Mathematician solves 140-year old mapping problem


The Schwarz-Christoffel formula is used to simplify complex shapes such as aircraft wings

Image by Tukka

A mathematician at Imperial College, London has come up with a solution to a problem that has remained unsolved for nearly 140 years. The Schwarz-Christoffel formula, which was developed in the mid-nineteenth century, is used by mathematicians, engineers and scientists for conformal mapping – the process of transforming a complicated shape into a simpler circular shape to make it easier to analyse. One field in which this technique is well-established is aerodynamics, since it allows the study of airflow patterns around aeroplane wings by simplifying the geometry of the problem. However, use of the Schwarz-Christoffel formula has previously been limited by the fact that it only worked for regular shapes that did not contain any holes.

Now applied mathematician Professor Darren Crowdy has extended the Schwarz-Christoffel formula so that it can be used for mapping more complicated shapes. He says of his work, "this formula is an essential piece of mathematical kit which is used the world over. Now, with my additions to it, it can be used in far more complex scenarios than before. In industry, for example, this mapping tool was previously inadequate if a piece of metal or other material was not uniform all over – for instance, if it contained parts of a different material, or had holes. With my extensions to this formula, you can take account of these differences and map them onto a simple disk shape for analysis in the same way as you can with less complex shapes without any of the holes." Professor Crowdy hopes that his work will increase the number of applications of this type of conformal mapping, both in industry and in academic fields of research.

Geometry used to analyse music

It was over 2,500 years ago that the Greek mathematician Pythagoras first discovered how simple mathematical ratios could be used to describe musical intervals that were pleasing to the ear. The connection between music and mathematics has fascinated scholars ever since, and now new research published in the journal *Science* takes this connection one step further. Three music professors – Clifton Callender of Florida State University, Ian Quinn of Yale University and Dmitri Tymoczko of Princeton University – have developed a tool called "geometrical music theory" which allows music to be analysed using the language of contemporary geometry. Their method involves assigning a mathematical structure to families of chords, rhythms and scales, so that they can be represented by points in complex geometrical spaces. Categorising collections of musical notes in different ways produces a diverse range of geometrical spaces, some of which mathematicians do not even have names for yet.

The researchers claim that their work could be used to create new kinds of musical instruments, toys or tools for visualising music, and that it could even change the way musicians are educated. The method could also be used to find out whether any new scales or chords exist that have yet to be discovered. "Our research offers a variety of tools for understanding and exploring music by drawing upon contemporary mathematics in natural and musically relevant ways," says Callender. "It also provides a way to compare chords, and represents all possible combinations of pitches, including those found in non-Western music and avant-garde works that don't conform to the traditional scales of Western music." Tymoczko