Implied correlation in CDO tranches: a Paradigm to be handled with care.

Roberto Torresetti   Damiano Brigo   Andrea Pallavicini

Credit Models, Banca IMI, Milan.

Milan, Nov 22, 2006

Available at www.ssrn.com

Abstract

We illustrate the two main types of implied correlation one may obtain from market CDO tranche spreads. Compound correlation is more consistent at single tranche level but for some market CDO tranche spreads cannot be implied. Base correlation is less consistent but more flexible and can be implied for a much wider set of CDO tranche market spreads. Furthermore, base correlation is more easily interpolated and leads to the possibility to price non-standard detachments. Even so, Base correlation may lead to negative expected tranche losses, thus violating basic no-arbitrage conditions. We illustrate these features with numerical examples.

JEL Classification: G16

Keywords: Implied Correlation, Base Correlation, Compound Correlation, Expected Tranche Loss, DJ iTraxx, CDX, CDO tranche, back-test, no-arbitrage conditions.

1. Compound Correlation.

Compound correlation is a first paradigm for implying credit default dependence from liquid market data. This approach consists in linking defaults across single names through a Gaussian copula where all the correlation parameters are collapsed to one. One then finds the value of the correlation parameter matching, for each quoted tranche attachment and detachment (strike), the relevant CDO tranche spread. When plotting such correlation against the strikes one obtains a (compound) correlation smile.

The market data we take as inputs, namely reference index term structure and 10y tranche spreads, are detailed in Table 1. An example of the compound correlation skew we imply from this set of market data is given below in Figure 1.

Notice that in Figure 1 there is no bar corresponding to the 6%-9% tranche: from the market spread of the tranche, given the reference index term, we cannot imply a compound correlation. We see in the following how this problem is not at all trivial, in that we face quite often market spreads where we cannot imply the compound correlation.

Table 1:
Index and Tranche Market Spread 3rd-aug-2005

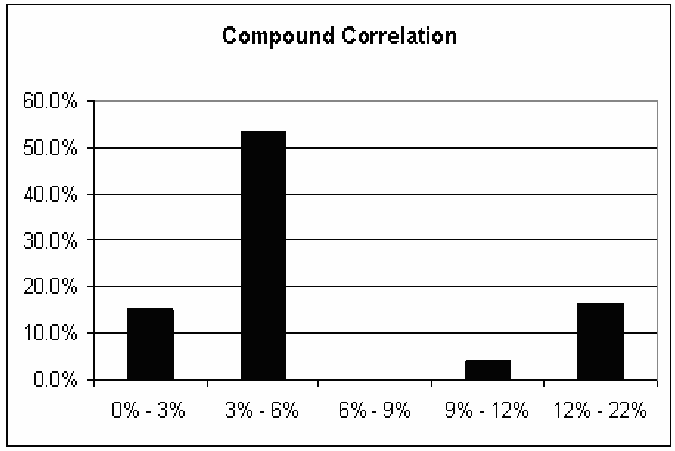
Reference Swap: Itraxx Europe S5

maturity	ref index
3y	21 bp
5y	36 bp
7y	46 bp
10y	56 bp

Tranche Spread: Itraxx Europe S5 10y

tranche	running	upfront
0-3	5.00%	49+.00%
3-6	3.60%	0.00%
6-9	0.82%	0.00%
9-12	0.46%	0.00%
12-22	0.31%	0.00%

