

M3P14 EXAMPLE SHEET 1

- 1a. Express 14 as an integer linear combination of 280 and 203.
- 1b. Find, with proof, all solutions to the linear diophantine equation $10x + 25y = 45$.
- 1c. Find a multiplicative inverse of 5 modulo 132.
- 1d. Find an integer congruent to 7 mod 25 and congruent to 1 mod 16.
- 1e. Find, with proof, the smallest nonnegative integer n such that $n \equiv 1 \pmod{3}$, $n \equiv 2 \pmod{5}$, and $n \equiv 3 \pmod{7}$.

2. Least Common Multiples

2a. Let a and b be nonzero integers. Show that there is a unique positive integer m with the following two properties:

- a and b divide m , and
- If n is any number divisible by both a and b , then $m|n$.

The number m is called the *least common multiple* of a and b .

2b. Show that the least common multiple of a and b is given by $\frac{|ab|}{(a,b)}$.

3. Show that the equation $ax \equiv b \pmod{n}$ has no solutions if b is not divisible by (a, n) , and exactly (a, n) solutions in \mathbb{Z}/n otherwise.

4. Let p be a prime, and a be any integer. Show that a^{p^2} is congruent to a modulo p .

5. Let n be a squarefree positive integer, and suppose that for all primes p dividing n , we have $(p-1)|(n-1)$. Show that for all integers a with $(a, n) = 1$, we have $a^n \equiv a \pmod{n}$.

6. Let n be a positive integer. Show that the sum $\sum_{d|n, d>0} \Phi(d)$ is equal to n .

7. (BONUS) Show that the sum $\sum_{p \text{ prime}, p > 0} \frac{1}{p}$ diverges.