Counterparty risk, collateral and funding across asset classes with arbitrage-free dynamical models

London Graduate School in Mathematical Finance - MF6 Course

London, Nov 6,7,13,14, 2012

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Agenda I

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   - http://www.damianobrigo.it/cvabook.pdf
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   - Market implied default probabilities
   - CDS and Defaultable Bonds: Intensity Models
   - Intensity models: Constant Intensity
   - Intensity models: Deterministic Intensity
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Agenda II

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- CVA Restructuring
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Agenda III

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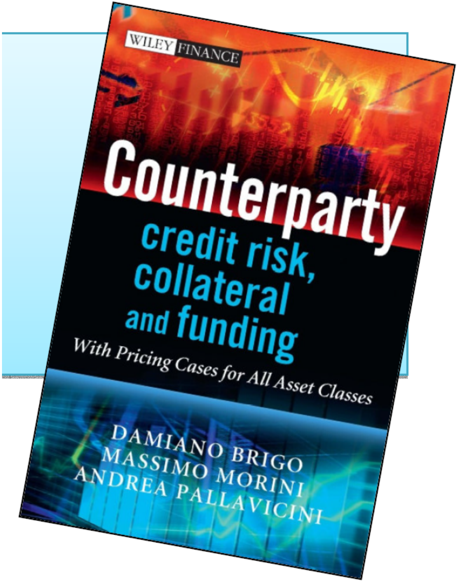
Agenda IV

Course based on 2002-13 Research and on Book



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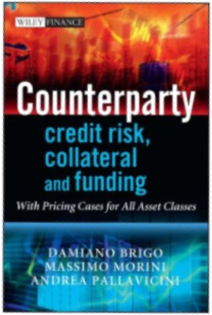
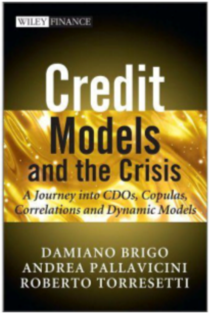
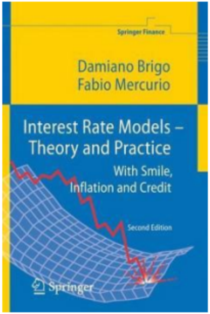






I have been working on Credit Risk and CVA since 2002.

Intro to Basic Credit Risk Products and Models

Before dealing with the current topical issues of Counterparty Credit Risk, CVA, DVA and Funding, we need to introduce some basic elements of Credit Risk Products and Credit Risk Modelling.

We now briefly look at:

- Products: Credit Default Swaps (CDS) and Defaultable Bonds
- Payoffs and prices of such products
- Market implied $\mathbb{Q}$ probabilities of default defined by such models
- Intensity models and probabilities of defaults as credit spreads
- Credit spreads as possibly constant, curved or even stochastic
- Credit spread volatility (stochastic credit spreads)

Defaultable (corporate) zero coupon bonds

We started this course by defining the zero coupon bond price $P(t, T)$. Similarly to $P(t, T)$ being one of the possible fundamental quantities for describing the interest-rate curve, we now consider a defaultable bond $\bar{P}(t, T)$ as a possible fundamental variable for describing the defaultable market.

DEFAULT FREE		
time t		time T
:	$\longleftarrow$	:
$P(t, T)$		1

with DEFAULT		
time t		time T :
:	$\longleftarrow$	NO DEFAULT: 1
$\bar{P}(t, T)$		DEFAULT: 0

When considering default, we have a random time τ representing the time at which the bond issuer defaults. $\boxed{\tau : \text{Default time of the issuer}}$

Defaultable (corporate) zero coupon bonds I

The value of a bond issued by the company and promising the payment of 1 at time T, as seen from time t, is the risk neutral expectation of the discounted payoff
BondPrice = Expectation[Discount x Payoff]

$$P(t,T) = \mathbb{E}\{D(t,T)\ 1\ |\mathcal{F}_t\}, \qquad \mathbf{1}_{\{\tau>t\}}\bar{P}(t,T) := \mathbb{E}\{D(t,T)\mathbf{1}_{\{\tau>T\}}|\mathcal{G}_t\}$$

where $\mathcal{G}_t$ represents the flow of information on whether default occurred before t and if so at what time exactly, and on the default free market variables (like for example the risk free rate r_t) up to t. The filtration of default-free market variables is denoted by $\mathcal{F}_t$. Formally, we assume

$$\mathcal{G}_t = \mathcal{F}_t \vee \sigma(\{\tau \le u\},\ 0 \le u \le t).$$

D is the stochastic discount factor between two dates, depending on interest rates, and represents discounting.

Defaultable (corporate) zero coupon bonds II

The "indicator" function $\mathbf{1}_{\text{condition}}$ is 1 if "condition" is satisfied and 0 otherwise. In particular, $\mathbf{1}_{\{\tau > T\}}$ reads 1 if default τ did not occur before T, and 0 in the other case.

We understand then that (ignoring recovery) $\mathbf{1}_{\{\tau > T\}}$ is the correct payoff for a corporate bond at time T: the contract pays 1 if the company has not defaulted, and 0 if it defaulted before T, according to our earlier stylized description.

Defaultable (corporate) zero coupon bonds

If we include a recovery amount REC to be paid at default τ in case of early default, we have as discounted payoff at time t

$$D(t,T)\mathbf{1}_{\{\tau>T\}} + \text{Rec}\, D(t,\tau)\mathbf{1}_{\{\tau\le T\}}$$

If we include a recovery amount REC paid at maturity T, we have as discounted payoff

$$D(t,T)\mathbf{1}_{\{\tau>T\}} + \text{Rec}\, D(t,T)\mathbf{1}_{\{\tau\le T\}}$$

Taking $\mathbb{E}[\cdot|\mathcal{G}_t]$ on the above expressions gives the price of the bond.

Fundamental Credit Derivatives: Credit Default Swaps

Credit Default Swaps are basic protection contracts that became quite liquid on a large number of entities after their introduction.

CDS's are now actively traded and have become a sort of basic product of the credit derivatives area, analogously to interest-rate swaps and FRA's being basic products in the interest-rate derivatives world.

As a consequence, the need is not to have a model to be used to value CDS's, but rather to consider a model that can be *calibrated* to CDS's, i.e. to take CDS's as model inputs (rather than outputs), in order to price more complex derivatives.

As for options, single name CDS options have never been liquid, as there is more liquidity in the CDS index options. We may expect models will have to incorporate CDS index options quotes rather than price them, similarly to what happened to CDS themselves.

Fundamental Credit Derivatives: CDS's

A CDS contract ensures protection against default. Two companies "A" (Protection buyer) and "B" (Protection seller) agree on the following.
If a third company "C" (Reference Credit) defaults at time τ, with $T_a < \tau < T_b$, "B" pays to "A" a certain (deterministic) cash amount L_{GD}.
In turn, "A" pays to "B" a rate R at times $T_{a+1}, \ldots, T_b$ or until default.
Set $\alpha_i = T_i - T_{i-1}$ and $T_0 = 0$.

Protection Seller B	$\rightarrow$ $\leftarrow$	protection L_{GD} at default τ_C if $T_a < \tau_C \leq T_b$ rate R at $T_{a+1}, \ldots, T_b$ or until default τ_C	$\rightarrow$ $\leftarrow$	Protection Buyer A

(protection leg and premium leg respectively). The cash amount L_{GD} is a *protection* for "A" in case "C" defaults. Typically $\mathrm{L}_{\mathrm{GD}} =$ notional, or "notional - recovery" $= 1 - \mathrm{R}_{\mathrm{EC}}$.

Fundamental Credit Derivatives: CDS's

A typical stylized case occurs when “A” has bought a corporate bond issued by “C” and is waiting for the coupons and final notional payment from “C”: If “C” defaults before the corporate bond maturity, “A” does not receive such payments. “A” then goes to “B” and buys some protection against this risk, asking “B” a payment that roughly amounts to the loss on the bond (e.g. notional minus deterministic recovery) that A would face in case “C” defaults.

Or again ”A” has a portfolio of several instruments with a large exposure to counterparty ”C”. To partly hedge such exposure, ”A” enters into a CDS where it buys protection from a bank ”B” against the default of ”C”.

Fundamental Credit Derivatives: CDS's

Protection Seller B	$\rightarrow$ $\leftarrow$	protection L_{GD} at default τ_C if $T_a < \tau_C \leq T_b$ rate R at $T_{a+1}, \ldots, T_b$ or until default τ_C	$\rightarrow$ $\leftarrow$	Protection Buyer A

Formally we may write the (Running) CDS discounted payoff to "B" at time $t < T_a$ as

$$\Pi_{\mathrm{RCDS}_{a,b}}(t) := D(t,\tau)(\tau - T_{\beta(\tau)-1})R\mathbf{1}_{\{T_a<\tau<T_b\}} + \sum_{i=a+1}^{b} D(t,T_i)\alpha_i R\mathbf{1}_{\{\tau>T_i\}}$$
$$-\mathbf{1}_{\{T_a<\tau\leq T_b\}} D(t,\tau)\,\mathrm{L}_{\mathrm{GD}}$$

where $T_{\beta(\tau)}$ is the first of the T_i's following τ.

CDS payout to Protection seller (receiver CDS)

The 3 terms in the payout are as follows (they are seen from the protection seller, receiver CDS):

- Discounted Accrued rate at default : This is supposed to compensate the protection seller for the protection he provided from the last T_i before default until default τ:

$$D(t,\tau)(\tau - T_{\beta(\tau)-1})R\mathbf{1}_{\{T_a<\tau<T_b\}}$$

- CDS Rate premium payments if no default: This is the premium received by the protection seller for the protection being provided

$$\sum_{i=a+1}^{b} D(t,T_i)\alpha_i R\mathbf{1}_{\{\tau>T_i\}}$$

- Payment of protection at default if this happens before final T_b

$$-\mathbf{1}_{\{T_a<\tau\leq T_b\}}D(t,\tau)\ \mathrm{L}_{\mathrm{GD}}$$

These are random discounted cash flows, not yet the CDS price.

CDS's: Risk Neutral Valuation Formula

Denote by $\mathrm{CDS}_{a,b}(t, R, \mathrm{L}_{\mathrm{GD}})$ the time t *price* of the above Running standard CDS's *payoffs*.

As usual, the price associated to a discounted payoff is its *risk neutral expectation*.

The resulting pricing formula depends on the assumptions on interest-rate dynamics and on the default time τ (reduced form models, structural models...).

CDS's: Risk Neutral Valuation

In general by risk-neutral valuation we can compute the CDS price at time 0 (or at any other time similarly):

$$\mathrm{CDS}_{a,b}(0, R, \mathrm{L}_{\mathrm{GD}}) = \mathbb{E}\{\Pi_{\mathrm{RCDS}_{a,b}}(0)\},$$

with the CDS discounted payoffs defined earlier. As usual, $\mathbb{E}$ denotes the risk-neutral expectation, the related measure being denoted by $\mathbb{Q}$.

However, we will not use the formulas resulting from this approach to price CDS that are already quoted in the market. *Rather, we will invert these formulas in correspondence of market CDS quotes to calibrate our models to the CDS quotes themselves. We will give examples of this later.*

Now let us have a look at some particular formulas resulting from the general risk neutral approach through some simplifying assumptions.

CDS Model-independent formulas

Assume the stochastic discount factors $D(s,t)$ to be independent of the default time τ for all possible $0 < s < t$. The price of the premium leg of the CDS at time 0 is:

$$\begin{aligned}\text{PremiumLeg}_{a,b}(R) &= \mathbb{E}[D(0,\tau)(\tau - T_{\beta(\tau)-1})R\mathbf{1}_{\{T_a<\tau<T_b\}}] + \\ &\quad + \sum_{i=a+1}^{b} \mathbb{E}[D(0,T_i)\alpha_i R\mathbf{1}_{\{\tau\ge T_i\}}] \\ &= \mathbb{E}\left[\int_{t=0}^{\infty} D(0,t)(t - T_{\beta(t)-1})R\mathbf{1}_{\{T_a<t<T_b\}}\delta_\tau(t)dt\right] \\ &\quad + \sum_{i=a+1}^{b} \mathbb{E}[D(0,T_i)]\alpha_i R\,\mathbb{E}[\mathbf{1}_{\{\tau\ge T_i\}}] =\end{aligned}$$

For those who don't know the theory of distributions (Dirac's delta etc), read $\delta_\tau(t)dt = \mathbf{1}_{\{\tau\in[t,t+dt]\}}$.

CDS Model-independent formulas

$$\text{PremiumLeg}_{a,b}(R) = \int_{t=T_a}^{T_b} \mathbb{E}[D(0,t)(t - T_{\beta(t)-1})R\,\delta_\tau(t)dt] +$$

$$+ \sum_{i=a+1}^{b} P(0,T_i)\alpha_i R\,\mathbb{Q}(\tau \geq T_i) =$$

$$= \int_{t=T_a}^{T_b} \mathbb{E}[D(0,t)](t - T_{\beta(t)-1})R\,\mathbb{E}[\delta_\tau(t)dt] + \sum_{i=a+1}^{b} P(0,T_i)\alpha_i R\,\mathbb{Q}(\tau \geq T_i)$$

$$= R\int_{t=T_a}^{T_b} P(0,t)(t - T_{\beta(t)-1})\mathbb{Q}(\tau \in [t, t+dt)) +$$

$$+R\sum_{i=a+1}^{b} P(0,T_i)\alpha_i \mathbb{Q}(\tau \geq T_i),$$

where we have used independence in factoring terms. Again, read $\delta_\tau(t)dt = \mathbf{1}_{\{\tau\in[t,t+dt]\}}$

CDS Model-independent formulas

We have thus, by rearranging terms and introducing a "unit-premium" premium leg (sometimes called "DV01", "Risky duration" or "annuity"):

$$\text{PremiumLeg}_{a,b}(R; P(0,\cdot), \mathbb{Q}(\tau > \cdot)) = R\ \text{PremiumLeg1}_{a,b}(P(0,\cdot), \mathbb{Q}(\tau > \cdot))$$

$$\text{PremiumLeg1}_{a,b}(P(0,\cdot), \mathbb{Q}(\tau > \cdot)) := -\int_{T_a}^{T_b} P(0,t)(t - T_{\beta(t)-1})d_t \boxed{\mathbb{Q}(\tau \geq t)}$$

$$+ \sum_{i=a+1}^{b} P(0,T_i)\alpha_i \boxed{\mathbb{Q}(\tau \geq T_i)}$$

This model independent formula uses the initial market zero coupon curve (bonds) at time 0 (i.e. $P(0,\cdot)$) and the survival probabilities $\mathbb{Q}(\tau \geq \cdot)$ at time 0 (terms in the boxes).

