ME. 341 (MEng. 3.6) Mathematics

Lecturer: Prof. Darren Crowdy
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Lecture times: Thursdays 11:00-13:00 (MENG 342) – Spring term

Tutorial arrangements: Mondays at 1pm in MENG 651/652. Postgraduate assistant: Ed Tucker (`e.tucker08@imperial.ac.uk`). Regular problem sheets.

Problems sheets: Regular problem sheets should be worked through as the course material is presented. The sheets will be made available at:
`www2.imperial.ac.uk/~dgcrowdy` (click on "Classes").
Solutions will be made available in due course.

Assessment: One 3-hour written examination at the end of session consisting of 6 questions. Students should answer 5 questions. Questions are marked out of 20.

Aims: To show how mathematics can be used to characterise and model the qualitative behaviour of a system without solving in detail the equations controlling the evolution of the system.
Objectives: By the end of the course, students will be able to :

- perform a critical point analysis;
- extract asymptotic behaviour from non-linear ordinary differential equations;
- use Fourier transforms to find particular solutions of linear ordinary differential equations;
- understand the basic ideas of chaos.

Recommended texts:

- Elementary differential equations and boundary-value problems,
  Boyce & DiPrima, (Wiley)
- Advanced engineering mathematics, Kreyszig, (Wiley)
- Advanced engineering mathematics, O'Neil, (Brooks/Cole)
- Mathematical methods for scientists & engineers, McQuarrie,
  (University Science)
- Oscillations, waves and chaos in chemical kinetics, Scott, (Oxford)
- The Fourier integral and its applications, Papoulis, (McGraw-Hill)

- Chaos in dynamical systems, Ott, (Cambridge)
- Chaotic vibrations, Moon, (Wiley)

Syllabus:

Linear Autonomous Systems

Examples including small oscillations of simple pendulum, damped and undamped; decaying and periodic oscillations. Use of the phase plane to visualize solutions. Classification of critical points using eigenvalues. Sketching of trajectories using information including eigenvectors.

Nonlinear Autonomous Systems

Large oscillations of simple pendulum. Use of phase plane to examine behaviour of trajectories near critical points. Trajectory laws. Role of Jacobian matrix. More complicated nonlinear examples relevant to a number of physical applications. Application to PDEs.

Fourier Transforms

Definition and relation to Fourier series. Elementary examples. Basic properties. Convolution theorem. The delta-function and its properties.

Finite Energy Signals

Definition. Parseval/energy theorem. Energy spectrum, autocorrelation, and the relationship between them. Autocorrelation properties. Examples including finite energy impulse train.

Finite Power Signals

Definition. Power spectrum, autocorrelation, and the relationship between them. Autocorrelation properties. Examples including infinite energy impulse train.

Introduction to Chaotic Systems

Pendulum with vertically oscillated pivot point: stability of critical points; numerical investigation showing period doubling phenomenon. 1D maps: fixed points and their stability; cobweb diagrams, period doubling. The Logistic Map as an example of a chaotic system: numerical experiments; analysis of period-doubling; transformation to the tent map.