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A NEW MODEL FOR A MODE-LOCKED SEMICONDUCTOR LASER

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We consider a new model for passive mode locking in a semiconductor laser comprising a set of delay differential equations. Bifurcations leading to the appearance and break-up of the mode-locking regime are studied numerically.

Passive mode locking is one of the most efficient methods for generating short optical pulses. In particular, mode-locked monolithic semiconductor lasers are cheap, compact, and reliable sources of pulses with high repetition rate, ideal for telecommunication applications [1]. The cavity length of such lasers can be sufficiently small due to the large gain of the semiconducting medium. Together with the small response time of the amplifying and absorbing media, this allows one to generate pulses with a repetition rate of several tens of GHz. The main physical mechanism of passive phase locking is well known. For the case where the absorber relaxation time is much larger than the pulse duration ("slow" absorber), this mechanism is as follows [2]. Upon arrival of a pulse, the absorbing medium gets saturated faster than the amplifying one and thus opens up a short amplification window necessary to compensate for losses and, hence, sustain the locking regime. Basic analytical theory of passive mode locking in lasers with slow absorber was developed more than 25 years ago by New and Haus [2–4]. Both authors used an approximation of small roundtrip losses and gain. In New's paper, spectral filtering of the pulse was neglected. This actually means that an infinite number of longitudinal cavity modes take part in the mode locking and, hence an infinitely short pulse is generated. The pulse energy remains finite and can be expressed in terms of laser parameters. In the paper by Haus, the parabolic approximation for the profile of the spectral-filtering coefficient was used. Haus showed that, even if the width of this profile goes to infinity, the boundaries of the mode-locking domain do not coincide with those obtained by New under the approximation of no spectral filtering. Under the approximation of weak saturation of absorption, Haus obtained an analytical expression for the mode-locked pulse shape in the form of hyperbolic secant. According to this formula, unlike the case of active mode locking, the pulse amplitude for passive mode locking decreases exponentially, and not by the Gaussian law, with the distance from the pulse center. This result was verified in experiments with a dye laser [5]. In the decades after the Haus paper, his model and its various modifications were developed in detail (see, e.g., [6–11]).

Despite certain advances related to the Haus model, its applicability to the rigorous description of mode locking in a semiconductor laser is doubtful. First, this is related to violation of the approximations used for derivation of this model, such as, e.g., small roundtrip losses and gain. Therefore, the passive mode locking in semiconductor lasers are now most often studied using direct numerical modeling [7]. Such an approach, although allowing fairly accurate account of different physical factors affecting the operation of a specific device, does not provide sufficient understanding of the physical processes underlying the mode locking.

In this paper, we propose a new model describing passive mode locking in a semiconductor laser. On the one hand, our model is much more general than the above-mentioned models by New and Haus. Unlike

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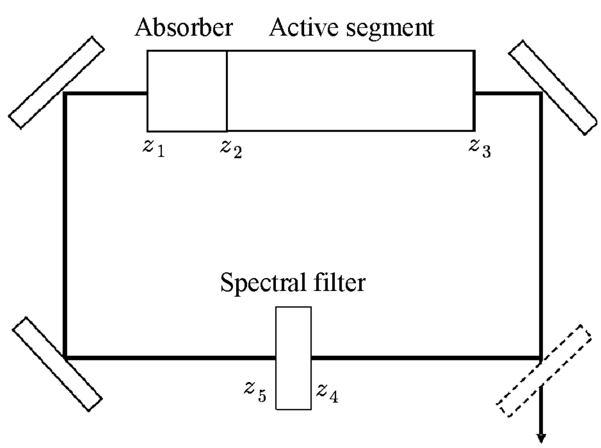

Fig. 1. Schematic picture of a ring laser. The coordinate z is measured along the resonator axis. The intervals $z_1 < z < z_2$ and $z_2 < z < z_3$ are filled with saturable absorber and active medium, respectively. The spectral filter is located in the interval $z_4 < z < z_5$. The intervals $z_3 < z < z_4$ and $z_5 < z < z_1 + L$ are filled with passive medium.