A similar formula holds for the protection leg, again under independence between default τ and interest rates.

CDS Model-independent formulas

$$
\begin{aligned}
\text{ProtecLeg}_{a,b}(\mathrm{L_{GD}}) &= \mathbb{E}[\mathbf{1}_{\{T_a<\tau\le T_b\}} D(0,\tau)\ \mathrm{L_{GD}}] \\
&= \mathrm{L_{GD}}\ \mathbb{E}\left[\int_{t=0}^{\infty} \mathbf{1}_{\{T_a<t\le T_b\}} D(0,t)\delta_\tau(t)dt\right] \\
&= \mathrm{L_{GD}}\left[\int_{t=T_a}^{T_b} \mathbb{E}[D(0,t)\delta_\tau(t)dt]\right] \\
&= \mathrm{L_{GD}}\int_{t=T_a}^{T_b} \mathbb{E}[D(0,t)]\mathbb{E}[\delta_\tau(t)dt] \\
&= \mathrm{L_{GD}}\int_{t=T_a}^{T_b} P(0,t)\mathbb{Q}(\tau\in[t,t+dt))
\end{aligned}
$$

(again interpret $\delta_\tau(t)dt = \mathbf{1}_{\{\tau\in[t,t+dt]\}}$)

CDS Model-independent formulas

so that we have, by introducing a "unit-notional" protection leg:

$$\mathrm{ProtecLeg}_{a,b}(\mathrm{L}_{\mathrm{GD}}; P(0,\cdot), \mathbb{Q}(\tau > \cdot)) = \mathrm{L}_{\mathrm{GD}}\ \mathrm{ProtecLeg1}_{a,b}(P(0,\cdot), \mathbb{Q}(\tau > \cdot)),$$

$$\mathrm{ProtecLeg1}_{a,b}(P(0,\cdot), \mathbb{Q}(\tau > \cdot)) := -\int_{T_a}^{T_b} P(0,t)\, d_t \boxed{\mathbb{Q}(\tau \geq t)}$$

This formula too is model independent given the initial zero coupon curve (bonds) at time 0 observed in the market and given the survival probabilities at time 0 (term in the box).

CDS Model-independent formulas

The total (Receiver) CDS price can be written as

$$\mathrm{CDS}_{a,b}(t, R, \mathrm{L_{GD}}; \mathbb{Q}(\tau > \cdot)) = R\,\mathrm{PremiumLeg1}_{a,b}(\mathbb{Q}(\tau > \cdot))$$

$$-\mathrm{L_{GD}}\ \mathrm{ProtecLeg1}_{a,b}(\mathbb{Q}(\tau > \cdot))$$

$$= R\left[-\int_{T_a}^{T_b} P(0,t)(t - T_{\beta(t)-1})d_t\boxed{\mathbb{Q}(\tau \geq t)} + \sum_{i=a+1}^{b} P(0,T_i)\alpha_i\boxed{\mathbb{Q}(\tau \geq T_i)}\right]$$

$$+\mathrm{L_{GD}}\left[\int_{T_a}^{T_b} P(0,t)\, d_t\boxed{\mathbb{Q}(\tau \geq t)}\right]$$

(Receiver) CDS Model-independent formulas

We may also use that $d_t\mathbb{Q}(\tau > t) = d_t(1 - \mathbb{Q}(\tau \leq t)) = -d_t\,\mathbb{Q}(\tau \leq t)$. We have

$$\mathrm{CDS}_{a,b}(t, R, \mathrm{L_{GD}}; \mathbb{Q}(\tau \leq \cdot)) = -\mathrm{L_{GD}}\left[\int_{T_a}^{T_b} P(0,t)\, d_t \boxed{\mathbb{Q}(\tau \leq t)}\right] +$$

$$R\left[\int_{T_a}^{T_b} P(0,t)(t - T_{\beta(t)-1}) d_t \boxed{\mathbb{Q}(\tau \leq t)} + \sum_{i=a+1}^{b} P(0,T_i)\alpha_i \boxed{\mathbb{Q}(\tau \geq T_i)}\right]$$

The integrals in the survival probabilities given in the above formulas can be valued as Stieltjes integrals in the survival probabilities themselves, and can easily be approximated numerically by summations through Riemann-Stieltjes sums, considering a low enough discretization time step.

CDS Model-independent formulas

The market quotes, at time 0, the fair $R = R_{0,b}^{\text{mkt MID}}(0)$ coming from bid and ask quotes for this fair R.

This fair R equates the two legs for a set of CDS with initial protection time $T_a = 0$ and final protection time $T_b \in \{1y, 2y, 3y, 4y, 5y, 6y.7y, 8y, 9y, 10y\}$, although often only a subset of the maturities $\{1y, 3y, 5y, 7y, 10y\}$ is available.

Solve then

$$\text{CDS}_{0,b}(t, R_{0,b}^{\text{mktMID}}(0), \text{L}_{\text{GD}}; \mathbb{Q}(\tau > \cdot)) = 0$$

in portions of $\mathbb{Q}(\tau > \cdot)$ starting from $T_b = 1y$, finding the market implied survival $\{\mathbb{Q}(\tau \geq t), t \leq 1y\}$; plugging this into the $T_b = 2y$ CDS legs formulas, and then solving the same equation with $T_b = 2y$, we find the market implied survival $\{\mathbb{Q}(\tau \geq t), t \in (1y, 2y]\}$, and so on up to $T_b = 10y$.

CDS Model-independent formulas

This is a way to strip survival (or equivalently default) probabilities from CDS quotes in a model independent way. No need to assume an intensity or a structural model for default here.

However, the market in doing the above stripping typically resorts to intensities (also called hazard rates), assuming existence of intensities associated with the default time.

We will refer to the method just highlighted as ”**CDS stripping**”.

CDS and Defaultable Bonds: Intensity Models

In intensity models the random default time τ is assumed to be exponentially distributed.

A strictly positive stochastic process $t \mapsto \lambda_t$ called *default intensity* (or hazard rate) is given for the bond issuer or the CDS reference name.

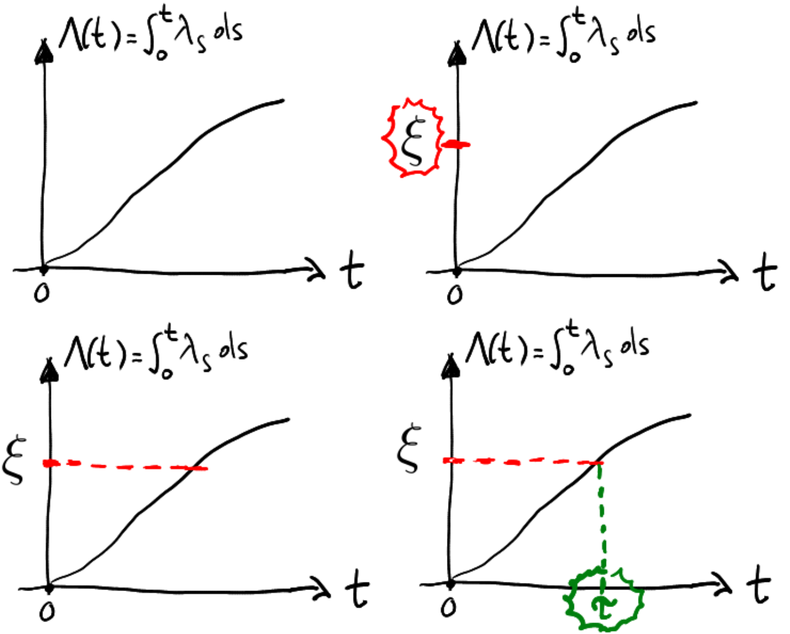
The *cumulated intensity* (or hazard function) is the process $t \mapsto \int_0^t \lambda_s \, ds =: \Lambda_t$. Since λ is positive, Λ is increasing in time.

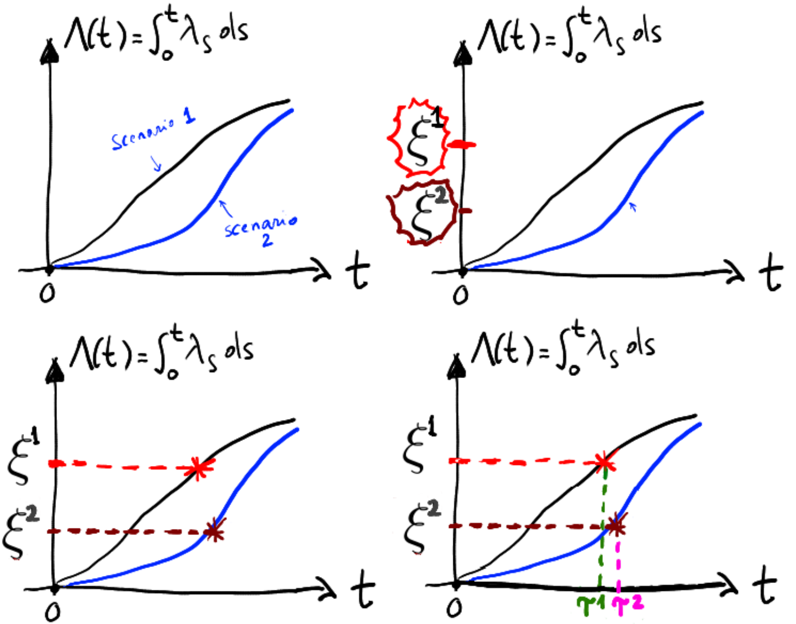
The default time is defined as the inverse of the cumulative intensity on an exponential random variable ξ with mean 1 and independent of λ

$$\tau = \Lambda^{-1}(\xi).$$

Recall that

$$\mathbb{Q}(\xi > u) = e^{-u}, \ \ \mathbb{Q}(\xi < u) = 1 - e^{-u}, \ \ \mathbb{E}(\xi) = 1.$$


$\Lambda(t) = \int_0^t \lambda_s \, ds$
t
0
$\Lambda(t) = \int_0^t \lambda_s \, ds$
ξ
t
0
$\Lambda(t) = \int_0^t \lambda_s \, ds$
ξ
t
0
$\Lambda(t) = \int_0^t \lambda_s \, ds$
ξ
τ
t
0


$\Lambda(t) = \int_0^t \lambda_s ds$
Scenario 1
scenario 2
ξ^1
ξ^2
τ^1
τ^2
0
t

CDS and Defaultable Bonds: Intensity Models

A few calculations: Probability of surviving time t:

$$\mathbb{Q}(\tau > t) = \mathbb{Q}(\Lambda^{-1}(\xi) > t) = \mathbb{Q}(\xi > \Lambda(t)) = \rightarrow$$

Let's use the tower property of conditional expectation and the fact that Λ is independent of ξ:

$$\rightarrow = \mathbb{E}[\mathbb{Q}(\xi > \Lambda(t)|\Lambda(t))] = \mathbb{E}[e^{-\Lambda(t)}] = \mathbb{E}[e^{-\int_0^t \lambda_s\, ds}]$$

This looks exactly like a bond price if we replace r by λ!

CDS and Defaultable Bonds: Intensity Models

Let's price a defaultable zero coupon bond with zero recovery. Assume that ξ is also independent of r.

$$\bar{P}(0,T) = \mathbb{E}[D(0,T)1_{\{\tau>T\}}] = \mathbb{E}[e^{-\int_0^T r_s\, ds}1_{\{\Lambda^{-1}(\xi)>T\}}] =$$

$$= \mathbb{E}[e^{-\int_0^T r_s\, ds}1_{\{\xi>\Lambda(T)\}}] = \mathbb{E}[\mathbb{E}\{e^{-\int_0^T r_s\, ds}1_{\{\xi>\Lambda(T)\}}|\Lambda, r\}]$$

$$= \mathbb{E}[e^{-\int_0^T r_s\, ds}\mathbb{E}\{1_{\{\xi>\Lambda(T)\}}|\Lambda, r\}]$$

$$= \mathbb{E}[e^{-\int_0^T r_s\, ds}\mathbb{Q}\{\xi > \Lambda(T)|\Lambda\}] = \mathbb{E}[e^{-\int_0^T r_s\, ds}e^{-\Lambda(T)}] =$$

$$= \mathbb{E}[e^{-\int_0^T r_s\, ds - \int_0^T \lambda_s\, ds} = \mathbb{E}[e^{-\int_0^T \boxed{(r_s+\lambda_s)}\, ds}]$$

So the price of a defaultable bond is like the price of a default-free bond *where the risk free discount short rate r has been replaced by r plus a spread λ.*

CDS and Defaultable Bonds: Intensity Models

This is why in intensity models, the intensity is interpreted as a credit spread.

Because of properties of the exponential random variable, one can also prove that

$$\mathbb{Q}(\tau \in [t, t + dt)|\tau > t, ``\lambda[0, t]") = \lambda_t \, dt$$

and the intensity $\lambda_t \, dt$ is also a local probability of defaulting around t.

So:
λ is an instantaneous credit spread or local default probability

ξ is pure jump to default risk

Intensity models and Interest Rate Models

As is now clear, the exponential structure of τ in intensity models makes the modeling of credit risk very similar to interest rate models.

The spread/intensity λ behaves exactly like an interest rate in discounting

Then it is possible to use a lot of techniques from interest rate modeling (short rate models for r, first choice seen earlier) for credit as well.

Intensity: Constant, time dependent or stochastic

- Constant λ_t: in this case $\lambda_t = \gamma$ for a deterministic constant credit spread (intensity);
- Time dependent deterministic intensity λ_t: in this case $\lambda_t = \gamma(t)$ for a deterministic curve in time $\gamma(t)$. This is a model with a term structure of credit spreads but without credit spread volatility.
- Time dependent and stochastic intensity λ_t: in this case λ_t is a full stochastic process. This allows us to model the term structue of credit spreads but also their volatility.

The case with constant intensity $\lambda_t = \gamma$: CDS

Assume as an approximation that the CDS premium leg pays continuously.

Instead of paying $(T_i - T_{i-1})R$ at T_i as the standard CDS, given that there has been no default before T_i, we approximate this premium leg by assuming that it pays "$dt\ R$" in $[t, t + dt)$ it there has been no default before $t + dt$.