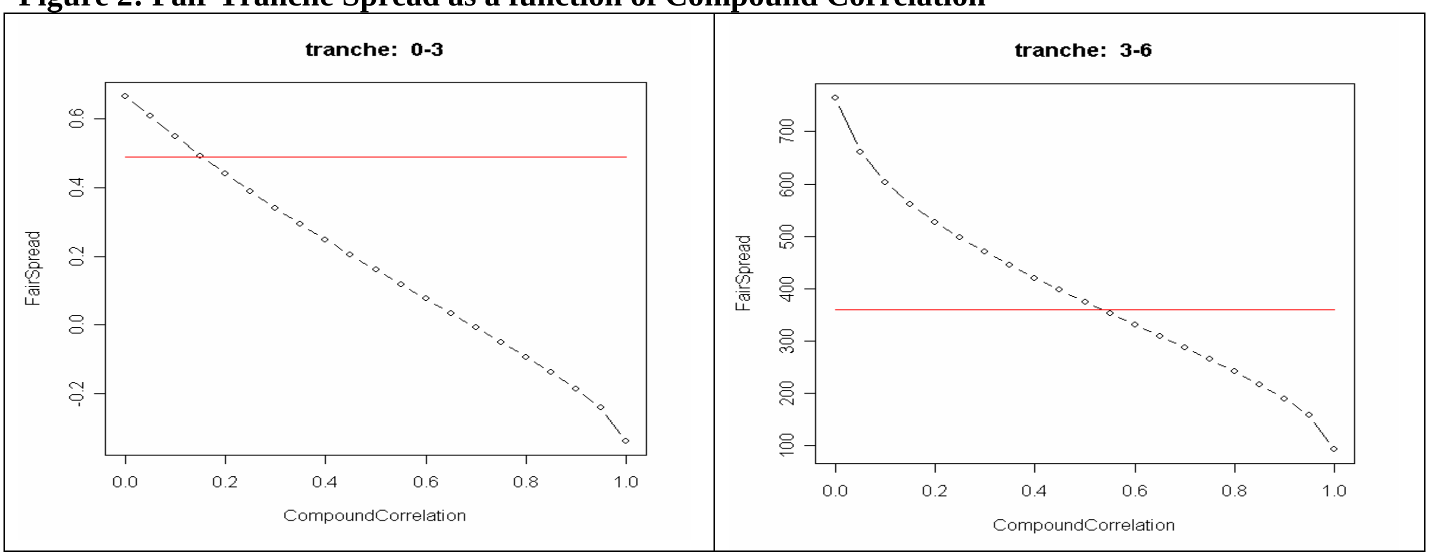
Figure 1: Compound Correlation Skew

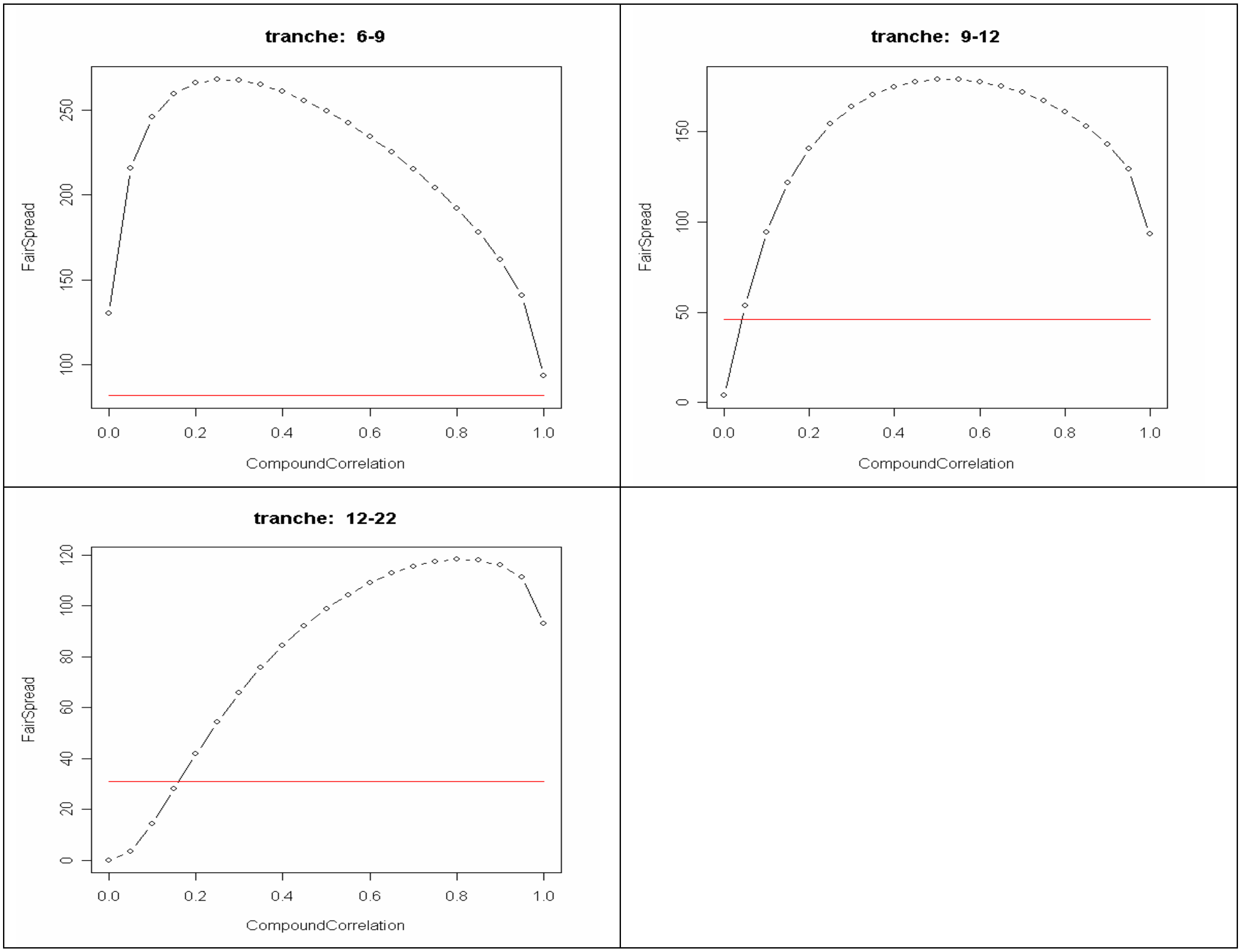


2. Existence and monotonicity of market spread as a function of compound correlation

We have just seen that on a particular date we cannot imply a compound correlation from the market spread of the 6-9 tranche. We investigate further this date plotting in Figure 2 the fair market spread as a function of the compound correlation: the equity tranche is quoted upfront (0.25 means 25%) and all other tranches are quoted in number of running basis points (123 means 1.23% per annum). The red flat line is the level of the market spread. The fair tranche spread is obtained dividing the NPV of the default leg by an annuity factor (the NPV of the premium leg of a tranche with spread equal to 100%).

Figure 2: Fair Tranche Spread as a function of Compound Correlation





We note that:

1) For certain tranches, from the unique market spread we can imply more than one compound correlation, although this does not happen in our example of Figure 2 (the flat red line crosses the dotted black line at most in one point).

