbs sampler is hampered by strong associations among the $3N+5$ parameters, where N is the number of stars. Simply evaluation of the posterior is difficult because of the complexity of the computer evolution model. To address these problems we we introduce decorrelation methods and interpolate within a tabulated form of the computer model.

Correlation Reduction: Because the data is uninformative for the masses of field stars, the cluster/field star indicator is highly correlated with mass. To improve the resulting poor mixing of the indicator, we replace the priors on the masses for field stars with an estimate of the posteriors for cluster stars obtain in an initial run. This strategy does not effect inference and greatly improves mixing. We also introduce a set of static non-linear and dynamic linear transformations to reduce correlations. The dynamic transformations are based on a set of linear regression analyses computed using a sample generated in an initial run. The combined result of all of these adjustments is a useful sampler with significantly improved mixing.

Analysis of the Hyades Cluster

Fig. 7: The Hyades cluster forms the nose of Taurus the Bull.

The *Hyades* is the nearest open cluster to the solar system and is visible to the unaided eye as the nose of Taurus the Bull. The distance of the cluster has been measured at about 151 light years using stellar parallax and the age of the cluster at 625 ± 50 million years using the main sequence turnoff or MSTO (using the fact that the most massive main sequence stars are the first to evolve into red giants), see Perryman et al. (ApJ, 1998). A second age estimate, based on white dwarf magnitudes is about 300 million years, see Weideman et al. (ApJ, 1992). *The two estimates should agree better.*

Our primary scientific goal is to compare MSTO estimate with an improved estimate based primarily on the white dwarf magnitudes. Thus, we remove red giants and stars near the turn off from the data. A secondary goal is to evaluate the underlying computer/physical models for stellar evolution. To do this, we compare existing external measures with those produced under our fit. The result is the most sophisticated empirical check of the computer models to date.

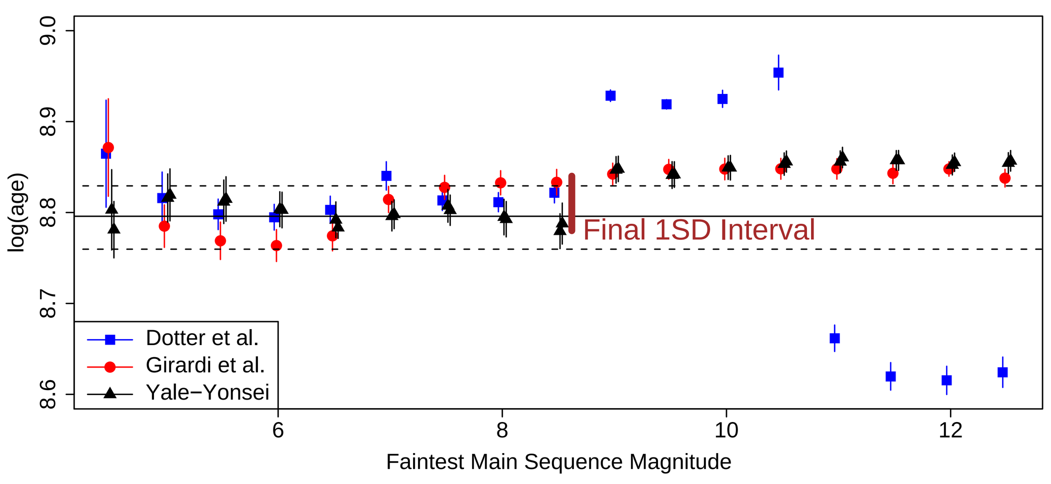

Fig. 8: Fitted age using different data depths and evolution models.

Because the stellar evolution models are known to be imprecise for the faintest main sequence stars, we compare fits using varying data depths. Fig. 8 illustrates the results using three main sequence models. Residual plots (not shown) indicate that the models are most reliable for stars at or below the 8.5th magnitude. Our final estimate for the age, 648 ± 45, is shown in brown. This estimate agrees with the main-sequence turn off estimate far better, see Fig. 9. This result helps to demonstrate the ability of our (bright) white dwarf technique to derive ages consistent with main sequence turn-off ages.

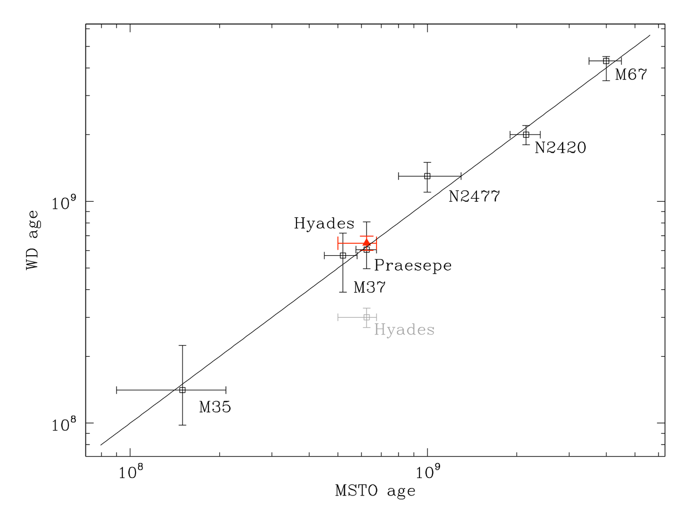

Fig. 9: The agreement between the MSTO age and the WD age for several clusters. The former best WD age for the Hyades is in grey.

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