The case with constant intensity $\lambda_t = \gamma$: CDS

This amounts to replace the original pricing formula of a CDS (receiver case, spot CDS with $T_a = 0 =$ today)

$$\mathrm{CDS}_{0,b}(0, R, \mathrm{L_{GD}}; \mathbb{Q}(\tau > \cdot)) = R\left[-\int_0^{T_b} P(0,t)(t - T_{\beta(t)-1}) d_t\mathbb{Q}(\tau \geq t)\right.$$

$$\left. + \sum_{i=1}^{b} P(0, T_i)\alpha_i \mathbb{Q}(\tau \geq T_i)\right] + \mathrm{L_{GD}}\left[\int_0^{T_b} P(0,t)\, d_t\, \mathbb{Q}(\tau \geq t)\right]$$

with (accrual term vanishes because payments continuous now)

$$R\int_0^{T_b} P(0,t)\mathbb{Q}(\tau \geq t)dt + \mathrm{L_{GD}}\int_0^{T_b} P(0,t)\, d_t\mathbb{Q}(\tau \geq t)$$

The case with constant intensity $\lambda_t = \gamma$: CDS

If the intensity is a constant γ we have

$$\mathbb{Q}(\tau > t) = e^{-\gamma t}, \quad d_t\mathbb{Q}(\tau > t) = -\gamma e^{-\gamma t} dt = -\gamma \mathbb{Q}(\tau > t) dt,$$

and the receiver CDS price we have seen earlier becomes

$$\mathrm{CDS}_{0,b}(t, R, \mathrm{L}_{\mathrm{GD}}; \mathbb{Q}(\tau > \cdot)) = -\mathrm{L}_{\mathrm{GD}} \left[\int_0^{T_b} P(0,t) \gamma \mathbb{Q}(\tau \geq t) dt \right]$$

$$+ R \left[\int_0^{T_b} P(0,t) \mathbb{Q}(\tau \geq t) dt \right]$$

If we insert the market CDS rate $R = R_{0,b}^{\text{mkt MID}}(0)$ in the premium leg, then the CDS present value should be zero. Solve

$$\mathrm{CDS}_{a,b}(t, R, \mathrm{L}_{\mathrm{GD}}; \mathbb{Q}(\tau > \cdot)) = 0 \quad \text{in} \quad R$$

to obtain

$$\boxed{\gamma = \frac{R_{0,b}^{\text{mkt MID}}(0)}{\mathrm{L}_{\mathrm{GD}}}}$$

The case with constant intensity $\lambda_t = \gamma$: CDS

from which we see that also **the CDS premium rate *R* is indeed a sort of CREDIT SPREAD, or INTENSITY.**

We can play with this formula with a few examples.

CDS of FIAT trades at 300bps for 5y, with recovery 0.3

What is a quick rough calcul for the risk neutral probability that FIAT survives 10 years?

$$\boxed{\gamma = \frac{R_{0,b}^{\text{mkt FIAT}}(0)}{\text{L}_{\text{GD}}}} = \frac{300/10000}{1-0.3} = 4.29\%$$

The case with constant intensity $\lambda_t = \gamma$: CDS

Survive 10 years:

$$\mathbb{Q}(\tau > 10y) = \exp(-\gamma 10) = \exp(-0.0429 * 10) = 65.1\%$$

Default between 3 and 5 years:

$$\mathbb{Q}(\tau > 3y) - \mathbb{Q}(\tau > 5y) = \exp(-\gamma 3) - \exp(-\gamma 5)$$

$$= \exp(-0.0429 * 3) - \exp(-0.0429 * 5) = 7.2\%$$

If R_{CDS} goes up and REC remains the same, γ goes up and survival probabilities go down (default probs go up)

If REC goes up and R_{CDS} remains the same, LGD goes down and γ goes up - default probabilities go up

The case with time dependent intensity $\lambda_t = \gamma(t)$: CDS

We consider now **deterministic time-varying** intensity $\gamma(t)$, which we assume to be a positive and piecewise continuous function. We define

$$\Gamma(t) := \int_0^t \gamma(u)du,$$

the **cumulated intensity, cumulated hazard rate,** or also **Hazard function.**

From the exponential assumption, we have easily

$$\mathbb{Q}\{s < \tau \leq t\} = \mathbb{Q}\{s < \Gamma^{-1}(\xi) \leq t\} = \mathbb{Q}\{\Gamma(s) < \xi \leq \Gamma(t)\} =$$

$$= \mathbb{Q}\{\xi > \Gamma(s)\} - \mathbb{Q}\{\xi > \Gamma(t)\} = \exp(-\Gamma(s)) - \exp(-\Gamma(t)) \text{ i.e.}$$

"prob of default between s and t is "$e^{-\int_0^s \gamma(u)du} - e^{-\int_0^t \gamma(u)du} \approx \int_s^t \gamma(u)du$" (where the final approximation is good ONLY for small exponents).

CDS Calibration and Implied Hazard Rates/Intensities

Reduced form models are the models that are most commonly used in the market to infer implied default probabilities from market quotes.

Market instruments from which these probabilities are drawn are especially CDS and Bonds.

We just implement the stripping algorithm sketched earlier for "CDS stripping", but now taking into account that the probabilities are expressed as exponentials of the deterministic intensity γ, that is assumed to be piecewise constant.

By adding iteratively CDS with longer and longer maturities, at each step we will strip the new part of the intensity $\gamma(t)$ associated with the last added CDS, while keeping the previous values of γ, for earlier times, that were used to fit CDS with shorter maturities.

A Case Study of CDS stripping: Lehman Brothers

Here we show an intensity model with piecewise constant λ obtained by CDS stripping.

We also show the AT1P structural / firm value model by Brigo et al (2004-2010). This will not be subject for this course, but in case of interest, for details on AT1P see

http://arxiv.org/abs/0912.3028
http://arxiv.org/abs/0912.3031
http://arxiv.org/abs/0912.4404

Otherwise ignore the AT1P and σ_i parts of the tables.

- **August 23, 2007**: Lehman announces that it is going to shut one of its home lending units (*BNC Mortgage*) and lay off 1,200 employees. The bank says it would take a $52 million charge to third-quarter earnings.
- **March 18, 2008**: Lehman announces better than expected first-quarter results (but profits have more than halved).
- **June 9, 2008**: Lehman confirms the booking of a $2.8 billion loss and announces plans to raise $6 billion in fresh capital by selling stock. Lehman shares lose more than 9% in afternoon trade.
- **June 12, 2008**: Lehman shakes up its management; its chief operating officer and president, and its chief financial officer are removed from their posts.
- **August 28, 2008**: Lehman prepares to lay off 1,500 people. The Lehman executives have been knocking on doors all over the world seeking a capital infusion.
- **September 9, 2008**: Lehman shares fall 45%.
- **September 14, 2008**: Lehman files for bankruptcy protection and hurtles toward liquidation after it failed to find a buyer.

Lehman Brothers CDS Calibration: July 10th, 2007

On the left part of this Table we report the values of the quoted CDS spreads before the beginning of the crisis. We see that the spreads are very low. In the middle of Table 1 we have the results of the exact calibration obtained using a *piecewise constant* intensity model.

T_i	R_i (bps)	λ_i (bps)	Surv (Int)	σ_i	Surv (AT1P)
10 Jul 2007			100.0%		100.0%
1y	16	0.267%	99.7%	29.2%	99.7%
3y	29	0.601%	98.5%	14.0%	98.5%
5y	45	1.217%	96.2%	14.5%	96.1%
7y	50	1.096%	94.1%	12.0%	94.1%
10y	58	1.407%	90.2%	12.7%	90.2%

Table: Results of calibration for July 10th, 2007.

Lehman Brothers CDS Calibration: June 12th, 2008

We are in the middle of the crisis. We see that the CDS spreads R_i have increased with respect to the previous case, but are not very high, indicating the fact that the market is aware of the difficulties suffered by Lehman but thinks that it can come out of the crisis. Notice that now the term structure of both R and *intensities* is inverted. This is typical of names in crisis

T_i	R_i (bps)	λ_i (bps)	Surv (Int)	σ_i	Surv (AT1P)
12 Jun 2008			100.0%		100.0%
1y	397	6.563%	93.6%	45.0%	93.5%
3y	315	4.440%	85.7%	21.9%	85.6%
5y	277	3.411%	80.0%	18.6%	79.9%
7y	258	3.207%	75.1%	18.1%	75.0%
10y	240	2.907%	68.8%	17.5%	68.7%

Table: Results of calibration for June 12th, 2008.

Lehman Brothers CDS Calibration: Sept 12th, 2008

In this Table we report the results of the calibration on September 12th, 2008, just before Lehman's default. We see that the spreads are now very high, corresponding to lower survival probability and higher intensities than before.

T_i	R_i (bps)	λ_i (bps)	Surv (Int)	σ_i	Surv (AT1P)
12 Sep 2008			100.0%		100.0%
1y	1437	23.260%	79.2%	62.2%	78.4%
3y	902	9.248%	65.9%	30.8%	65.5%
5y	710	5.245%	59.3%	24.3%	59.1%
7y	636	5.947%	52.7%	26.9%	52.5%
10y	588	6.422%	43.4%	29.5%	43.4%

Table: Results of calibration for September 12th, 2008.

Stochastic Intensity. The CIR++ model

We have seen in detail CDS calibration in presence of **deterministic** and **time varying** intensity or hazard rates, $\gamma(t)dt = \mathbb{Q}\{\tau \in dt | \tau > t\}$

As explained, this accounts for credit spread structure but not for **volatility**.

The latter is obtained moving to stochastic intensity (Cox process). The deterministic function $t \mapsto \gamma(t)$ is replaced by a stochastic process $t \mapsto \lambda(t) = \lambda_t$. The Hazard function $\Gamma(t) = \int_0^t \gamma(u)du$ is replaced by the Hazard process (or cumulated intensity) $\Lambda(t) = \int_0^t \lambda(u)du$.

CIR++ stochastic intensity λ

We model the stochastic intensity as follows: consider

$$\lambda_t = y_t + \psi(t;\beta)\ , \quad t \geq 0,$$

where the intensity has a random component y and a deterministic component ψ to fit the CDS term structure. For y we take a Jump-CIR model

$$dy_t = \kappa(\mu - y_t)dt + \nu\sqrt{y_t}dZ_t + dJ_t,\ \beta = (\kappa, \mu, \nu, y_0),\ 2\kappa\mu > \nu^2.$$

Jumps are taken themselves independent of anything else, with exponential arrival times with intensity η and exponential jump size with a given parameter.

In this course we will focus on the case with no jumps J, see B and El-Bachir (2006) or B and M (2006) for the case with jumps.

CIR++ stochastic intensity λ.

Calibrating Implied Default Probabilities

With no jumps, y follows a noncentral chi-square distribution; Very important: $y > 0$ as must be for an intensity model (Vasicek would not work). This is the CIR++ model we have seen earlier for interest rates.

About the parameters of CIR:

$$dy_t = \kappa(\mu - y_t)dt + \nu\sqrt{y_t}dZ_t$$

κ: speed of mean reversion
μ: long term mean reversion level
ν: volatility.

CIR++ stochastic intensity λ. I

Calibrating Implied Default Probabilities

$$E[\lambda_t] = \lambda_0 e^{-\kappa t} + \mu(1 - e^{-\kappa t})$$

$$\mathrm{VAR}(\lambda_t) = \lambda_0 \frac{\nu^2}{\kappa}(e^{-\kappa t} - e^{-2\kappa t}) + \mu \frac{\nu^2}{2\kappa}(1 - e^{-\kappa t})^2$$

After a long time the process reaches (asymptotically) a stationary distribution around the mean μ and with a corridor of variance $\mu\nu^2/2\kappa$. The largest κ, the fastest the process converges to the stationary state. So, ceteris paribus, increasing κ kills the volatility of the credit spread. The largest μ, the highest the long term mean, so the model will tend to higher spreads in the future in average.

The largest ν, the largest the volatility. Notice however that κ and ν fight each other as far as the influence on volatility is concerned.

CIR++ stochastic intensity λ. II

Calibrating Implied Default Probabilities

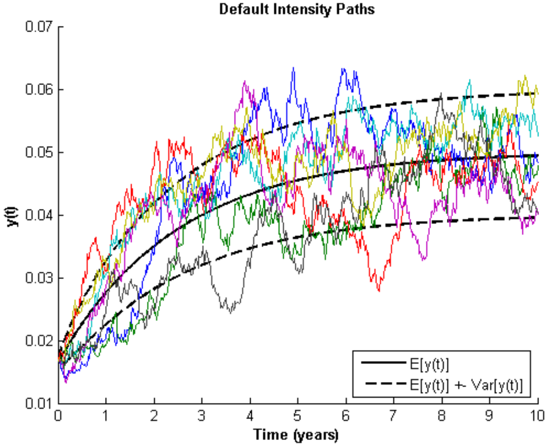


Figure: $y0 = 0.0165, \kappa = 0.4, \mu = 0.05, \nu = 0.04$

EXERCISE: The CIR model

Assume we are given a stochastic intensity process of CIR type,

$$dy_t = \kappa(\mu - y_t)dt + \nu\sqrt{y_t}dW(t)$$

where y_0, κ, μ, ν are positive constants. W is a brownian motion under the risk neutral measure.
a) Increasing κ increases or decreases randomness in the intensity? And ν?
b) The mean of the intensity at future times is affected by k? And by ν?
c) What happens to mean of the intensity when time grows to infinity?
d) Is it true that, because of mean reversion, the variance of the intensity goes to zero (no randomness left) when time grows to infinity?
e) Can you compute a rough approximation of the percentage volatility in the intensity?

EXERCISE: The CIR model

f) Suppose that $y_0 = 400bps = 0.04$, $\kappa = 0.3$, $\nu = 0.001$ and $\mu = 400bps$. Can you guess the behaviour of the future random trajectories of the stochastic intensity after time 0?
g) Can you guess the spread of a CDS with 10y maturity with the above stochastic intensity when the recovery is 0.35?

EXERCISE Solutions. I

a) We can refer to the formulas for the mean and variance of y_T in a CIR model as seen from time 0, at a given T. The formula for the variance is known to be (see for Example Brigo and Mercurio (2006))

$$\mathrm{VAR}(y_T) = y_0 \frac{\nu^2}{\kappa}(e^{-\kappa T} - e^{-2\kappa T}) + \mu \frac{\nu^2}{2\kappa}(1 - e^{-\kappa T})^2$$

whereas the mean is

$$E[y_T] = y_0 e^{-\kappa T} + \mu(1 - e^{-\kappa T})$$

We can see that for k becoming large the variance becomes small, since the exponentials decrease in k and the division by k gives a small value for large k. In the limit

$$\lim_{\kappa \to +\infty} \mathrm{VAR}(y_T) = 0$$

so that for very large κ there is no randomness left.