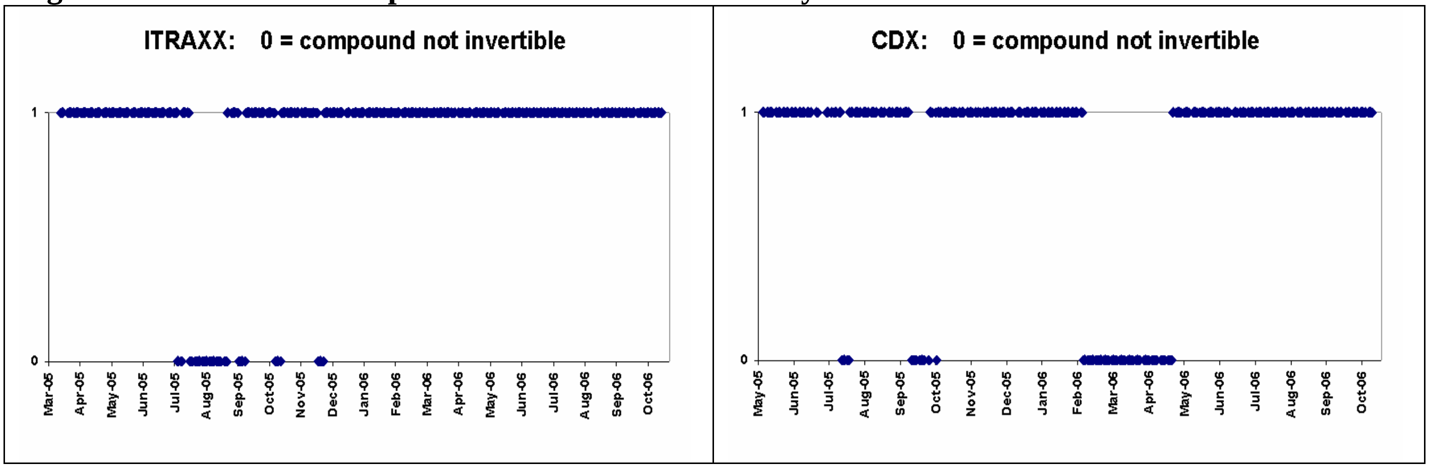
2) Given a market spread we are not always guaranteed we can imply a compound correlation, as we see for example in the 6-9 tranche of Figure 2 (there is no intersection between the flat red line and the dotted black line).

3. Historical Relevance of the Invertibility Limitations of the Compound Correlation

We have seen before that on 3rd-aug-2005 we cannot imply a compound correlation for the 6-9 tranche. We now check how often in the past this kind of problem occurred, in that we look for past tranche spreads from which we cannot imply a compound correlation.

In figure 3 we plot the dates where the 10y tranches on iTraxx and CDX are not invertible. For the iTraxx all cases of non-invertibility are imputable to the 6%-9% tranche, whereas for the CDX all cases of non-invertibility are imputable to the 7%-10% tranche. In all cases the market spread is too small to be inverted: the same problem we had for the 6-9 tranche in Figure 2.