these models, we do not assume the smallness of roundtrip gain and losses, weak saturation, and infinite width of the spectral-filtering profile. The only approximations we adopt are based on the widely-used assumptions of the ring geometry of the cavity and the Lorentz shape of the spectral-filtering profile. On the other hand, the proposed model, represented by a system of delay differential equations, is significantly simpler than the presently used numerical models describing passive mode locking. Our model admits clear physical interpretation of the obtained results in terms of the quantities used in the Haus and New models. It can be shown that these two models can be obtained from our model as their specific cases [12]. Another advantage of our model is the possibility of its studying by the methods developed for analysis of bifurcations of the delay differential equations.

Consider a ring laser with saturable absorber (see Fig. 1). Let us assume that one of the counterpropagating waves is suppressed, i.e., that the lasing takes place in a single direction. Let z be the coordinate along the laser axis. The laser has five segments. The first ($z_1 < z < z_2$) and second ($z_2 < z < z_3$) segments comprise the absorbing and amplifying media, respectively. The third ($z_3 < z < z_4$) and fifth ($z_5 < z < z_1 + L$) segments are passive. The fourth segment ($z_4 < z < z_5$) operates as a spectral filter. The evolution of the amplitude of a traveling electromagnetic wave in the amplifying and absorbing segments can be described by the following system of partial differential equations:

$$\frac{\partial E(t,z)}{\partial z} + \frac{1}{v}\frac{\partial E(t,z)}{\partial t} = \frac{g_{\mathrm{r}}\Gamma_{\mathrm{r}}}{2}(1 - i\alpha_{\mathrm{r}})\left[N_{\mathrm{r}}(t,z) - N_{\mathrm{r}}^{\mathrm{tr}}\right] E(t,z), \tag{1}$$

$$\frac{\partial N_{\mathrm{r}}(t,z)}{\partial t} = J_{\mathrm{r}} - \gamma_{\mathrm{r}} N_{\mathrm{r}}(t,z) - v g_{\mathrm{r}}\Gamma_{\mathrm{r}}\left[N_{\mathrm{r}}(t,z) - N_{\mathrm{r}}^{\mathrm{tr}}\right] |E(t,z)|^2. \tag{2}$$

Here, the subscript $\mathrm{r} = \mathrm{g}$ ($\mathrm{r} = \mathrm{q}$) refers to the amplifying (absorbing) segments, respecitvely, $E(z,t)$ is the slowly varying complex amplitude of the electric field, the quantities $N_{\mathrm{g}}(z,t)$ and $N_{\mathrm{q}}(z,t)$ describe the carrier densities in the amplifying and absorbing laser segments, $N_{\mathrm{g}}^{\mathrm{tr}}$ and $N_{\mathrm{q}}^{\mathrm{tr}}$ are the carrier densities at the transparency level, v is the group velocity of light, which is assumed the same in all four segments, and the parameters α_{g} and α_{q}, g_{g} and g_{q}, and $\gamma_{\mathrm{g}} = 1/T_{\mathrm{g}}$ and $\gamma_{\mathrm{q}} = 1/T_{\mathrm{q}}$ describe the alpha factors, differential gain/losses, and relaxation rates for the amplifying and absorbing segments, respectively. The factors Γ_{g} and Γ_{q} are added to take into account the transverse field distribution in the corresponding segment. The parameter J_{g} specifies the injection current in the amplifying segment. For the absorbing segment, $J_{\mathrm{q}} = 0$.

The electric-field amplitude in the passive segments satisfies Eq. (1) with zero right-hand side:

$$\frac{\partial E(t,z)}{\partial z} + \frac{1}{v}\frac{\partial E(t,z)}{\partial t} = 0. \tag{3}$$

The last segment responsible for spectral filtering ($z_4 < z < z_5$) is assumed infinitely thin, i.e., $z_4 = z_5$. The transformation of the electric-field amplitude in this segment is described by the relation

$$\hat{E}(\omega, z_5) = \hat{f}(\omega)\hat{E}(\omega, z_4), \tag{4}$$

where $\hat{E}(\omega, z_4)$ and $\hat{E}(\omega, z_5)$ are the Fourier components of the amplitudes $E(t, z_4)$ and $E(t, z_5)$, respectively. The function $\hat{f}(\omega)$ specifies the spectral line profile of the filtering element.