EXERCISE Solutions. II

We can instead see that VAR(y_T) is proportional to ν^2, so that if ν increases randomness increases, as is obvious from $\nu\sqrt{y_t}$ being the instantaneous volatility in the process y.

b) As the mean is

$$E[y_T] = y_0 e^{-\kappa T} + \mu(1 - e^{-\kappa T})$$

we clearly see that this is impacted by κ (indeed, "speed of mean reversion") and by μ clearly ("long term mean") but not by the instantaneous volatility parameter ν.
c) As T goes to infinity, we get for the mean

$$\lim_{T \to +\infty} y_0 e^{-\kappa T} + \mu(1 - e^{-\kappa T}) = \mu$$

so that the mean tends to μ (this is why μ is called "long term mean").

EXERCISE Solutions. III

d) In the limit where time goes to infinity we get, for the variance

$$\lim_{T\to+\infty}[y_0\frac{\nu^2}{\kappa}(e^{-\kappa T}-e^{-2\kappa T})+\mu\frac{\nu^2}{2\kappa}(1-e^{-\kappa T})^2]=\mu\frac{\nu^2}{2\kappa}$$

So this does not go to zero. Indeed, mean reversion here implies that as time goes to infinite the mean tends to μ and the variance to the constant value $\mu\frac{\nu^2}{2\kappa}$, but not to zero.

EXERCISE Solutions. IV

e) Rough approximations of the percentage volatilities in the intensity would be as follows. The instantaneous variance in dy_t, conditional on the information up to t, is (remember that $VAR(dW(t)) = dt$)

$$VAR(dy_t) = \nu^2 y_t dt$$

The percentage variance is

$$VAR\left(\frac{dy_t}{y_t}\right) = \frac{\nu^2 y_t}{y_t^2} dt = \frac{\nu^2}{y_t} dt$$

and is state dependent, as it depends on y_t. We may replace y_t with either its initial value y_0 or with the long term mean μ, both known. The two rough percentage volatilities estimates will then be, for $dt = 1$,

$$\sqrt{\frac{\nu^2}{y_0}} = \frac{\nu}{\sqrt{y_0}}, \quad \sqrt{\frac{\nu^2}{\mu}} = \frac{\nu}{\sqrt{\mu}}$$

EXERCISE Solutions. V

These however do not take into account the important impact of κ in the overall volatility of finite (as opposed to instantaneous) credit spreads and are therefore relatively useless.

EXERCISE Solutions. VI

f) First we check if the positivity condition is met.

$$2\kappa\mu = 2 \cdot 0.3 \cdot 0.04 = 0.024; \quad \nu^2 = 0.001^2 = 0.000001$$

hence $2\kappa\mu > \nu^2$ and trajectories are positive. Then we observe that the variance is very small: Take $T = 5y$,

$$\mathrm{VAR}(y_T) = y_0 \frac{\nu^2}{\kappa}(e^{-\kappa T} - e^{-2\kappa T}) + \theta \frac{\nu^2}{2\kappa}(1 - e^{-\kappa T})^2 \approx 0.0000006.$$

Take the standard deviation, given by the square root of the variance:

$$\mathrm{STDEV}(y_T) \approx \sqrt{0.0000006} = 0.00077.$$

which is much smaller of the level 0.04 at which the intensity refers both in terms of initial value and long term mean. Therefore there is almost no randomness in the system as the variance is very small compared to the initial point and the long term mean.

EXERCISE Solutions. VII

Hence there is almost no randomness, and since the initial condition y_0 is the same as the long term mean $\mu_0 = 0.04$, the intensity will behave as if it had the value 0.04 all the time. All future trajectories will be very close to the constant value 0.04.

g) In a constant intensity model the CDS spread can be approximated by

$$y = \frac{R_{CDS}}{1 - REC} \Rightarrow R_{CDS} = y(1 - REC) = 0.04(1 - 0.35) = 260 bps$$

CIR++ stochastic intensity λ. I

Calibrating Implied Default Probabilities

For restrictions on the β's that keep ψ and hence λ positive, **as is required in intensity models**, we may use the results in B. and M. (2001) or (2006). We will often use the hazard process $\Lambda(t) = \int_0^t \lambda_s ds$, and also $Y(t) = \int_0^t y_s ds$ and $\Psi(t,\beta) = \int_0^t \psi(s,\beta)ds$.

If we can read from the market some implied risk-neutral default probabilities, and associate to them implied hazard functions Γ^{Mkt} (as we have done in the Lehman example), we may wish our stochastic intensity model to agree with them. By recalling that survival probabilities look exactly like bonds formulas in short rate models for r, we see that our model agrees with the market if

$$\exp(-\Gamma^{\mathrm{Mkt}}(t)) = \exp\left(-\Psi(t,\beta)\right) \mathbb{E}[e^{-\int_0^t y_s ds}]$$

CIR++ stochastic intensity λ. II

Calibrating Implied Default Probabilities

IMPORTANT 1: This is possible only if λ is strictly positive;
IMPORTANT 2: It is fundamental, if we aim at calibrating default probabilities, that the last expected value can be computed analytically.
The only known diffusion model used in interest rates satisfying both constraints is CIR++

CIR++ stochastic intensity λ

Calibrating Implied Default Probabilities

$$\exp(-\Gamma^{\text{Mkt}}(t)) = \mathbb{Q}\{\tau > t\} = \exp\left(-\Psi(t,\beta)\right)\mathbb{E}[e^{-\int_0^t y_s ds}]$$

Now notice that $\mathbb{E}[e^{-\int_0^t y_s ds}]$ is simply the bond price for a CIR interest rate model with short rate given by y, so that it is known analytically. We denote it by $P^y(0,t,y_0;\beta)$.

Similarly to the interest-rate case, λ is calibrated to the market implied hazard function Γ^{Mkt} if we set

$$\Psi(t,\beta) := \Gamma^{\text{Mkt}}(t) + \ln(P^y(0,t,y_0;\beta))$$

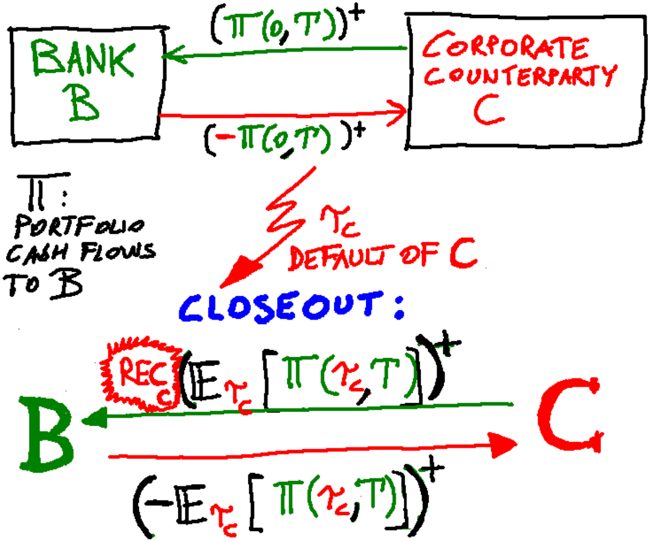
where we choose the parameters β in order to have a positive function ψ, by resorting to the condition seen earlier.

This concludes our introduction to Defaultable Bonds, CDS, credit spreads and intensity models.

We now turn to using such tools in one of the problems the industry is facing right now:

Pricing of counterparty credit risk, leading to the notion of Credit Valuation Adjustment (CVA)

Context



$(\Pi(0,T))^+$
BANK
B
CORPORATE
COUNTERPARTY
C
$(-\Pi(0,T))^+$
Π :
PORTFOLIO
CASH FLOWS
TO B
τ_C
DEFAULT OF C
CLOSEOUT:
$REC_C \left(\mathbb{E}_{\tau_C}[\Pi(\tau_C,T)]\right)^+$
B
C
$\left(-\mathbb{E}_{\tau_C}[\Pi(\tau_C,T)]\right)^+$

Q & A: What is Counterparty Credit Risk?

Q What is counterparty risk in general?

A *The risk taken on by an entity entering an OTC contract with a counterparty having a relevant default probability. As such, the counterparty might not respect its payment obligations.*

> *The counterparty credit risk is defined as the risk that the counterparty to a transaction could default before the final settlement of the transaction's cash flows. An economic loss would occur if the transactions or portfolio of transactions with the counterparty has a positive economic value at the time of default.*
>
> [Basel II, Annex IV, 2/A]

Q & A: Credit VaR and CVA

Q What is the difference between Credit VaR and CVA?

A *They are both related to credit risk.*

- *Credit VaR is a Value at Risk type measure, a Risk Measure. it measures a potential loss due to counterparty default.*
- *CVA is a price, it stands for Credit Valuation Adjustment and is a price adjustment. CVA is obtained by pricing the counterparty risk component of a deal, similarly to how one would price a credit derivative.*

Q & A: Credit VaR and CVA

Q What is the difference in practical use?

A *Credit VaR answers the question:*

- *”How much can I lose of this portfolio, within (say) one year, at a confidence level of 99%, due to default risk and exposure?”*
- *CVA instead answers the question:*
  ”How much discount do I get on the price of this deal due to the fact that you, my counterparty, can default? I would trade this product with a default free party. To trade it with you, who are default risky, I require a discount.”

Clearly, a price needs to be more precise than a risk measure, so the techniques will be different.

Q & A: Credit VaR and CVA

Q Different? Are the methodologies for Credit VaR and CVA not similar?

A *There are analogies but CVA needs to be more precise in general. Also, Credit VaR should use statistics under the physical measure whereas CVA should use statistics under the pricing measure*

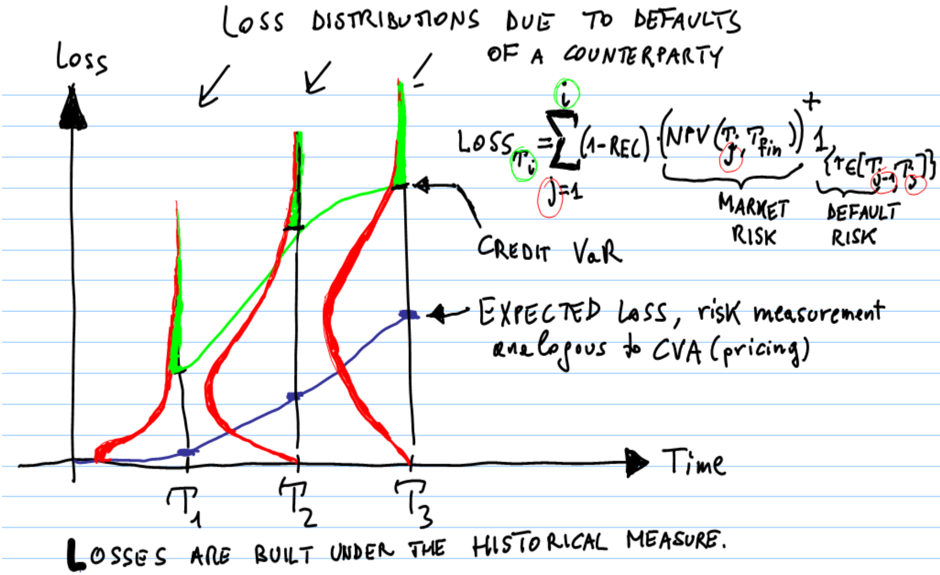
Q What are the regulatory bodies involved?

A *There are many, for Credit VaR type measures it is mostly Basel II and now III, whereas for CVA we have IAS, FASB and ISDA. But the picture is now blurring since Basel III is quite interested in CVA too*

Q What is the focus of this presentation?

A *We will focus on CVA.*

Q & A: Credit VaR and CVA



Q & A: CVA and Model Risk, WWR

Q What impacts counterparty risk CVA?

A *The OTC contract's underlying volatility, the correlation between the underlying and default of the counterparty, and the counterparty credit spreads volatility.*

Q Is it model dependent?

A *It is highly model dependent even if the original portfolio without counterparty risk was not. There is a lot of model risk.*

Q What about *wrong way risk*?

A *The amplified risk when the reference underlying and the counterparty are strongly correlated in the wrong direction.*

Q & A: Collateral

Q What is collateral?

A *It is a guarantee (liquid and secure asset, cash) that is deposited in a collateral account in favour of the investor party facing the exposure. If the depositing counterparty defaults, thus not being able to fulfill payments associated to the above mentioned exposure, Collateral can be used by the investor to offset its loss.*

Q & A: Netting

Q What is netting?

A *This is the agreement to net all positions towards a counterparty in the event of the counterparty default. This way positions with negative PV can be offset by positions with positive PV and counterparty risk is reduced. This has to do with the option on a sum being smaller than the sum of the options. CVA is typically computed on netting sets.*

Q & A: Basel III and CVA Risk

Q What is happening with Basel III?

A *Basel noticed that during the crisis only one third of losses due to counterparty risk were due to actual defaults. The remaining losses have been due to CVA mark to market losses. Hence the pricing of counterparty risk has been twice as dangerous as the risk itself.*

Q Then we should ”risk-measure” CVA itself?

A *Indeed there is a lot of discussion around Value at Risk of CVA. This is not traditional credit VaR of course. It is something much more sophisticated. It is a percentile on future possible losses due to future averse movements of the PRICING of counterparty risk*

Q & A: Collateral and CVA. Gap Risk

Q But collateral should spare one the pains of this?

A *Collateral/CSA Margining is only an imperfect remedy to Counterparty risk, mostly due to Gap Risk, the risk of sudden mark to market changes and defaults between margining dates. This can be dramatic for assets that are subject to strong contagion under systemic risk. Also, we have re-hyphotecation, where collateral is not kept segregated as a guarantee. More generally, there are collateral disputes.*

Q & A: CVA Restructuring

Q So to manage CVA risk it is either Collateral (with the above caveats) or large capital requirements. Isn't this possibly creating a liquidity strain and depress the economy further?

A *A third possibility would be a macroeconomically healthy way of securitizing counterparty risk, a way recognized by regulators that banks could adopt to "buy" counterparty risk protection in the market. This is quite delicate. Floating margin lending, based on a notion of floating CVA, might be interesting from this point of view. More traditional fixed-premium cash CDO-type securitization mechanisms have failed so far.*

Q & A: CVA - Unilateral or Bilateral?