Figure 3: Back-test of Compound Correlation Invertibility



4. Base Correlation

The tranched loss can be written as:

[1] $$L(A,B) = \min[B - A, \max[L - A, 0]]/(B - A)$$

where L is the portfolio Loss at maturity and A and B are the attachment and detachment points.

With a little manipulation we can write:

[2] $$\begin{aligned} L(A,B) &= (\max[L - A, 0] - \max[L - B, 0])/(B - A) \\ &= (-\min[0, A - L] + \min[0, B - L])/(B - A) \\ &= (-\min[L, A] + L + \min[L, B] - L)/(B - A) \\ &= (B \cdot L(0,B) - A \cdot L(0,A))/(B - A) \end{aligned}$$

Thus the tranched loss can be rewritten as the difference between two tranched equity losses. Iteratively, given the base correlation on the detachment A, we look for the base correlation on the detachment B such that the net present value of the A-B tranche is set to 0.

The only problem left with this approach is that we are using a different correlation parameters in the calculation of the expected loss for the tranches $L(0, A)$ and $L(0, B)$ concurring to the same payoff. Valuing different parts of the same payoff with different model parameters (correlations) clearly leads to inconsistencies. This means that we are not guaranteed that this expectation is a strictly increasing function of time.

5. Is Base Correlation immune from inconsistencies in practice?

In the left hand graph in Figure 4 we plot the Base Correlation calibrated to the market data in Table 1. In the right hand graph in figure 4 we plot the Expected Equity Tranche Loss for the various detachment points as a function of time.

$$E[L(0,B)] \quad , \quad B = 3\%, 6\%, 9\%, 12\%, 22\% .$$

From these expectations, using equations [2], we can compute the Expected Tranche Loss, plotted in Figure 5, as a function of time:

$$E[L(A,B)] = (B \cdot E[L(0,B)] - A \cdot E[L(0,A)]) / (B - A)$$

Figure 4: Base Correlation and Expected Equity Tranche Loss

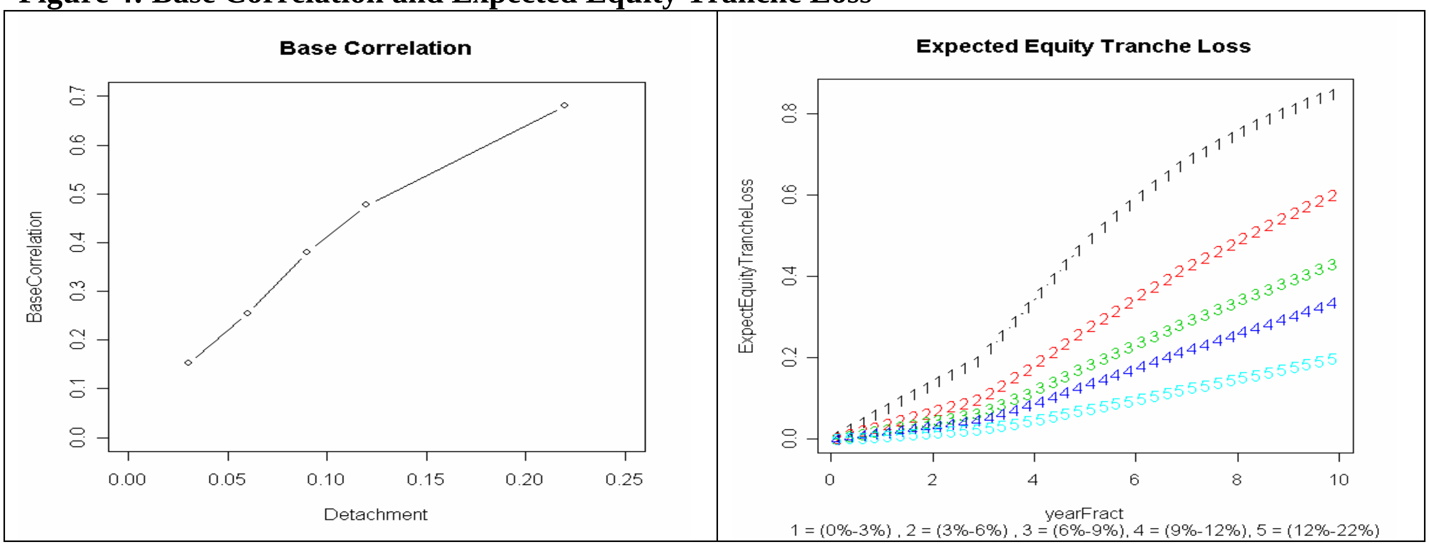
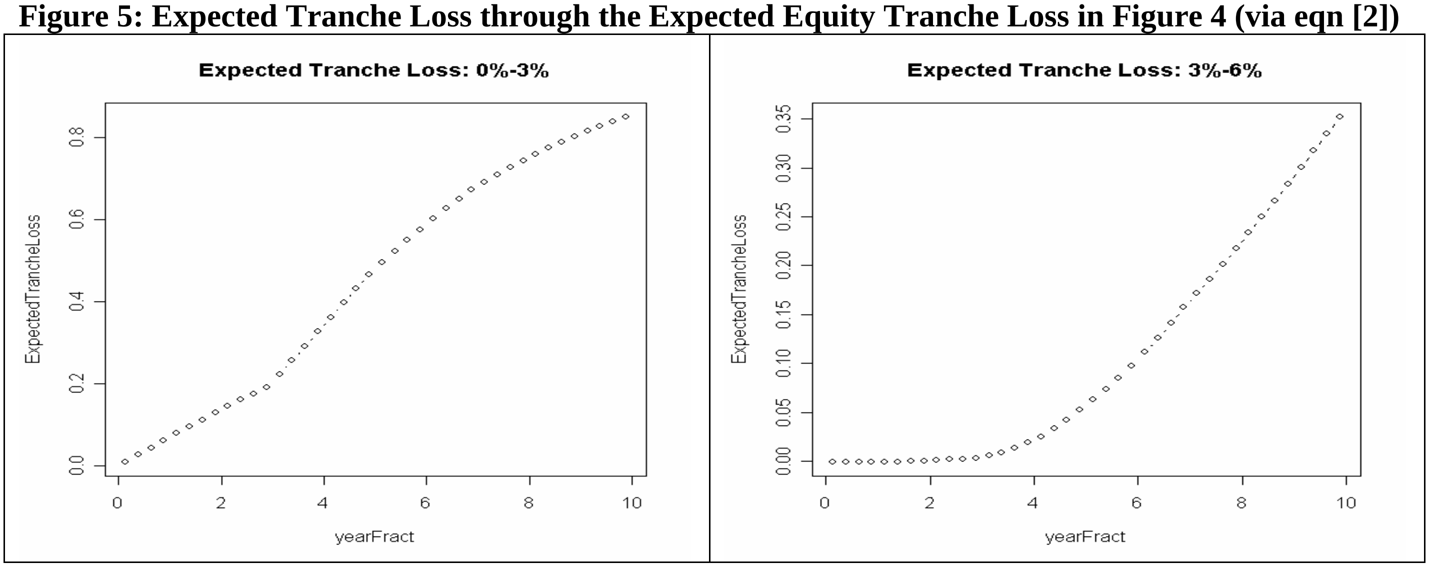
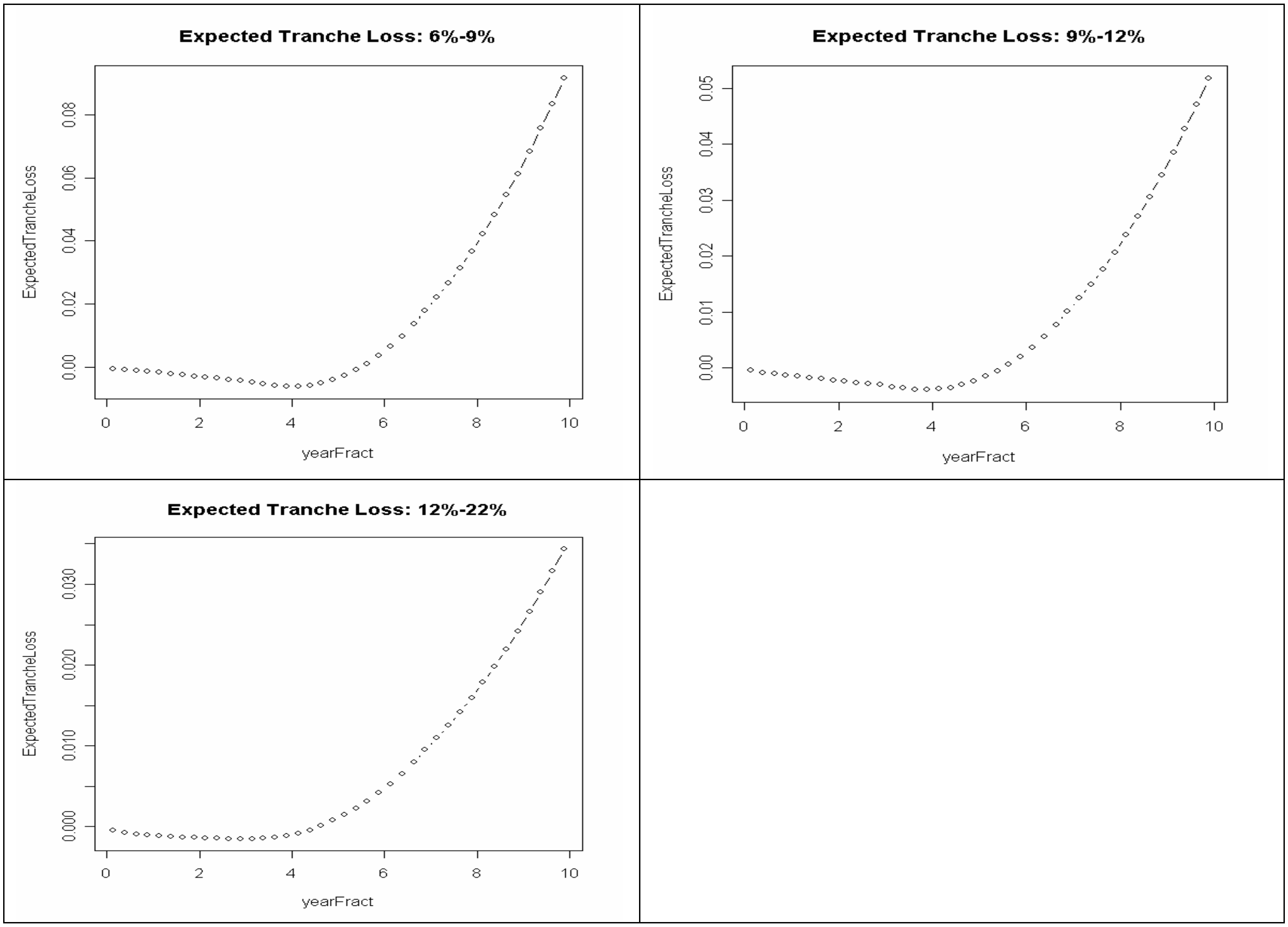