In a ring laser, the electric-field amplitude satisfies the periodic boundary conditions

$$E(t, z + L) = E(t, z), \tag{5}$$

where L is the cavity length.

After the coordinate transformation $(t, z) \to (\tau, z)$, where $\tau = t - z/v$, Eqs. (1) and (2) for the amplifying and absorbing segments take the form

$$\frac{\partial A(\tau, z)}{\partial z} = \frac{1}{2}(1 - i\alpha_{\rm g})\, n_{\rm g}(\tau, z) A(\tau, z), \tag{6}$$

$$\frac{\partial n_{\rm g}(\tau, z)}{\partial \tau} = j_{\rm g} - \gamma_{\rm g} n_{\rm g}(\tau, z) - n_{\rm g}(\tau, z)\, |A(\tau, z)|^2; \tag{7}$$

$$\frac{\partial A(\tau, z)}{\partial z} = -\frac{1}{2}(1 - i\alpha_{\rm q})\, n_{\rm q}(\tau, z) A(\tau, z), \tag{8}$$

$$\frac{\partial n_{\rm q}(\tau, z)}{\partial \tau} = -j_{\rm q} - \gamma_{\rm q} n_{\rm q}(\tau, z) - s n_{\rm q}(\tau, z)\, |A(\tau, z)|^2, \tag{9}$$

where $A(\tau, z) = E(t, z)\sqrt{v g_{\rm g}\Gamma_{\rm g}}$, $n_{\rm g}(\tau, z) = g_{\rm g}\Gamma_{\rm g}\,[N_{\rm g}(\tau, z) - N_{\rm g}^{\rm tr}]$, $n_{\rm q}(\tau, z) = g_{\rm q}\Gamma_{\rm q}\,[N_{\rm q}(\tau, z) - N_{\rm q}^{\rm tr}]$, $j_{\rm g} = g_{\rm g}\Gamma_{\rm g} J_{\rm g} - \gamma_{\rm g} N_{\rm g}^{\rm tr}$ and $j_{\rm q} = \gamma_{\rm q} N_{\rm q}^{\rm tr}$. The parameter $s = g_{\rm q}\Gamma_{\rm q}/(g_{\rm g}\Gamma_{\rm g})$ is the ratio of the saturation intensities in the amplifying and absorbing segments.

Similarly, for the passive segments we obtain:

$$\frac{\partial A(\tau, z)}{\partial z} = 0. \tag{10}$$

Now, using Eqs. (6)–(10) and (4), we describe the transformation of the field after its pass through each of the five cavity segments.

1. Saturable absorber ($z_1 < z < z_2$). The relation between the input and output field amplitudes is as follows:

$$A(\tau, z_2) = \exp\left[-\frac{1 - i\alpha_{\rm q}}{2} Q(\tau)\right] A(\tau, z_1), \tag{11}$$

where $Q(\tau)$ is the dimensionless integral carrier density in the absorbing segment:

$$Q(\tau) = \int\limits_{z_1}^{z_2} n_{\rm q}(\tau, z)\, {\rm d}z. \tag{12}$$

Integrating Eq. (9) over z from z_1 to z_2 and using the relation

$$\int\limits_{z_1}^{z_2} n_{\rm q}(z, \tau)\, |A(\tau, z)|^2\, {\rm d}z = -|A(z_2, \tau)|^2 + |A(z_1, \tau)|^2, \tag{13}$$

which follows from Eq. (6), we obtain the differential equation for the integral carrier density:

$${\rm d}Q(\tau)/{\rm d}\tau = -q_0 - \gamma_{\rm q} Q(\tau) + s\, |A(\tau, z_2)|^2 - s\, |A(\tau, z_1)|^2, \tag{14}$$

where $q_0 = \int_{z_1}^{z_2} j_{\rm q}\, {\rm d}z$.