Basel II on **bilateral** counterparty risk:

> *Unlike a firm's exposure to credit risk through a loan, where the exposure to credit risk is unilateral and only the lending bank faces the risk of loss, the counterparty credit risk creates a bilateral risk of loss: the market value of the transaction can be positive or negative to either counterparty to the transaction.* [Basel II, Annex IV, 2/A]

Q & A: CVA - Unilateral or Bilateral?

Q When is valuation of counterparty risk CVA symmetric?

A *When we include the possibility that also the entity computing the counterparty risk adjustment may default, besides the counterparty itself.*

Q When is valuation of counterparty risk CVA asymmetric?

A *When the entity computing the counterparty risk adjustment considers itself default-free, and only the counterparty may default.*

Q Which one is computed usually for valuation adjustments?

A *Pre-crisis it used to be the asymmetric one; At the moment there is quite a debate*

Q & A: DVA

Q What happens in the symmetric case?

A *We have a new quantity called Debit Valuation Adjustment, or DVA.*

Q What is that?

A *It answers the question: "I recognize that I am default risky, so in trading this position with you, I accept to be charged more for this product than if I were default free, since you, my counterparty, are taking additional risk due to my possible default. DVA is then the increase in value I need to pay to enter this deal with you."*

Q & A: DVA

Q Looks like CVA seen from the other side. Is this why now pricing is symmetric?

A *Indeed, on side "B" you have*

$$DVA_{B,C} - CVA_{B,C}.$$

On the other side you have that $DVA_{B,C}$ becomes $CVA_{C,B}$ and $CVA_{B,C}$ becomes $DVA_{C,B}$, so you have exactly

$$-(DVA_{B,C} - CVA_{B,C}) = DVA_{C,B} - CVA_{C,B}.$$

Symmetry as in a swap. However, there are a few caveats

Q & A: DVA

Q Meaning?

A *DVA increases when the credit quality of the calculating entity worsens, because it becomes less likely that the calculating entity will have to repay its debt. However this is a profit that can only be realized by defaulting. Should it be accounted for?*

Q How would one hedge DVA?

A *One would have to sell protection of oneself (issue and then buy back bonds? Proxy Hedging?). Very difficult. Without hedging, is it really a price? However, it is from the other side, since it is CVA. Perspectival*

Q & A: DVA

Q Is this why you said DVA is debated?

A *Yes, regulators are fighting. FASB approved it. Basel does not recognize it, "perverse incentive". This makes CVA capital charges larger, since in future P&L simulations there will be no DVA balancing CVA.*

Q & A: First default

Q Does bilateral counterparty risk pricing, namely DVA - CVA, consider closeout? Namely, that at the **first** default the deal is liquidated or replaced?

A *Only if you take into account the first to default time in valuation. Correct CVA and DVA account for that. However first to default involves knowing the default "correlation" between the two entities in the deal. It may be difficult. Hence often the industry uses a formula ignoring first to default. This however involves double counting.*

Q & A: Closeout

Q And again on closeout, how is exactly the value of the residual deal computed at the closeout time?

A *You may have a risk free closeout, where the residual deal is priced at mid market without any residual credit risk, or you may have a replacement closeout, where the remaining deal is priced by taking into account the credit quality of the surviving party and of the party that replaces the defaulted one (so the new DVA - CVA at default).*

Q & A: Closeout

Q Does it make a big difference?

A *It does.*

- *Part of the market argues that if you are closing the deal at the closeout time, why should you worry about residual credit risk until the final maturity?*
- *On the other hand, if before the first default you were marking to market the deal including CVA and DVA, and all of a sudden at the first default you take CVA and DVA out, you create a discontinuity.*
- *It has been found that Risk Free closeout penalizes borrowers, whereas Replication closeout penalizes lenders, and the effect depends on the default correlation between parties.*

ISDA is not very assertive on closeout.

Q & A: CVA and Payout Risk

Q So we have:

- DVA Yes/No?
- First to Default time or not?
- Risk Free closeout or replication closeout?

It looks like not even the precise payout of CVA is clear, let alone model risk.

A *Yes, there is a lot of payout risk. In an interview to Risk Magazine, top tier 1 banks complained towards smaller banks by saying that the latter were more aggressive in CVA assumptions, thus taking clients that would otherwise work with the top tier 1. This aggressive pricing has been interpreted by tier 1 banks as using the cheapest form of CVA payout for the client.*
It has also been said that 5 banks may compute CVA in 15 different ways across functions and deals.

Q & A: Funding Costs I

Q There is a further topic I keep hearing around. Its the inclusion of Cost of Funding into the valuation framework. What is that?

A A When you manage a trading position, you need to obtain cash in order to do a number of operations:

- hedging the position,
- posting collaterals or paying interests on them,
- paying coupons or notional amounts,
- set reserves in place...
- You may obtain cash from your Treasury department or in the market. You may also receive cash as a consequence of being in the position:
  - a coupon, a notional reimbursement, a positive mark-to-market move, getting some collateral, a closeout payment. . .

Q & A: Funding Costs II

Q Why should I to consider such flows?

A All such flows need to be remunerated:

- if you are borrowing, this will have a cost,
- and if you are lending, this will provide you with some revenues.

Including the cost of funding into your valuation framework means to properly account for such features.

Q & A: Funding Costs III

As we will see, the inclusion of a simple and additive ”Funding Valuation Adjustments” (FVA) is not as straightforward as CVA and DVA are (even with all their problems).

Proper inclusion of funding leads to a recursive pricing problem where credit and funding risk interact in a complex and nonlinear/ non decomposable way

We can compute a total adjustment for funding and credit risk but not separate adjustments

Q & A. CCPs

Q And what about Central Counterparty Clearing houses (CCP's)?

A *CCPs are commercial entities that, ideally, would interpose themselves between the two parties in a trade.*

- *Each party will post collateral margins say daily, every time the mark to market goes against that party.*
- *Collateral will be held by the CCP as a guarantee for the other party.*
- *If a party in the deal defaults and the mark to market is in favour of the other party, then the surviving party will obtain the collateral from the CCP and will not be affected, in principle, by counterparty risk.*
- *Moreover, there is also an initial margin that is supposed to cover for additional risks like deteriorating quality of collateral, gap risk, wrong way risk, etc.*

Q & A: CCPs

Q It looks pretty safe. With the current regulation and law pushing firms to trade through central clearing, will all this analysis of credit, liquidity and funding risk be a moot point? Are CCP's going to be the end of CVA/DVA/FVA problems?

A *CCP's will reduce risk in many cases but are not a panacea. They also require daily margining, and one may question the fees and initial margins they charge. Also, they could become too big to fail. And finally, too many CCPs makes netting unefficient, whereas too few creates concentration risk.*

Q & A: CCPs

Q So CCP's are not really a panacea. Other issues with CCPs?

A *The following points are worth keeping in mind:*[a]

- *CCPs are usually highly capitalised. All clearing members post collateral (asymmetric "CSA"). Initial margin means clearing members are overcollateralised all the time.*
- *TABB Group says extra collateral could be about 2 $ Trillion.*[b]
- *CCPs can default and did default. Defaulted ones - 1974: Caisse de Liquidation des Affaires en Marchandises; 1983: Kuala Lumpur Commodity Clearing House; 1987: Hong Kong Futures Exchange. The ones that were close to default- 1987: CME and OCC, US; 1999: BM&F, Brazil.*

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[a] See for example Piron, B. (2012). Why collateral and CCPs can be bad for your wealth. SunGard's Adaptive White Paper.

[b] Rhode, W. (2011). European Credit and Rates Dealers 2011 – Capital, Clearing and Central Limit Order Books. TABB Group Research Report

Q & A: CVA Desks? "Best practices"?

Q In terms of active and concrete CVA (DVA? FVA?) management, what is the "best practice" banks follow? I hear about "CVA Desks", what does that mean?

A *The idea is to move Counterparty Risk management away from classic asset classes trading desks by creating a specific counterparty risk trading desk, or "CVA desk". Under a lot of simplifying assumptions, this would allow "classical" traders to work in a counterparty risk-free world in the same way as before the counterparty risk crisis exploded.*

Q & A: CVA Desks? "Best practices"?

Q How would this CVA desk help classical trading desks, more in detail?[a]

A *It would free the classical traders from the need to:*

- *develop advanced credit models to be coupled with classical asset classes models (FX, equity, rates, commodities...);*
- *know the whole netting sets trading portfolios; traders would have to worry only about their specific deals and asset classes, as the CVA desk takes care of "options on whole portfolios" embedded in counterparty risk pricing and hedging;*
- *Hedge counterparty credit risk, which is very complicated.*

[a] See for example "CVA Desk in the Bank Implementation", *Global Market Solutions* white paper

Q & A: CVA Desks? "Best practices"?

Q Is this working?

A *Of course the idea of being able to relegate all CVA(/DVA/FVA) issues to a single specialized trading desk is a little delusional.*

- *WWR makes isolating CVA from other activities quite difficult.*
- *In particular WWR means that the idea of hedging CVA and the pure classical risks separately is not effective.*
- *CVA calculations may depend on the collateral policy, which does not depend on the CVA desk or even on the trading floor.*
- *We have seen FVA and CVA interact*

In any case a CVA desk can have different levels of sophistication and effectiveness.

Q & A: CVA Desks? "Best practices"?

Q What do "classical traders" think about this?

A *Clearly, being P&L sensitive this is rather delicate. There are mixed feelings.*

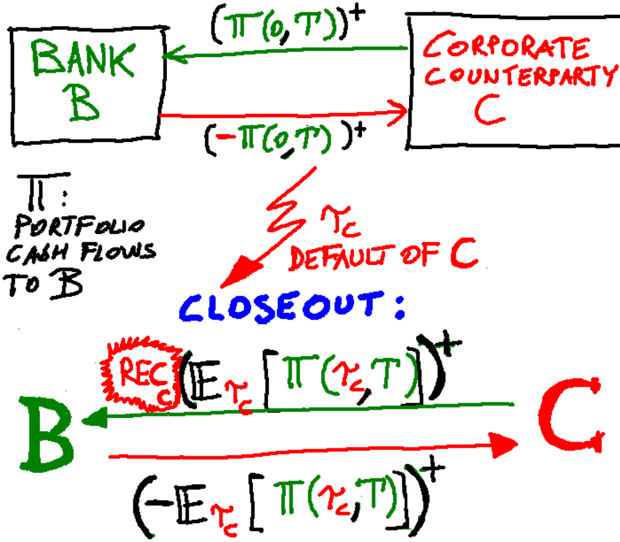
- *Because CVA is hard to hedge (especially the jump to default risk and WWR), occasionally classical traders feel that the CVA desk does not really hedge their counterparty risk effectively and question the validity of the CVA fees they pay to the CVA desk.*
- *Other traders are more optimistic and feel protected by the admittedly approximate hedges implemented by the CVA desk.*
- *There is also a psychological component of relief in delegating management of counterparty risk elsewhere.*

For an introductory dialogue on Counterparty Risk see

CVA Q&A

D. Brigo (2012). Counterparty Risk Q&A: Credit VaR, CVA, DVA, Closeout, Netting, Collateral, Re-hypothecation, Wrong Way Risk, Basel, Funding, and Margin Lending. SSRN.com and arXiv.org.

Context


BANK
B
$(\Pi(0,T))^+$
CORPORATE
COUNTERPARTY
C
$(-\Pi(0,T))^+$
Π :
PORTFOLIO
CASH FLOWS
TO B
τ_C
DEFAULT OF C
CLOSEOUT:
REC
$\mathrm{REC}_C (\mathbb{E}_{\tau_C}[\Pi(\tau_C,T)])^+$
B
C
$(-\mathbb{E}_{\tau_C}[\Pi(\tau_C,T)])^+$

General Notation

- We will call “Bank” or sometimes the ”investor” the party interested in the counterparty adjustment. This is denoted by “B”
- We will call “counterparty” the party with whom the Bank is trading, and whose default may affect negatively the Bank. This is denoted by “C”.
- “1” will be used for the underlying name/risk factor(s) of the contract
- The counterparty’s default time is denoted with τ_C and the recovery rate for unsecured claims with R_{EC_C} (we often use $\text{L}_{\text{GD}_C} := 1 - \text{R}_{\text{EC}_C}$).
- $\Pi_B(t, T)$ is the discounted payout without default risk seen by ‘B’ (sum of all future cash flows between t and T, discounted back at t). $\Pi_C(t, T) = -\Pi_B(t, T)$ is the same quantity but seen from the point of view of ‘C’. When we omit the index B or C we mean ‘B’.

General Notation

- We define $NPV_B(t,T) = \mathbb{E}_t[\Pi(t,T)]$. When T is clear from the context we omit it and write NPV(t).
- $$\Pi(s,t) + D(s,t)\Pi(t,u) = \Pi(s,u)$$
- $$\mathbb{E}_0[D(0,u)NPV(u,T)] = \mathbb{E}_0[D(0,u)\mathbb{E}_u[\Pi(u,T)]] =$$
$$= \mathbb{E}_0[D(0,u)\Pi(u,T)] = NPV(0,T) - \mathbb{E}_0[\Pi(0,u)]$$
$$= NPV(0,T) - NPV(0,u)$$

Unilateral counterparty risk

We now look into unilateral counterparty risk.

This is a situation where counterparty risk pricing is computed by assuming that only the counterparty can default, whereas the investor or bank doing the calculation is assumed to be default free.

Hence we will only consider here the default time τ_C of the counterparty. We will address the bilateral case later on.

The mechanics of Evaluating unilateral counterparty risk

payoff under counterparty default risk

counterparty defaults after final maturity

counterparty defaults before final maturity

original payoff of the instrument

all cash flows before default
$\oplus$ recovery of the residual NPV at default if positive
$\ominus$ Total residual NPV at default if negative

General Formulation under Asymmetry

$$\Pi^D_B(t,T) = \mathbf{1}_{\tau_C > T}\Pi_B(t,T)$$
$$+\mathbf{1}_{t<\tau_C\leq T}\left[\Pi_B(t,\tau_C) + D(t,\tau_C)\left(REC_C\,(\mathrm{NPV}_B(\tau_C))^+ - (-\mathrm{NPV}_B(\tau_C))^+\right.\right.$$

This last expression is the general payoff seen from the point of view of 'B' (Π_B, NPV_B) under unilateral counterparty default risk. Indeed,

1. if there is no early default, this expression reduces to first term on the right hand side, which is the payoff of a default-free claim.
2. In case of early default of the counterparty, the payments due before default occurs are received (second term)
3. and then if the residual net present value is positive only the recovery value of the counterparty REC_C is received (third term),
4. whereas if it is negative it is paid in full by the investor/ Bank (fourth term).

