Mechanical Engineering Mathematics ME3.6
Problem Sheet 1: Linear Autonomous Systems

1. Find and classify the critical points of the following systems of linear ODEs. In each case find the general solutions for $x(t)$ and $y(t)$.

Find the lines along which $dy/dx = 0$ and $dy/dx = \infty$. Also determine the sign of dy/dx on the x and y axes.

Putting all of this information together, sketch a few trajectories of each system in the $x - y$ phase plane and use arrows to indicate the direction of increasing t.

(i) $dx/dt = 2x + 5y,\ dy/dt = x - 2y$;

(ii) $dx/dt = -3x + y,\ dy/dt = -x - 3y$;

(iii) $dx/dt = -4x + 3y,\ dy/dt = -2x + y$;

(iv) $dx/dt = -2x - 2y,\ dy/dt = 3x + 2y$;

(v) $dx/dt = -2x - 2y + 3,\ dy/dt = 3x + 2y - 4$.

2. A spring of mass m and stiffness k has damping coefficient γ. Show that the equation of motion can be written in the form

$$dx/dt = y,\ dy/dt = -(\gamma/m)y - (k/m)x.$$

Show that dy/dx is infinite along the x –axis, and dy/dx is a negative constant along the y –axis. Show also that $dy/dx = 0$ when $y = -(k/\gamma)x$.

Classify the critical point of this system, distinguishing between the cases $(\gamma/m)^2 > 4(k/m)$ and $(\gamma/m)^2 < 4(k/m)$.

Sketch the phase plane for the two specific cases:

(i) $(\gamma/m) = 2,\ (k/m) = 3/4$;

(ii) $(\gamma/m) = 1,\ (k/m) = 3/4$.

What is the essential difference between them in terms of the motion of the spring?