Figure 5: Expected Tranche Loss through the Expected Equity Tranche Loss in Figure 4 (via eqn [2])





From Figure 4 we note that the base correlation is a much smoother function of detachments than compound correlation. Also, to price a non-standard tranche, say a 4%-15% tranche, we can interpolate the non-standard attachment (4%) and detachment (15%) whereas with the compound correlation we do not know exactly what to interpolate (since with compound correlation there is a unique correlation associated to each tranche, i.e. correlation is associated with two points rather than a single one).

As we can see from our examples, also the base correlation approach is not immune from inconsistencies. In fact in Figure 5 we note that for the 6-9, 9-12 and 12-22 tranches the expected tranche loss becomes initially slightly negative. This inconsistency arises from the different base correlations we use in equation [2] to compute the two expected tranche loss terms in A and B.

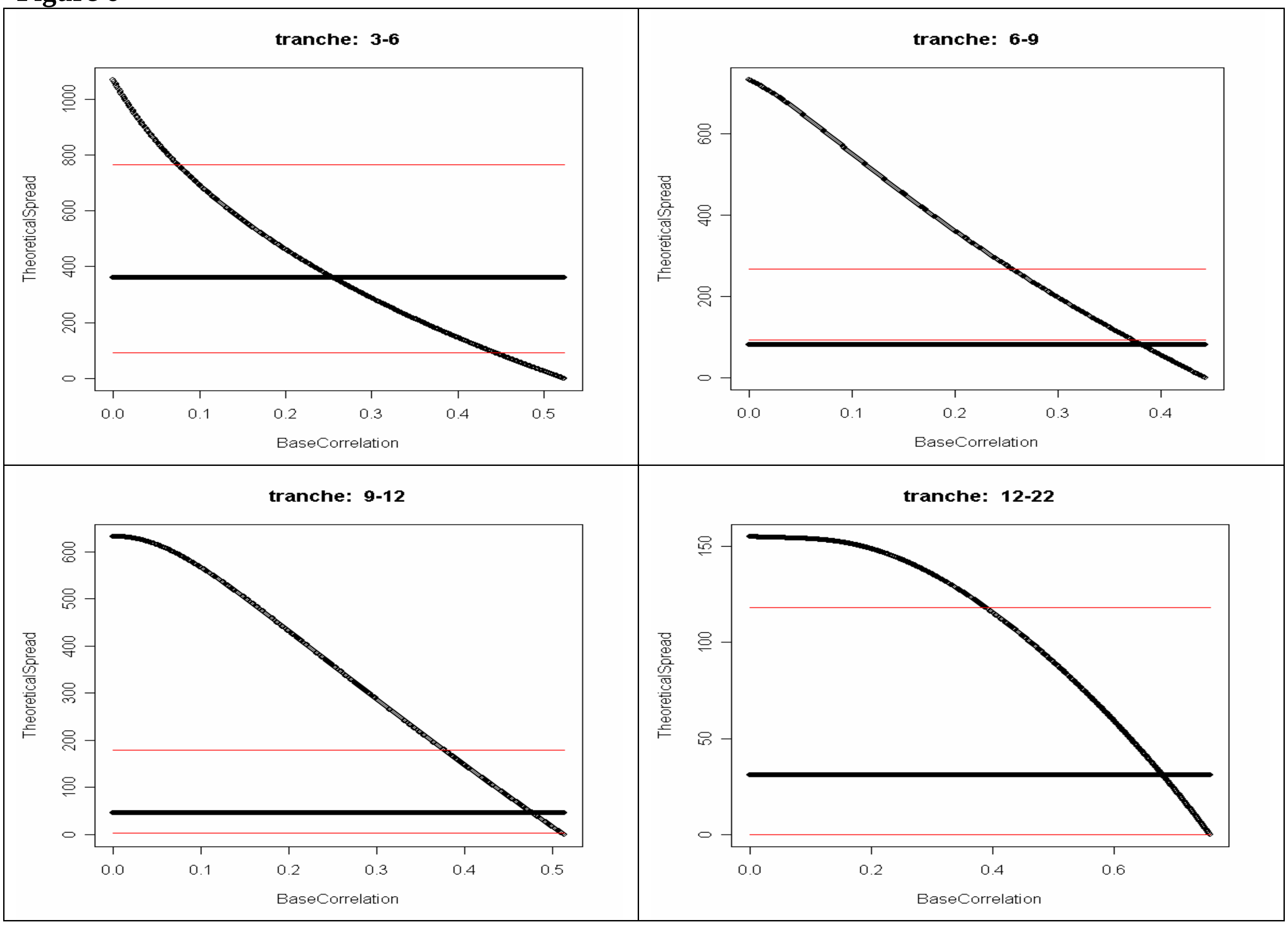
6. Is Base Correlation a solution to the inconsistencies of Compound Correlation?

The answer is in the affirmative and this can be clearly seen for example in Figure 6 where we plot the fair tranche spread as a function of the base correlation on the detachment point for each tranche, given the base correlation on the attachment point set equal to the calibrated base in Figure 4.

This gives us an idea of the range of the tranche spread we can calibrate using base correlation. These plots of Figure 6 can be compared with the plots in Figure 2, showing the fair tranche spread as a function of compound correlation.   In Figure 6 the thick black line is flat at the level of the market spread for the tranche. The two thin red lines are the minimum and maximum spread we are able to obtain by varying compound correlation.

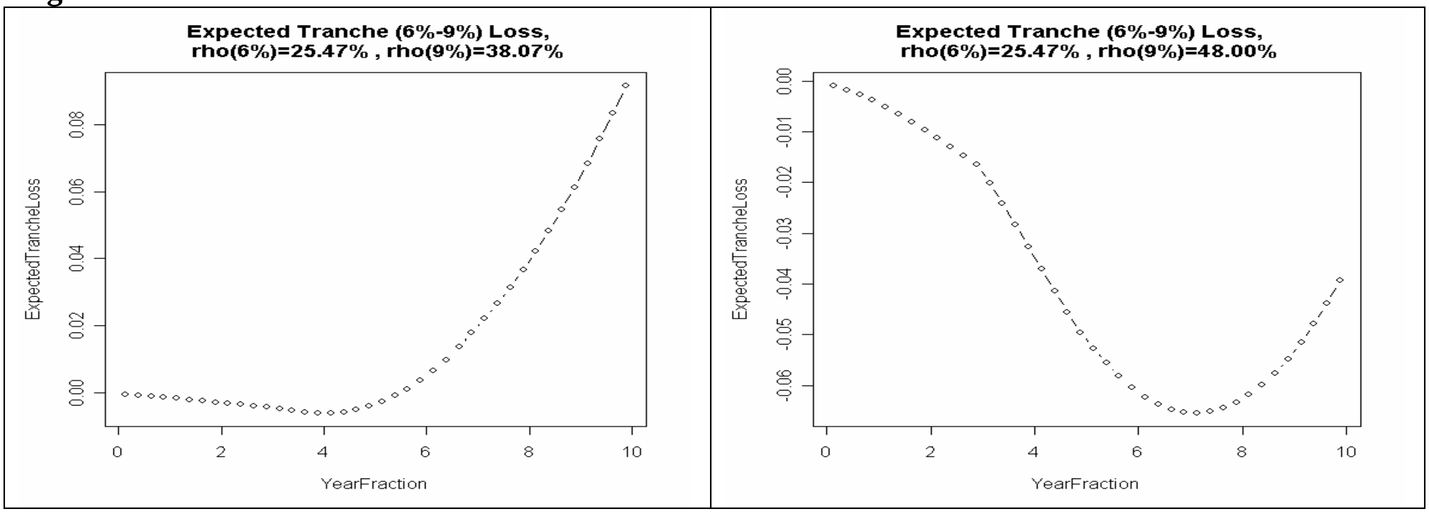
We note that for each tranche the fair spread is a monotonic function of the base correlation on the detachment point and also the range of market spread that can be attained by varying base correlation is much wider than the corresponding one for compound correlation. Consider for example the 6-9 tranche: from Figure 2 the tranche spread that can be inverted in a compound correlation setting lies between 93 and 268 bps, whereas from Figure 6 the tranche spread that can be inverted in a base correlation setting lies in the wider range between 0 and 732 bps.