2. The amplification segment ($z_2 < z < z_3$). Equations for this segment are similar to those for the absorption segment and have the form

$$A(\tau, z_3) = \exp\left[\frac{1 - i\alpha_{\rm g}}{2} G(\tau)\right] A(\tau, z_2) \tag{15}$$

and

$$\mathrm{d}G(\tau)/\mathrm{d}\tau = g_0 - \gamma_{\mathrm{g}} G(\tau) - |A(\tau, z_3)|^2 + |A(\tau, z_2)|^2. \tag{16}$$

Here,

$$G(\tau) = \int_{z_2}^{z_3} n_{\mathrm{g}}(\tau, z)\,\mathrm{d}z, \qquad g_0 = \int_{z_2}^{z_3} j_{\mathrm{g}}\,\mathrm{d}z. \tag{17}$$

3. Passive segments ($z_3 < z < z_4$ and $z_5 < z < z_1 + L$). The field transformation in these segments is given by the relations

$$A(z_4, \tau) = \sqrt{\kappa} A(z_3, \tau), \qquad A(z_1 + L, \tau) = A(z_5, \tau). \tag{18}$$

Here, the factor $\sqrt{\kappa} < 1$ describes the total linear nonresonant losses. Without loss of generality, one can assume that they are confined in the first passive segment.

4. Spectral filter ($z_4 < z < z_5$). In the time domain, Eq. (4) takes the form

$$A(\tau, z_5) = \int_{-\infty}^{\tau} f(\tau - \theta) A(\theta, z_4)\,\mathrm{d}\theta, \tag{19}$$

where $f(\tau)$ goes to zero at $\tau \to \infty$ suffuciently fast to ensure convergence of the integral in Eq. (19).

Substituting Eqs. (11), (15), and (18) into (19) and using the periodic boundary conditions (5), which take the form

$$A(\tau, z + L) = A(\tau + T, z) \tag{20}$$

in terms of the variables (z, τ), we obtain the transformation of the electric-field amplitude at the input to the absorbing segment after a cavity roundtrip:

$$A(\tau + T) = \sqrt{\kappa} \int_{-\infty}^{\tau} f(\tau - \theta) \exp\left[\frac{1 - i\alpha_{\mathrm{g}}}{2} G(\theta) - \frac{1 - i\alpha_{\mathrm{q}}}{2} Q(\theta)\right] A(\theta)\,\mathrm{d}\theta. \tag{21}$$

Here, we introduce the notation $A(\tau) \equiv A(\tau, z_1)$, and $T = L/v$ is the time of cavity roundtrip.

Equations describing the time evolution of $G(\tau)$ and $Q(\tau)$ are obtained from Eqs. (16) and (14). Using Eqs. (11) and (15) to express $A(\tau, z_2)$ and $A(\tau, z_3)$ via $A(\tau, z_1) \equiv A(\tau)$, we obtain

$$\mathrm{d}G(\tau)/\mathrm{d}\tau = g_0 - \gamma_{\mathrm{g}} G(\tau) - \exp[-Q(\tau)]\,(\exp[G(\tau)] - 1)\,|A(\tau)|^2, \tag{22}$$

$$\mathrm{d}Q(\tau)/\mathrm{d}\tau = q_0 - \gamma_{\mathrm{q}} Q(\tau) - s\,(1 - \exp[-Q(\tau)])\,|A(\tau)|^2. \tag{23}$$

The system of integro-differential equations (21)–(23) describes passive mode locking in a ring laser with arbitrary line profile of the spectral filter, which is specified by the function $f(\tau)$. Consider the Lorentz profile

$$f(\tau - \theta) = \gamma \exp[-\gamma\,(\tau - \theta)]. \tag{24}$$

In this case, Eqs. (21)–(23) can be replaced by a system of delay differential equations:

$$\mathrm{d}A(\tau)/\mathrm{d}\tau = -\gamma\left[A(\tau) - \sqrt{\kappa} \exp\left[\frac{1 - i\alpha_{\mathrm{g}}}{2} G(\tau - T) - \frac{1 - i\alpha_{\mathrm{q}}}{2} Q(\tau - T)\right] A(\tau - T)\right], \tag{25}$$

$$\mathrm{d}G(\tau)/\mathrm{d}\tau = g_0 - \gamma_{\mathrm{g}} G(\tau) - \exp[-Q(\tau)]\,(\exp[G(\tau)] - 1)\,|A(\tau)|^2, \tag{26}$$

$$\mathrm{d}Q(\tau)/\mathrm{d}\tau = -q_0 - \gamma_{\mathrm{q}} Q(\tau) - s\,(1 - \exp[-Q(\tau)])\,|A(\tau)|^2. \tag{27}$$