General Formulation under Asymmetry

If one simplifies the cash flows and takes the risk neutral expectation, one obtains the fundamental formula for the valuation of counterparty risk when the investor/ Bank B is default free:

$$\mathbb{E}_t\left\{\Pi_B^D(t,T)\right\} =$$
$$\mathbf{1}_{\{\tau_C>t\}}\mathbb{E}_t\left\{\Pi_B(t,T)\right\} - \mathbb{E}_t\left\{\mathrm{LGD_C}\mathbf{1}_{\{t<\tau_C\le T\}}\mathrm{D}(t,\tau_C)\left[\mathrm{NPV_B}(\tau_C)\right]^+\right\} \quad (*)$$

- First term : Value without counterparty risk.
- Second term : Unilateral Counterparty Valuation Adjustment
- $\mathrm{NPV}(\tau_C) = \mathbb{E}_{\tau_C}\left[\Pi(\tau_C,T)\right]$ is the value of the transaction on the counterparty default date. LGD = 1 - REC_counterparty.

$$\mathrm{UCVA}_0 = \mathbb{E}_t\left\{\mathrm{LGD_C}\mathbf{1}_{\{t<\tau_C\le T\}}\mathrm{D}(t,\tau_C)\left[\mathrm{NPV_B}(\tau_C)\right]^+\right\}$$

Proof of the formula

In the proof we omit indices: $\tau = \tau_C$, REC=REC$_C$, LGD=LGD$_C$, NPV=NPV$_B$, $\Pi = \Pi_B$. The proof is obtained easily putting together the following steps. Since

$$1_{\{\tau>t\}}\Pi(t,T) = 1_{\{\tau>T\}}\Pi(t,T) + 1_{\{t<\tau\le T\}}\Pi(t,T)$$

we can rewrite the terms inside the expectation in the right hand side of the simplified formula (*) as

$$\begin{aligned}
& \mathbf{1}_{\{\tau>t\}}\Pi(t,T) - \left\{LGD\mathbf{1}_{\{t<\tau\le T\}}D(t,\tau)\left[\mathrm{NPV}(\tau)\right]^+\right\} \\
& = 1_{\{\tau>T\}}\Pi(t,T) + 1_{\{t<\tau\le T\}}\Pi(t,T) \\
+ \quad & \{(\mathrm{Rec}-1)[1_{\{t<\tau\le T\}}D(t,\tau)(\mathrm{NPV}(\tau))^+]\} \\
= \quad & 1_{\{\tau>T\}}\Pi(t,T) + 1_{\{t<\tau\le T\}}\Pi(t,T) \\
+ \quad & \mathrm{Rec}\ \ 1_{\{t<\tau\le T\}}D(t,\tau)(\mathrm{NPV}(\tau))^+ - 1_{\{t<\tau\le T\}}D(t,\tau)(\mathrm{NPV}(\tau))^+
\end{aligned}$$

Conditional on the information at τ the second and the fourth terms are equal to

Proof (cont'd)

$$
\begin{aligned}
& E_\tau[1_{\{t<\tau\leq T\}}\Pi(t,T) - 1_{\{t<\tau\leq T\}}D(t,\tau)(\text{NPV}(\tau))^+] \\
= \; & E_\tau[1_{\{t<\tau\leq T\}}[\Pi(t,\tau) + D(t,\tau)\Pi(\tau,T) - D(t,\tau)(E_\tau[\Pi(\tau,T)])^+]] \\
= \; & 1_{\{t<\tau\leq T\}}[\Pi(t,\tau) + D(t,\tau)E_\tau[\Pi(\tau,T)] - D(t,\tau)(E_\tau[\Pi(\tau,T)])^+] \\
= \; & 1_{\{t<\tau\leq T\}}[\Pi(t,\tau) - D(t,\tau)(E_\tau[\Pi(\tau,T)])^-] \\
= \; & 1_{\{t<\tau\leq T\}}[\Pi(t,\tau) - D(t,\tau)(E_\tau[-\Pi(\tau,T)])^+] \\
= \; & 1_{\{t<\tau\leq T\}}[\Pi(t,\tau) - D(t,\tau)(-\text{NPV}(\tau))^+]
\end{aligned}
$$

since

$$1_{\{t<\tau\leq T\}}\Pi(t,T) = 1_{\{t<\tau\leq T\}}\{\Pi(t,\tau) + D(t,\tau)\Pi(\tau,T)\}$$

and $f = f^+ - f^- = f^+ - (-f)^+$.

Proof (cont'd)

Then we can see that after conditioning the whole expression of the original long payoff on the information at time τ and substituting the second and the fourth terms just derived above, the expected value with respect to $\mathcal{F}_t$ coincides exactly with the one in our simplified formula (*) by the properties of iterated expectations by which $\mathbb{E}_t[X] = \mathbb{E}_t[\mathbb{E}_\tau[X]]$.

What we can observe

- Including counterparty risk in the valuation of an otherwise default-free derivative $\Longrightarrow$ credit (hybrid) derivative.

- The inclusion of counterparty risk adds a level of optionality to the payoff.
  In particular, model independent products become model dependent also in the underlying market.
  $\Longrightarrow$ **Counterparty Risk analysis incorporates an opinion about the underlying market dynamics and volatility.**

The point of view of the counterparty "C"

The deal from the point of view of 'C', while staying in a world where only 'C" may default.

$$\Pi_C^D(t,T) = \mathbf{1}_{\tau_C>T}\Pi_C(t,T)$$
$$+\mathbf{1}_{t<\tau_C\leq T}\left[\Pi_C(t,\tau_C) + D(t,\tau_C)\left((\mathrm{NPV}_C(\tau_C))^+ - REC_C\left(-\mathrm{NPV}_C(\tau_C)\right)^+\right.\right.$$

This last expression is the general payoff seen from the point of view of 'C' (Π_C, NPV_C) under unilateral counterparty default risk. Indeed,

1. if there is no early default, this expression reduces to first term on the right hand side, which is the payoff of a default-free claim.
2. In case of early default of the counterparty 'C", the payments due before default occurs go through (second term)
3. and then if the residual net present value is positive to the defaulted 'C', it is received in full from 'B' (third term),
4. whereas if it is negative, only the recovery fraction REC_C it is paid to 'B' (fourth term).

The point of view of the counterparty "C"

The above formula simplifies to

$$\mathbb{E}_t \left\{ \Pi_C^D(t,T) \right\} =$$
$$\mathbf{1}_{\tau_C > t} \mathbb{E}_t \left\{ \Pi_C(t,T) \right\} + \mathbb{E}_t \left\{ \mathrm{LGD}_C \mathbf{1}_{t < \tau_C \leq T} D(t,\tau_C) \left[-\mathrm{NPV}_C(\tau_C) \right]^+ \right\}$$

and the adjustment term with respect to the risk free price $\mathbb{E}_t \left\{ \Pi_C(t,T) \right\}$ is called

UNILATERAL DEBIT VALUATION ADJUSTMENT

$$\mathrm{UDVA}_C(t) = \mathbb{E}_t \left\{ \mathrm{LGD}_C \mathbf{1}_{\{t < \tau_C \leq T\}} D(t,\tau_C) \left[-\mathrm{NPV}_C(\tau_C) \right]^+ \right\}$$

We note that $\mathrm{UDVA}_C = \mathrm{UCVA}_B$.
Notice also that in this universe $\mathrm{UDVA}_B = \mathrm{UCVA}_C = 0$.

Including the investor/ Bank default or not?

Often the investor, when computing a counterparty risk adjustment, considers itself to be default-free. This can be either a unrealistic assumption or an approximation for the case when the counterparty has a much higher default probability than the investor.

If this assumption is made when no party is actually default-free, the unilateral valuation adjustment is asymmetric: if “C” were to consider itself as default free and “B” as counterparty, and if “C” computed the counterparty risk adjustment, this would not be the opposite of the one computed by “B” in the straight case.

Also, the total NPV including counterparty risk is similarly asymmetric, in that the total value of the position to “B” is not the opposite of the total value of the position to “C”. There is no *cash conservation.*

Including the investor/ Bank default or not?

We get back symmetry if we allow for default of the investor/ Bank in computing counterparty risk. This also results in an adjustment that is cheaper to the counterparty “C”.

The counterparty “C” may then be willing to ask the investor/ Bank “B” to include the investor default event into the model, when the Counterparty risk adjustment is computed by the investor

The case of symmetric counterparty risk

Suppose now that we allow for both parties to default. Counterparty risk adjustment allowing for default of "B"?
"B" : the investor; "C": the counterparty;
("1": the underlying name/risk factor of the contract).
τ_B, τ_C: default times of "B" and "C". T: final maturity
We consider the following events, forming a partition

Four events ordering the default times

$$\begin{aligned}
\mathcal{A} &= \{\tau_B \le \tau_C \le T\} \quad & E = \{T \le \tau_B \le \tau_C\} \\
\mathcal{B} &= \{\tau_B \le T \le \tau_C\} \quad & F = \{T \le \tau_C \le \tau_B\} \\
\mathcal{C} &= \{\tau_C \le \tau_B \le T\} & \\
\mathcal{D} &= \{\tau_C \le T \le \tau_B\} &
\end{aligned}$$

Define $\mathrm{NPV}_{\{B,C\}}(t) := \mathbb{E}_t[\Pi_{\{B,C\}}(t,T)]$, and recall $\Pi_B = -\Pi_C$.

The case of symmetric counterparty risk

$$\Pi^D_B(t,T) = \mathbf{1}_{E\cup F}\Pi_B(t,T)$$
$$+\mathbf{1}_{\mathcal{C}\cup\mathcal{D}}\left[\Pi_B(t,\tau_C) + D(t,\tau_C)\left(REC_C\left(\mathrm{NPV}_B(\tau_C)\right)^+ - \left(-\mathrm{NPV}_B(\tau_C)\right)^+\right)\right]$$
$$+\mathbf{1}_{\mathcal{A}\cup\mathcal{B}}\left[\Pi_B(t,\tau_B) + D(t,\tau_B)\left(\left(\mathrm{NPV}_B(\tau_B)\right)^+ - REC_B\left(-\mathrm{NPV}_B(\tau_B)\right)^+\right)\right]$$

1. If no early default $\Rightarrow$ payoff of a default-free claim (1st term).
2. In case of early default of the counterparty, the payments due before default occurs are received (second term),
3. and then if the residual net present value is positive only the recovery value of the counterparty REC_C is received (third term),
4. whereas if negative, it is paid in full by the investor/ Bank (4th term).
5. In case of early default of the investor, the payments due before default occurs are received (fifth term),
6. and then if the residual net present value is positive it is paid in full by the counterparty to the investor/ Bank (sixth term),
7. whereas if it is negative only the recovery value of the investor/ Bank REC_B is paid to the counterparty (seventh term).

The case of symmetric counterparty risk

$$\mathbb{E}_t\left\{\Pi^D_B(t,T)\right\} = \mathbb{E}_t\left\{\Pi_B(t,T)\right\} + \mathrm{DVA}_B(t) - \mathrm{CVA}_B(t)$$

$$\mathrm{DVA}_B(t) = \mathbb{E}_\mathrm{t}\left\{\mathrm{LGD_B}\cdot\mathbf{1}(\mathrm{t} < \tau^{1\mathrm{st}} = \tau_\mathrm{B} < \mathrm{T})\cdot\mathrm{D}(\mathrm{t},\tau_\mathrm{B})\cdot[-\mathrm{NPV_B}(\tau_\mathrm{B})]^+\right\}$$

$$\mathrm{CVA}_B(t) = \mathbb{E}_\mathrm{t}\left\{\mathrm{LGD_C}\cdot\mathbf{1}(\mathrm{t} < \tau^{1\mathrm{st}} = \tau_\mathrm{C} < \mathrm{T})\cdot\mathrm{D}(\mathrm{t},\tau_\mathrm{C})\cdot[\mathrm{NPV_B}(\tau_\mathrm{C})]^+\right\}$$

$$1(A\cup B) = 1(t < \tau^{1st} = \tau_B < T),\;\; 1(C\cup D) = 1(t < \tau^{1st} = \tau_C < T)$$

- Obtained simplifying the previous formula and taking expectation.
- 2nd term : adj due to scenarios $\tau_B < \tau_C$. This is positive to the investor/ Bank B and is called ”Debit Valuation Adjustment” (DVA)
- 3d term : Counterparty risk adj due to scenarios $\tau_C < \tau_B$
- Bilateral Valuation Adjustment as seen from B: $\mathrm{BVA}_B = \mathrm{DVA}_B - \mathrm{CVA}_B$.
- If computed from the opposite point of view of “C” having counterparty “B”, $\mathrm{BVA}_C = -\mathrm{BVA}_B$. Symmetry.

The case of symmetric counterparty risk

Strange consequences of the formula new mid term, i.e. DVA

- credit quality of investor WORSENS $\Rightarrow$ books POSITIVE MARK TO MKT
- credit quality of investor IMPROVES $\Rightarrow$ books NEGATIVE MARK TO MKT
- Citigroup in its press release on the first quarter revenues of 2009 reported a *positive* mark to market due to its *worsened* credit quality: "Revenues also included [...] a net 2.5$ billion positive CVA on derivative positions, excluding monolines, mainly due to the widening of Citi's CDS spreads"

The case of symmetric counterparty risk: DVA?

`October 18, 2011, 3:59 PM ET, WSJ. Goldman Sachs Hedges Its Way to Less Volatile Earnings.`

Goldman's DVA gains in the third quarter totaled $450 million [...] That amount is comparatively smaller than the $1.9 billion in DVA gains that J.P. Morgan Chase and Citigroup each recorded for the third quarter. Bank of America reported $1.7 billion of DVA gains in its investment bank. Analysts estimated that Morgan Stanley will record $1.5 billion of net DVA gains when it reports earnings on Wednesday [...]

Is DVA real? **DVA Hedging**. Buying back bonds? Proxying?

DVA hedge? One should sell protection on oneself, impossible, unless one buys back bonds that he had issued earlier. Very Difficult. Most times: proxying. Instead of selling protection on oneself, one sells protection on a number of names that one thinks are highly correlated to oneself.

The case of symmetric counterparty risk: DVA?

