M4A32 – Vortex dynamics: Syllabus

The list below provide a broad outline of possible topics to be covered in the course. Topics will be chosen according to student interest, time constraints etc.

1. **Preliminaries:** A derivation of the governing Euler/vorticity equations is given and introduction of general concepts. Streamfunction-vorticity relations in different geometries, as well as the Biot-Savart integral will be considered.

    - conservation of mass, incompressibility, ideal and barotropic fluids
    - dynamics; derivation of the Euler and vorticity equations
    - streamfunction-vorticity relations in various geometries
    - Biot-Savart integrals
    - Kelvin's circulation theorem
    - Helmholtz laws of vortex motion

2. **Point vortex motion:** a study of the simplest model of vorticity, its equilibria, stability and dynamics as well as its Hamiltonian structure; generalizations such as point vortex motion in geometrically complicated domains as well as on the surface of a sphere (this has geophysical and astrophysical applications).

    - equations of motion
    - equilibria and relative equilibria (Thomson vortex arrays, von Karman vortex streets); integrability
    - general dynamics (e.g. vortex leap-frogging, vortex collapse);
    - stability of (relative) equilibria; Kelvin's variational principle
    - Hamiltonian structure; Poisson brackets; symplectic integration
    - vortex motion with boundaries (Kirchhoff-Routh theory); extensions to multiply connected geometries
    - vortex motion on a spherical surface (applications to geophysics and astrophysics)

3. **Vortex patch models:** A study of the next-simplest model of vorticity, this time "distributed" but uniform. Analytical as well as numerical approaches will be discussed.

    - Rankine vortex
    - Kirchhoff elliptical vortex and Moore-Saffman/Kida generalizations

- Point-patch methods; multipolar vortices
- vortex interactions; elliptical vortex model of vortex interactions
- vortex merger
- contour dynamics and numerical methods; filamentation

4. **Vortex sheets:** A study of vortex sheets, important in unsteady potential flows and for understanding aerodynamics and the flight mechanisms.

   - Birkhoff-Rott equation
   - Kelvin-Helmholtz instability
   - ill-posedness of vortex sheets; Moore's analysis
   - unsteady potential flows; basic aerodynamics; Brown-Michael equation; models of shed vortex structures; forces/torques on aerofoils in unsteady potential flow
   - vortex sheet roll-up
   - Kaden spiral; similarity solutions

5. **Vortex rings:** A study of axisymmetric vortex rings (which are ubiquitous in vortex dynamics from smoke rings to jellyfish propulsion) including exact solutions and analytical generalizations.

   - Hill's spherical vortex;
   - Norbury generalizations
   - thin-cored vortex rings

6. **Vortex filaments:** The study of concentrated regions of vorticity with very thin cores.

   - local induction approximation
   - the cut-off method
   - nonlinear Schrödinger equation; soliton solutions
   - Kelvin waves on a vortex filament

7. **Viscous effects:** A study of the effect of viscosity on the vortex structures studied so far.

   - Lamb-Oseen vortex and viscous cores
   - aircraft trailing vortices
   - Burgers vortex and stability
   - Lundgren transformation