Indeed, the solution of Eq. (25) has the form

$$A(\tau) = \exp(-\gamma\tau)A(0) + \gamma\sqrt{\kappa}\int\limits_0^{\tau} \exp\left[\gamma\,(\theta-\tau) + \frac{1-i\alpha_{\rm g}}{2}G(\theta-T) - \frac{1-i\alpha_{\rm q}}{2}Q(\theta-T)\right] A(\theta-T)\,{\rm d}\theta \quad (28)$$

and concides with Eqs. (21) and (24) if

$$A(0) = \gamma\sqrt{\kappa}\int\limits_{-\infty}^{0} \exp\left[\gamma\theta + \frac{1-i\alpha_{\rm g}}{2}G(\theta-T) - \frac{1-i\alpha_{\rm q}}{2}Q(\theta-T)\right] A(\theta-T)\,{\rm d}\theta. \quad (29)$$

Note that even in the case where Eq. (29) is not satisfied, the first term in Eq. (28) goes to zero at $\tau \to \infty$. Therefore, at this limit the solution of Eq. (25) coincides with Eqs. (21) and (24).

Equations (25)–(27) are the basic model proposed in this paper. These equations are obtained for the case where the center of the transmission line of a spectral filter exactly coincides with the frequency of one of the cavity modes. If this condition is not satisfied, then equations describing passive mode locking can be obtained from Eqs. (25)–(27) by applying the ansatz $\sqrt{\kappa} \to \sqrt{\kappa}\,\exp(i\phi)$ in Eq. (25), where the phase ϕ is determined by the frequency mismatch between the line center of a spectral filter and the closest cavity mode. Below, we consider only the case $\phi = 0$.

It is noteworthy that an approach similar to the one used here was employed earlier by Gurevich and Khanin [13–16] for describing the passive mode locking in a solid-state laser. The system of delay differential equations obtained by these authors comprises, instead of the saturable gain G and saturable absorption Q, two other variables specifying the electromagnetic-field amplitude in different cross sections of the laser. The model given by Eqs. (25)–(27), unlike the Gurevich-Khanin model, has no singularity at points where the electromagnetic-field amplitude goes to zero, which we consider an advantage of our model. This makes it possible to study our model by methods similar to those developed by New and Haus for the description of passive mode locking in lasers with slow absorber. This category comprises also lasers for which the relaxation time scale of the saturable absorption is much larger than the duration of generated pulse, in particular, semiconductor and solid-state lasers operated in the regime of mode locking. The New and Haus methods are based on dividing the solution for the time-periodic intensity into two parts. One part describes the so-called slow motion and corresponds to the interval between pulses, when the electromagnetic-field amplitude is practically zero and the gain and absorption slowly relax to non-saturated values. Detailed description of analytical methods for studying Eqs. (25)–(27) in the case of slow absorber is given in [12].

The case of Lorentzian spectral filtering is not the only one for which Eq. (21) can be replaced by a delay differential equation. Another case takes place if the function $f(\tau)$ has the form

$$f(\tau) = \frac{\gamma}{2}[\mathrm{sgn}(\tau) - \mathrm{sgn}(\tau - \gamma^{-1})]. \quad (30)$$