Figure 6



In Figure 6 we did not plot the market spread for all base correlations between 0 and 1 because beyond a certain point the fair tranche spread becomes negative. Recall once again that in Equation [2] we use two different correlation parameters for different parts of the same payoff: when these two correlations are very different from each other (the detachment correlation is much higher than the attachment one) the inconsistency of a negative expected tranche loss becomes more evident.

Figure 7



Consider for example the 6-9 tranche. In Figure 7 we plot in the abscissas the year fraction of the tranche payment dates and in the ordinates the Expected Tranche (6-9) Loss. For both graphs in Figure 7 the tranche attachment correlation is the calibrated base on the 6% detachment. The tranche detachment (9%) correlation is set to the calibrated base on the left hand graph (38.07%) and to an arbitrarily high level (48%) on the right hand graph.

7. Conclusions

Calibrating base correlations by tranching the loss as shown in equation [2] without calibrating first single compound correlations solves the two main issues concerning compound correlation calibration.

Indeed, as illustrated in Figures 1 and 7 by calibrating directly base correlations:

1) we get a monotonic mapping of the base correlation parameter into tranche spreads;
2) we can invert a wider range of tranche spreads into a base correlation parameter;
3) we can price bespoke detachments by interpolating the base correlation across detachments.

Even so, base correlation needs to be handled with care, since it may lead to negative loss distributions, thus violating basic no-arbitrage constraints.

Alternative copula specifications are possible. Indeed, Hull and White (2004) show that on a particular date the "double-t copula" can consistently reproduce tranche spreads without skew in the correlation parameter. Independent tests of ours show that the skew resurfaces at later dates (from May 2005 on).

A more model independent approach to tranche interpolation and pricing consists in implying expected tranche losses without assuming any model, see for example Walker (2006) or Torresetti Brigo and Pallavicini (2006b). An explicit model implying a dynamics for dependence across defaults, absent in the copula case, is given in Brigo Pallavicini and Torresetti (2006). Finally, we note that the loss distribution of the pool under the risk neutral measure, which has been discussed in this paper, is different from the actual loss distribution in the objective measure, as pointed out for example in Torresetti, Brigo and Pallavicini (2006a).

Acknowledgement

We are grateful to Martin Krekel for helpful comments on an earlier draft.

References

Brigo, D., Pallavicini, A., and Torresetti, R. (2006). Calibration of CDO Tranches with the dynamical Generalized-Poisson Loss model.
Working paper available at SSRN, http://ssrn.com/abstract=900549

Hull, J., White, A., (2004). Valuation of a CDO and an n-th to Default CDS Without Monte Carlo Simulation, Journal of Derivatives, 12, 2.

Li, D., (2000) On Default Correlation: A Copula Function Approach.
Journal of Fixed Income.

McGinty, L., Ahluwalia, R., (2004) A Model for Base Correlation Calculation, JP Morgan technical document.

Torresetti, R., Brigo, D., and Pallavicini, A. (2006a). Risk Neutral versus Objective Loss Distribution and CDO Tranches Valuation.
Working paper available at SSRN, http://ssrn.com/abstract=900784

Torresetti, R., Brigo, D., and Pallavicini, A. (2006b). Implied Expected Tranched Loss Surface from CDO Data.
Available at http://papers.ssrn.com/sol3/papers.cfm?abstract_id=933291

Walker, M. (2006). CDO models. Towards the next generation: incomplete markets and term structure.
Available at http://defaultrisk.com/pp_crdrv109.htm