Again from the WSJ article above:

[...] Goldman Sachs CFO David Viniar said Tuesday that the company attempts to hedge [DVA] using a basket of different financials.
A Goldman spokesman confirmed that the company did this by selling CDS on a range of financial firms. [...] Goldman wouldn't say what specific financials were in the basket, but Viniar confirmed [...] that the basket contained 'a peer group.'

This can approximately hedge the spread risk of DVA, but not the jump to default risk. Merrill hedging DVA risk by selling protection on Lehman would not have been a good idea. Worsens systemic risk.

DVA or no DVA? Accounting VS Capital Requirements

NO DVA: Basel III, page 37, July 2011 release

This CVA loss is calculated without taking into account any offsetting debit valuation adjustments which have been deducted from capital under paragraph 75.

YES DVA: FAS 157

Because nonperformance risk (the risk that the obligation will not be fulfilled) includes the reporting entitys credit risk, the reporting entity should consider the effect of its credit risk (credit standing) on the fair value of the liability in all periods in which the liability is measured at fair value under other accounting pronouncements FAS 157 (see also IAS 39)

DVA or no DVA? Accounting VS Capital Requirements

Stefan Walter says:

”The potential for perverse incentives resulting from profit being linked to decreasing creditworthiness means capital requirements cannot recognise it, says Stefan Walter, *secretary-general of the Basel Committee*: The main reason for not recognising DVA as an offset is that it would be inconsistent with the overarching supervisory prudence principle under which we do not give credit for increases in regulatory capital arising from a deterioration in the firms own credit quality.”

The case of symmetric counterparty risk: DVA?

When allowing for the investor to default: symmetry

- DVA: One more term with respect to the unilateral case.
- depending on credit spreads and correlations, the total adjustment to be subtracted (CVA-DVA) can now be either positive or negative. In the unilateral case it can only be positive.
- Ignoring the symmetry is clearly more expensive for the counterparty and cheaper for the investor.
- Hedging DVA is difficult. Hedging “by peers” ignores jump to default risk
- We assume the unilateral case in most of the numerical presentations
- WE TAKE THE POINT OF VIEW OF ‘B” from now on, so we omit the subscript ‘B’. We denote the counterparty as ‘C”.

Closeout: Replication (ISDA?) VS Risk Free

When we computed the bilateral adjustment formula from

$$\Pi_B^D(t,T) = \mathbf{1}_{E\cup F}\Pi_B(t,T)$$
$$+\mathbf{1}_{C\cup D}\left[\Pi_B(t,\tau_C) + D(t,\tau_C)\left(REC_C\,(\mathrm{NPV}_B(\tau_C))^+ - (-\mathrm{NPV}_B(\tau_C))^+\right)\right]$$
$$+\mathbf{1}_{A\cup B}\left[\Pi_B(t,\tau_B) + D(t,\tau_B)\left((-\mathrm{NPV}_C(\tau_B))^+ - REC_B\,(\mathrm{NPV}_C(\tau_B))^+\right)\right]$$

(where we now substituted $\mathrm{NPV}_B = -\mathrm{NPV}_C$ in the last two terms) we used the risk free NPV upon the first default, to close the deal. But what if upon default of the first entity, the deal needs to be valued by taking into account the credit quality of the surviving party? What if we make the substitutions

$$\mathrm{NPV}_B(\tau_C) \to \mathrm{NPV}_B(\tau_C) + \mathrm{UDVA}_B(\tau_C)$$

$$\mathrm{NPV}_C(\tau_B) \to \mathrm{NPV}_C(\tau_B) + \mathrm{UDVA}_C(\tau_B)?$$

Closeout: Replication (ISDA?) VS Risk Free

ISDA (2009) Close-out Amount Protocol.

”In determining a Close-out Amount, the Determining Party may consider any relevant information, including, [...] quotations (either firm or indicative) for replacement transactions supplied by one or more third parties that **may take into account the creditworthiness of the Determining Party** at the time the quotation is provided”

This makes valuation more continuous: upon default we still price including the DVA, as we were doing before default.

Closeout: Substitution (ISDA?) VS Risk Free

The final formula with subsitution closeout is quite complicated:

$$\Pi_B^D(t,T) = \mathbf{1}_{E\cup F}\Pi_B(t,T)$$
$$+\mathbf{1}_{C\cup D}\Big[\Pi_B(t,\tau_C) + D(t,\tau_C)$$
$$\cdot\left(REC_C\,(\mathrm{NPV}_B(\tau_C) + \mathrm{UDVA}_B(\tau_C))^+ - (-\mathrm{NPV}_B(\tau_C) - \mathrm{UDVA}_B(\tau_C))^+\right)$$
$$+\mathbf{1}_{A\cup B}\Big[\Pi_B(t,\tau_B) + D(t,\tau_B)$$
$$\cdot\left((-\mathrm{NPV}_C(\tau_B) - \mathrm{UDVA}_C(\tau_B))^+ - REC_B\,(\mathrm{NPV}_C(\tau_B) + \mathrm{UDVA}_C(\tau_B))^+\right)$$

Closeout: Substitution (ISDA?) VS Risk Free

B. and Morini (2010)

We analyze the Risk Free closeout formula in Comparison with the Replication Closeout formula for a Zero coupon bond when:
1. Default of ‘B’ and ‘C” are independent
2. Default of ‘B’ and ‘C” are co-monotonic

Suppose ‘B’ (the lender) holds the bond,
and ‘C’ (the borrower) will pay the notional 1 at maturity T.

The risk free price of the bond at time 0 to ’B’ is denoted by $P(0, T)$.

Closeout: Replication (ISDA?) VS Risk Free

Suppose 'B' (the lender) holds the bond, and 'C' (the borrower) will pay the notional 1 at maturity T.
The risk free price of the bond at time 0 to 'B' is denoted by $P(0, T)$.
If we assume deterministic interest rates, the above formulas reduce to

$$P^{D,Repl}(0,T) = P(0,T)[\mathbb{Q}(\tau_C > T) + REC_C \mathbb{Q}(\tau_C \leq T)]$$

$$\begin{aligned} P^{D,Free}(0,T) = P(0,T)[&\mathbb{Q}(\tau_C > T) + \mathbb{Q}(\tau_B < \tau_C < T) \\ &+ REC_C \mathbb{Q}(\tau_C \leq \min(\tau_B, T))] \end{aligned}$$

$$= P(0,T)[\mathbb{Q}(\tau_C > T) + REC_C \mathbb{Q}(\tau_C \leq T) + LGD_C \mathbb{Q}(\tau_B < \tau_C < T)]$$

Risk Free Closeout and Credit Risk of the Lender

The adjusted price of the bond DEPENDS ON THE CREDIT RISK OF THE LENDER 'B' IF WE USE THE RISK FREE CLOSEOUT. This is counterintuitive and undesirable.

Closeout: Replication (ISDA?) VS Risk Free

Co-Monotonic Case

If we assume the default of B and C to be co-monotonic, and the spread of the lender ‘B” to be larger, we have that the lender ‘B” defaults first in ALL SCENARIOS (e.g. ‘C’ is a subsidiary of ‘B’, or a company whose well being is completely driven by ‘B’: ‘C’ is a trye factory whose only client is car producer ‘B”). In this case

$$P^{D,Repl}(0,T) = P(0,T)[\mathbb{Q}(\tau_C > T) + REC_C\mathbb{Q}(\tau_C \leq T)]$$

$$P^{D,Free}(0,T) = P(0,T)[\mathbb{Q}(\tau_C > T) + \mathbb{Q}(\tau_C < T)] = P(0,T)$$

Risk free closeout is correct. Either ‘B” does not default, and then ‘C” does not default either, or if ‘B” defaults, at that precise time C is solvent, and B recovers the whole payment. Credit risk of ‘C” should not impact the deal.

Closeout: Substitution (ISDA?) VS Risk Free

Contagion. What happens at default of the Lender

$$P^{D,Subs}(t,T) = P(t,T)[\mathbb{Q}_t(\tau_C > T) + REC_C \mathbb{Q}_t(\tau_C \le T)]$$

$$P^{D,Free}(t,T) = P^{D,Subs}(t,T) + P(t,T) LGD_C \mathbb{Q}_t(\tau_B < \tau_C < T)$$

We focus on two cases:

- τ_B and τ_C are independent. Take $t < T$.

$$\mathbb{Q}_{t-\Delta t}(\tau_B < \tau_C < T) \mapsto \quad \{\tau_B = t\} \quad \mapsto \mathbb{Q}_{t+\Delta t}(\tau_C < T)$$

  and this effect can be quite sizeable.

- τ_B and τ_C are comonotonic. Take an example where $\tau_B = t < T$ implies $\tau_C = u < T$ with $u > t$. Then

$$\mathbb{Q}_{t-\Delta t}(\tau_C > T) \mapsto \quad \{\tau_B = t, \tau_C = u\} \quad \mapsto 0$$

$$\mathbb{Q}_{t-\Delta t}(\tau_C \le T) \mapsto \quad \{\tau_B = t, \tau_C = u\} \quad \mapsto 1$$

$$\mathbb{Q}_{t-\Delta t}(\tau_B < \tau_C < T) \mapsto \quad \{\tau_B = t, \tau_C = u\} \quad \mapsto 1$$

Closeout: Substitution (ISDA?) VS Risk Free

Let us put the pieces together:

- τ_B and τ_C are independent. Take $t < T$.

$$P^{D,Subs}(t-\Delta t, T) \mapsto \quad \{\tau_B = t\} \mapsto \quad \text{no change}$$

$$P^{D,Free}(t-\Delta t, T) \mapsto \quad \{\tau_B = t\} \mapsto \quad \text{add } \mathbb{Q}_{t-\Delta t}(\tau_B > \tau_C, \tau_C < T)$$

  and this effect can be quite sizeable.

- τ_B and τ_C are comonotonic. Take an example where $\tau_B = t < T$ implies $\tau_C = u < T$ with $u > t$. Then

$$P^{D,Subs}(t-\Delta t, T) \mapsto \quad \{\tau_B = t\} \mapsto \quad \text{subtract } X$$

$$X = LGD_C P(t,T)\mathbb{Q}_{t-\Delta t}(\tau_C > T)$$

$$P^{D,Free}(t-\Delta t, T) \mapsto \quad \{\tau_B = t\} \mapsto \quad \text{no change}$$

Closeout: Replication (ISDA?) VS Risk Free

The independence case: Contagion with Risk Free closeout

The Risk Free closeout shows that *upon default of the lender*, the mark to market to the lender itself jumps up, or equivalently **the mark to market to the borrower jumps down**. The effect can be quite dramatic.
The Replication closeout instead shows no such contagion, as the mark to market does not change upon default of the lender.

The co-monotonic case: Contagion with Replication closeout

The Risk Free closeout behaves nicely in the co-monotonic case, and there is no change upon default of the lender.
Instead the Replication closeout shows that *upon default of the lender* the mark to market to the lender jumps down, or equivalently **the mark to market to the borrower jumps up**.

Closeout: Replication (ISDA?) VS Risk Free

Impact of an early default of the Lender

Dependence→ Closeout↓	independence	co-monotonicity
Risk Free	Negatively affects Borrower	No contagion
Replication	No contagion	Further Negatively affects Lender

For a numerical case study and more details see Brigo and Morini (2010, 2011).

A simplified formula without τ^{1st} for bilateral VA

- The simplified formula is only a simplified representation of bilateral risk and ignores that upon the first default closeout proceedings are started, thus involving a degree of double counting
- It is attractive because it allows for the construction of a bilateral counterparty risk pricing system based only on a unilateral one.
- The correct formula involves default dependence between the two parties through τ^{1st} and allows no such incremental construction
- A simplified bilateral formula is possible also in case of substitution closeout, but it turns out to be identical to the simplified formula of the risk free closeout case.
- We analyze the impact of default dependence between investor 'B' and counterparty 'C' on the difference between the two formulas by looking at a zero coupon bond and at an equity forward.

A simplified formula without τ^{1st} for bilateral VA

One can show easily that the difference between the full correct formula and the simplified formula is

$$\begin{aligned} & E_0[1_{\{\tau_B<\tau_C<T\}} LGD_C D(0,\tau_C)(E_{\tau_C}(\Pi(\tau_C,T)))^+] \\ - \quad & E_0[1_{\{\tau_C<\tau_B<T\}} LGD_B D(0,\tau_B)(-E_{\tau_B}(\Pi(\tau_B,T)))^+]. \end{aligned} \tag{2}$$

A simplified formula without τ^{1st}: The case of a Zero Coupon Bond

We work under deterministic interest rates. We consider $P(t, T)$ held by 'B" (lender) who will receive the notional 1 from 'C"(borrower) at final maturity T if there has been no default of 'C".

The difference between the correct bilateral formula and the simplified one is, under risk free closeout,

$$LGD_C P(0, T)\mathbb{Q}(\tau_B < \tau_C < T).$$

The case with substitution closeout is instead trivial and the difference is null. For a bond, the simplified formula coincides with the full substitution closeout formula.

Therefore the difference above is the same difference between risk free closeout and substitution closeout formulas, and has been examined earlier, also in terms of contagion.

A simplified formula without τ^{1st}: The case of an Equity forward

In this case the payoff at maturity time T is given by $S_T - K$

where S_T is the price of the underlying equity at time T and K the strike price of the forward contract (typically $K = S_0$, ‘at the money’, or $K = S_0/P(0,T)$, ‘at the money forward’).

We compute the difference D^{BC} between the correct bilateral risk free closeout formula and the simplified one.

A simplified formula without τ^{1st}: The case of an Equity forward

$D^{BC} := A_1 - A_2$, where

$$A_1 = E_0 \left\{ 1_{\{\tau_B < \tau_C < T\}} LGD_C D(0, \tau_C)(S_{\tau_C} - P(\tau_C, T)K)^+ \right\}$$
$$A_2 = E_0 \left\{ 1_{\{\tau_C < \tau_B < T\}} LGD_B D(0, \tau_B)(P(\tau_B, T)K - S_{\tau_B})^+ \right\}$$

The worst cases will be the ones where the terms A_1 and A_2 do not compensate. For example assume there is a high probability that $\tau_B < \tau_C$ and that the forward contract is deep in the money. In such case A_1 will be large and A_2 will be small.

Similarly, a case where $\tau_C < \tau_B$ is very likely and where the forward is deep out of the money will lead to a large A_2 and to a small A_1.

However, we show with a numerical example that even when the forward is at the money the difference can be relevant. For more details see Brigo and Buescu (2011).