The Fourier transform of Eq. (30) is written as

$$\hat{f}(\omega) = \frac{1}{\sqrt{2\pi}}\exp\left(\frac{i\omega}{2\gamma}\right)\left(\frac{\omega}{2\gamma}\right)^{-1}\sin\left(\frac{\omega}{2\gamma}\right), \quad (31)$$

which corresponds to reflection from a small-amplitude Bragg lattice. In this case, Eq. (21) can be replaced by an equation with two delays:

$$\begin{aligned}{\rm d}A(\tau)/{\rm d}\tau = \gamma\sqrt{\kappa}\,\Bigg[&\exp\left[\frac{1-i\alpha_{\rm g}}{2}G(\tau-T) - \frac{1-i\alpha_{\rm q}}{2}Q(\tau-T)\right] A(\tau-T) \\ &- \exp\left[\frac{1-i\alpha_{\rm g}}{2}G(\tau-T_1) - \frac{1-i\alpha_{\rm q}}{2}Q(\tau-T_1)\right] A(\tau-T_1)\Bigg], \quad (32)\end{aligned}$$

where $T_1 = T + \gamma^{-1}$. The solution of Eq. (32) has the form

$$A(\tau) = c + \gamma\sqrt{\kappa} \int_{\tau-T_1}^{\tau-T} \exp\left[\frac{1-i\alpha_{\rm g}}{2}G(\theta) - \frac{1-i\alpha_{\rm q}}{2}Q(\theta)\right] A(\theta)\,{\rm d}\theta, \tag{33}$$

where c is an arbitrary constant. If $c = 0$, then Eq. (33) coincides with Eq. (21) in which the function $f(\tau)$ is specified by Eq. (30). Substituting $\tau = 0$ and $c = 0$ into Eq. (33), we obtain the following initial condition for the field amplitude:

$$A(0) = \gamma\sqrt{\kappa} \int_{-T_1}^{-T} \exp\left[\frac{1-i\alpha_{\rm g}}{2}G(\theta) - \frac{1-i\alpha_{\rm q}}{2}Q(\theta)\right] A(\theta)\,{\rm d}\theta. \tag{34}$$

Note, however, that, as follows from Eq. (33), the initial condition for Eqs. (32), (26), and (27), unlike Eqs. (25)–(27), does not decay exponentially in time. Therefore, to obtain the correct result when solving this system of equations numerically, it is necessary to choose the correct initial condition satisfying Eq. (34). The meaning of this condition is as follows. Replacing integral equation (21) by delay differential equation (32) is equivalent to replacing integral relation (19), which determines the electromagnetic-field transformation in the spectral filter, by the differential equation which determines the field $A(\tau, z_5)$ at the output of the spectral filter via the input field $A(\tau, z_4)$ with accuracy up to an arbitrary constant. To eliminate such an ambiguity, it is necessary to use initial condition (34).

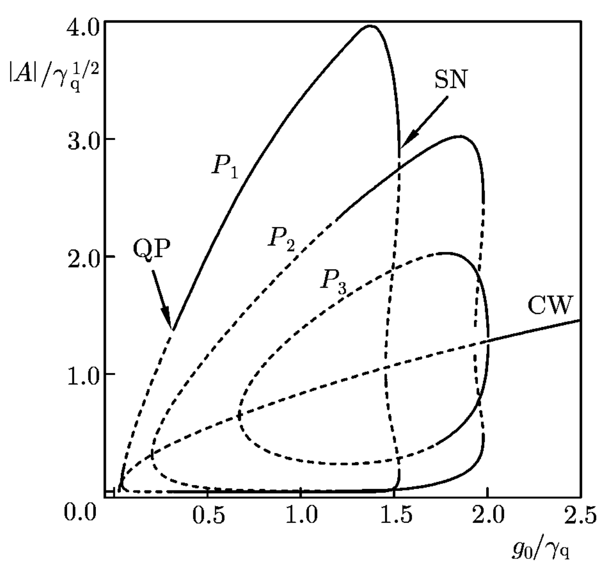

Fig. 2. Solutions of system (25)–(27) with constant and periodic time dependences of the laser-field intensities for $q_0 = 2\gamma_{\rm q}$, $T = 25$ ps, $\gamma^{-1} = 0.4$ ps, $\alpha_{\rm g} = \alpha_{\rm q} = 0$, $s = 5$, $\gamma_{\rm g}^{-1} = 1$ ps, $\gamma_{\rm q}^{-1} = 10$ ps, and $\kappa = 0.5$. Periodic solutions P_1, P_2, and P_3 bifurcate from the stationary solution (marked as CW) at the Hopf bifurcation points. Stable and unstable solutions are denoted by solid and dashed lines, respectively.

