

Analytic Number Theory

Test No 2, 17 March 2016

Question 1. Find the Euler product representation of the Dirichlet series $\sum_{n=1}^{\infty} f(n)n^{-s}$ where

$$f(n) = \begin{cases} (-1)^{(n-1)/2} & \text{for } n \text{ odd} \\ 0 & \text{for } n \text{ even} \end{cases}$$

What is the half-plane of validity of the formula?

Question 2. By using Euler products, or otherwise, show that for every s with $\operatorname{Re}(s) > 3$ we have

$$\frac{\zeta^3(s)}{\zeta(2s)} = \sum_{n=1}^{\infty} d(n^2)n^{-s},$$