Finally, we show results of calculation of the domains of passive mode locking for Eqs. (25)–(27), obtained using the software package DDE-BIFTOOL for numerical analysis of bifurcations of delay differential equations [17]. These results are shown in Fig. 2 for the case where the alpha factors of the amplifying and absorbing media are equal to zero: $\alpha_{\rm g} = \alpha_{\rm q} = 0$. The case of nonzero alpha factors is more complicated and will be considered in another publication. For nonzero alpha factors, a pulse after a cavity roundtrip acquires a phase shift dependent on the electromagnetic-field intensity and thus varying over the pulse. Preliminary calculations show that the presence of such a phase shift has a negative effect on the mode locking and can lead to its break-up at sufficiently large alpha factors. However, since the quantities G and Q enter Eq. (25) for the electromagnetic field with opposite signs, the phase shifts introduced by the amplifying and absorbing segments have opposite signs and, hence, partially compensate for each other if the signs of $\alpha_{\rm g}$ and $\alpha_{\rm q}$ are the same. Numerical calculations show that, for each value of $\alpha_{\rm g}$, there is a certain value of $\alpha_{\rm q}$, for which such compensation is maximum. This situation is most favorable for mode locking and qualitatively resembles the case where the alpha factors are zero.

The pump parameter g_0 of the amplifying medium was chosen as a bifurcation parameter in Fig. 2. Stable and unstable solutions are plotted by solid and dashed lines, respectively. The curve marked CW corresponds to the stationary lasing, i.e., the solution with time-independent intensity of the electromagnetic field. This solution is stable in a small vicinity near the linear threshold of lasing, $g_0/\gamma_{\rm g} = q_0/\gamma_{\rm q} - \ln\kappa$, and in the region of large g_0, where the contribution of the amplifying medium dominates that of the absorbing medium. In the intermediate domain of g_0 values, where the stationary lasing is unstable, solutions with periodic and quasiperiodic temporal dependences of the field intensity are observed. Periodic solutions in Fig. 2 are denoted P_1, P_2, and P_3. The solution P_1 has a period close to the time T of pass through the cavity. Therefore, it corresponds to a fundamental regime of passive mode locking. The solutions P_2 and P_3 have pulse-repetition periods approximately 2 and 3 times smaller than T, respectively. These solutions describe passive mode locking with two and three pulses coexisting in the cavity. The domain of stability of the fundamental mode-locking regime is bounded by two bifurcation points denoted as SN and QP in Fig. 2. The first point corresponds to a saddle-node bifurcation, where two periodic solutions (stable and unstable) merge and disappear. The second point corresponds to the bifurcation of a periodic solution into the one with quasiperiodic electric-field intensity. The latter solution corresponds to the mode locking modulated by the frequency of relaxation oscillations, which, for semiconductor lasers, is usually several times smaller than the repetition rate of mode-locked pulses. The modulation depth increases as the system goes to the left from the point QP. The bifurcation diagram shown in Fig. 2 is in qualitative agreement with the experimental data obtained for a monolithic semiconductor laser [18].

To summarize, we propose a new model, which describes passive mode locking in a semiconductor laser, in the form of a system of delay differential equations (25)–(27). This model is more convenient for analytical study than the earlier model by Gurevich and Khanin [13–16]. It is easy to generalize our model to the case of active or hybrid mode locking. The main approximations at which our model is based are the assumptions of the ring laser geometry and Lorentz line profile of the spectral filter. The approximations of small roundtrip gain and losses, as well as any assumptions on the relation between the pulse length and relaxation timescales of the amplifying and absorbing media, are not used. Therefore, the applicability domain of the model is significantly wider than for the well-known models by New and Haus. For the case of zero alpha factors, we have numerically studied the bifurcations responsible for the appearance and disappearance of the mode locking.

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