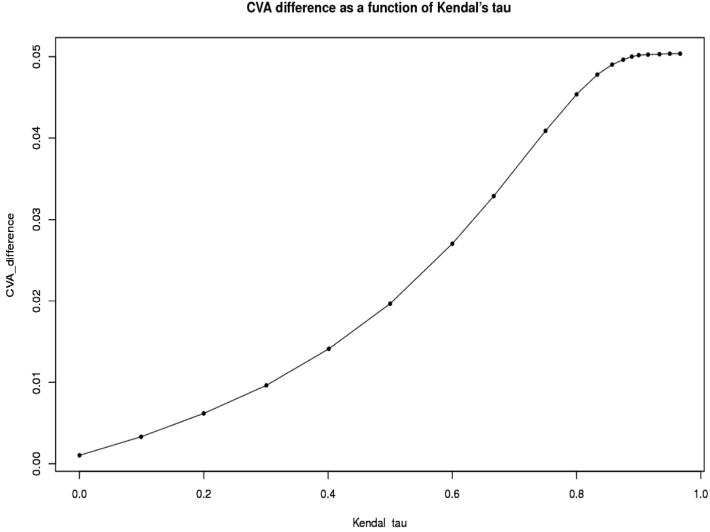


Figure: D^{BC} plotted against Kendall's tau between τ_B and τ_C, all other quantities being equal: $S_0 = 1$, $T = 5$, $\sigma = 0.4$, $K = 1$, $\lambda_B = 0.1$, $\lambda_C = 0.05$.

PAYOFF RISK

The exact payout corresponding with the Credit and Debit valuation adjustment is not clear.

- DVA or not?
- Which Closeout?
- First to default risk or not?
- How are collateral and funding accounted for exactly?

Worse than model risk: Payout risk. WHICH PAYOUT?

At a recent industry panel (WBS) on CVA it was stated that 5 banks might compute CVA in 15 different ways.

Methodology

1. Assumption: The *Bank/investor* enters a transaction with a *counterparty* and, when dealing with Unilateral Risk, the investor considers itself default free.
   Note : All the payoffs seen from the point of view of the *investor*.
2. We model and calibrate the default time of the *counterparty* using a stochastic intensity default model, except in the equity case where we will use a firm value model.
3. We model the transaction underlying and estimate the deal NPV at default.
4. We allow for the counterparty default time and the contract underlying to be correlated.
5. We start however from the case when such correlation can be neglected.

Approximation: Default Bucketing

General Formulation

1. Model (underlying) to estimate the NPV of the transaction.
2. Simulations are run allowing for correlation between the credit and underlying models, to determine the counterparty default time and the underlying deal NPV respectively.

Approximated Formulation under default bucketing

$$
\begin{aligned}
\mathbb{E}_0 \Pi^D(0,T) &:= \mathbb{E}_0 \Pi(0,T) - \text{L}_{\text{GD}} \mathbb{E}_0 [1_{\{\tau < T_b\}} D(0,\tau)(\mathbb{E}_\tau \Pi(\tau,T))^+] \\
&= \mathbb{E}_0 \Pi(0,T) - \text{L}_{\text{GD}} \mathbb{E}_0 [(\sum_{j=1}^{b} 1\{\tau \in (T_{j-1}, T_j]\})\, D(0,\tau)(\mathbb{E}_\tau \Pi(\tau,T))^+] \\
&= \mathbb{E}_0 \Pi(0,T) - \text{L}_{\text{GD}} \sum_{j=1}^{b} \mathbb{E}_0 [1\{\tau \in (T_{j-1}, T_j]\}\, D(0,\tau)(\mathbb{E}_\tau \Pi(\tau,T))^+] \\
&\approx \mathbb{E}_0 \Pi(0,T) - \text{L}_{\text{GD}} \sum_{j=1}^{b} \mathbb{E}_0 [1\{\tau \in (T_{j-1}, T_j]\}\, D(0,T_j)(\mathbb{E}_{T_j} \Pi(T_j,T))^+]
\end{aligned}
$$

Approximation: Default Bucketing and Independence

1. In this formulation defaults are bucketed but we still need a joint model for τ and the underlying Π including their correlation.
2. Option model for Π is implicitly needed in τ scenarios.

Approximated Formulation under independence (and 0 correlation)

$$\mathbb{E}_0 \Pi^D(0,T) := \mathbb{E}_0 \Pi(0,T)$$

$$-\mathrm{L}_{\mathrm{GD}} \sum_{j=1}^{b} \boxed{\mathbb{Q}\{\tau \in (T_{j-1}, T_j]\}} \mathbb{E}_0[D(0,T_j)(\mathbb{E}_{T_j}\Pi(T_j,T))^+]$$

1. In this formulation defaults are bucketed and only survival probabilities are needed (no default model).
2. Option model is STILL needed for the underlying of Π.

Ctrparty default model: CIR++ stochastic intensity

If we cannot assume independence, we need a default model.
Counterparty instantaneous credit spread: $\lambda(t) = y(t) + \psi(t;\beta)$

1. $y(t)$ is a CIR process with possible jumps

$$dy_t = \kappa(\mu - y_t)dt + \nu\sqrt{y_t}dW_t^y + dJ_t,\ \ \tau_C = \Lambda^{-1}(\xi),\ \ \Lambda(T) = \int_0^T \lambda(s)ds$$

2. $\psi(t;\beta)$ is the shift that matches a given CDS curve
3. ξ is standard exponential independent of all brownian driven stochastic processes
4. In CDS calibration we assume deterministic interest rates.
5. Calibration : Closed form Fitting of model survival probabilities to counterparty CDS quotes
6. B and El Bachir (2010) (Mathematical Finance) show that this model with jumps has closed form solutions for CDS options.

4 cases: Rates, Credit, Commodities and Equity

Impact of dynamics, volatilities, correlations, wrong way risk

- **Interest Rate Swaps and Derivatives Portfolios** (B. Masetti (2005), B. Pallavicini 2007, 2008, B. Capponi P. Papatheodorou 2011, B. C. P. P. 2012 with collateral and gap risk)
- **Commodities swaps (Oil)** (B. and Bakkar 2009)
- **Credit: CDS on a reference credit** (B. and Chourdakis 2009, B. C. Pallavicini 2012 Mathematical Finance)
- **Equity Return Swaps** (B. and Tarenghi 2004, B. T. Morini 2011)
- Equity uses AT1P firm value model of B. and T. (2004) (barrier options with time-inhomogeneous GBM) and extensions (random barriers for risk of fraud).

Further asset classes are studied in the literature. For example see Biffis et al (2011) for CVA on **longevity swaps.**

4 cases: Interest Rates, Credit, Commodities and Equity

We now examine UCVA with WWR for:

- Interest Rate Swaps and Derivatives Portfolios
- Commodities swaps (Oil)
- Credit: CDS on a reference credit
- Equity: Equity Return Swaps

Interest Rates Swap Case

Formulation for IRS under independence (no correlation)

$$\mathrm{IRS}^D(t,K) = \mathrm{IRS}(t.K)$$
$$-\textsc{Lgd} \sum_{i=a+1}^{b-1} \mathbb{Q}\{\tau \in (T_{i-1}, T_i]\}\, \textsc{Swaption}_{i,b}(t; K, S_{i,b}(t), \sigma_{i,b})$$

Modeling Approach with corr.

Gaussian 2-factor G2++ short-rate $r(t)$ model:
$r(t) = x(t) + z(t) + \varphi(t;\alpha), r(0) = r_0$

$$dx(t) = -ax(t)dt + \sigma dW_x$$
$$dz(t) = -bz(t)dt + \eta dW_z$$

$$dW_x dW_z = \rho_{x,z} dt$$

$$\alpha = [r_0, a, b, \sigma, \eta, \rho_{1,2}]$$

$$dW_x dW_y = \rho_{x,y} dt, dW_z dW_y = \rho_{z,y} dt$$

Calibration

- The function $\varphi(\cdot;\alpha)$ is deterministic and is used to calibrate the initial curve observed in the market.
- We use swaptions and zero curve data to calibrate the model.
- The r factors x and z and the intensity are taken to be correlated.

Interest Rates Swap Case

Total Correlation Counterparty default / rates

$$\bar{\rho} = \mathrm{Corr}(dr_t, d\lambda_t) = \frac{\sigma\rho_{x,y} + \eta\rho_{z,y}}{\sqrt{\sigma^2 + \eta^2 + 2\sigma\eta\rho_{x,z}}\sqrt{1 + \frac{2\beta\gamma^2}{\nu^2 y_t}}}.$$

where β is the intensity of arrival of λ jumps and γ is the mean of the exponentially distributed jump sizes.

Without jumps ($\beta = 0$)

$$\bar{\rho} = \mathrm{Corr}(dr_t, d\lambda_t) = \frac{\sigma\rho_{x,y} + \eta\rho_{z,y}}{\sqrt{\sigma^2 + \eta^2 + 2\sigma\eta\rho_{x,z}}}.$$

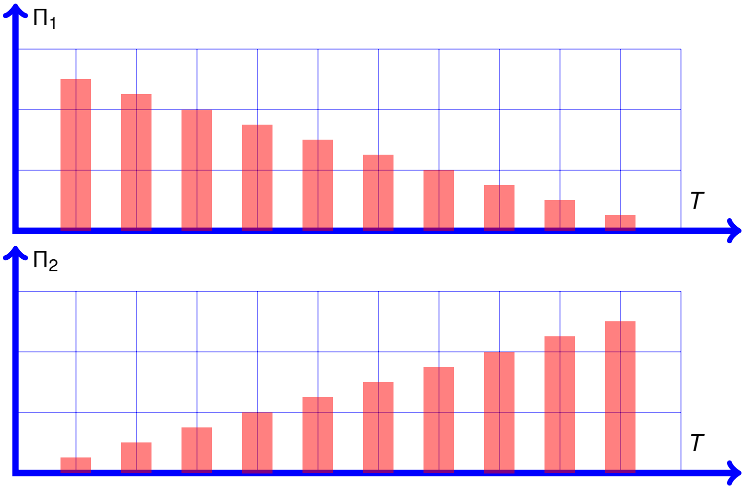
IRS: Case Study

1) Single Interest Rate Swaps (IRS)
At-the-money fix-receiver forward interest-rate-swap (IRS) paying on the EUR market.
The IRS's fixed legs pay annually a 30E/360 strike rate, while the floating legs pay LIBOR twice per year.
2) Netted portfolios of IRS.
- Portfolios of at-the-money IRS either with different starting dates or with different maturities.

1. (Π1) annually spaced dates $\{T_i : i = 0 \dots N\}$, T_0 two business days from trade date; portfolio of swaps maturing at each T_i, with $i > 0$, all starting at T_0.
2. (Π2) portfolio of swaps starting at each T_i all maturing at T_N.

Can also do exotics (Ratchets, CMS spreads, Bermudan)

IRS Case Study: Payment schedules



IRS Results

Counterparty risk price for netted receiver IRS portfolios Π1 and Π2 and simple IRS (maturity 10Y). Every IRS, constituting the portfolios, has unit notional and is at equilibrium. Prices are in bps.

λ	correlation $\bar{\rho}$	Π1	Π2	IRS
3%	-1	-140	-294	-36
	0	-84	-190	-22
	1	-47	-115	-13
5%	-1	-181	-377	-46
	0	-132	-290	-34
	1	-99	-227	-26
7%	-1	-218	-447	-54
	0	-173	-369	-44
	1	-143	-316	-37

Compare with "Basel 2" deduced adjustments

Basel 2, under the "Internal Model Method", models wrong way risk by means of a 1.4 multiplying factor to be applied to the zero correlation case, even if banks have the option to compute their own estimate of the multiplier, which can never go below 1.2 anyway.

Is this confirmed by our model?

$$(140 - 84)/84 \approx 66\% > 40\%$$

$$(54 - 44)/44 \approx 23\% < 40\%$$

So this really depends on the portfolio and on the situation.

A bilateral example and correlation risk

Finally, in the bilateral case for Receiver IRS, 10y maturity, high risk counterparty and mid risk investor, we notice that depending on the correlations

$$\bar{\rho}_0 = \mathrm{Corr}(dr_t, d\lambda_t^0), \quad \bar{\rho}_2 = \mathrm{Corr}(dr_t, d\lambda_t^2), \quad \rho_{0,2}^{Copula} = 0$$

the DVA - CVA or Bilateral CVA does change sign, and in particular for portfolios Π1 and IRS the sign of the adjustment follows the sign of the correlations.

$\bar{\rho}_2$	$\bar{\rho}_0$	**Π1**	**Π2**	**10×IRS**
-60%	**0%**	-117(7)	-382(12)	-237(16)
-40%	**0%**	-74(6)	-297(11)	-138(15)
-20%	**0%**	-32(6)	-210(10)	-40(14)
0%	**0%**	-1(5)	-148(9)	31(13)
20%	**0%**	24(5)	-96(9)	87(12)
40%	**0%**	44(4)	-50(8)	131(11)
60%	**0%**	57(4)	-22(7)	159(11)

Payer vs Receiver

- Counterparty Risk (CR) has a relevant impact on interest-rate payoffs prices and, in turn, correlation between interest-rates and default (intensity) has a relevant impact on the CR adjustment.
- The (positive) CR adjustment to be subtracted from the default free price **decreases with correlation for receiver payoffs**. Natural: If default intensities increase, with high positive correlation their correlated interest rates will increase more than with low correlation, and thus a receiver swaption embedded in the adjustment decreases more, reducing the adjustment.
- The adjustment for payer payoffs increases with correlation.

Further Stylized Facts

- As the default probability implied by the counterparty CDS increases, the size of the adjustment increases as well, but the impact of correlation on it decreases.
- Financially reasonable: Given large default probabilities for the counterparty, fine details on the dynamics such as the correlation with interest rates become less relevant
- **The conclusion is that we should take into account interest-rate/ default correlation in valuing CR interest-rate payoffs.**
- In the bilateral case correlation risk can cause the adjustment to change sign

Exotics

For examples on exotics, including Bermudan Swaptions and CMS spread Options, see

Papers with Exotics and Bilateral Risk

- Brigo, D., and Pallavicini, A. (2007). Counterparty Risk under Correlation between Default and Interest Rates. In: Miller, J., Edelman, D., and Appleby, J. (Editors), Numerical Methods for Finance, Chapman Hall.
- Brigo, D., Pallavicini, A., and Papatheodorou, V. (2009). Bilateral counterparty risk valuation for interest-rate products: impact of volatilities and correlations. Available at Defaultrisk.com, SSRN and arXiv

Commodities and WWR

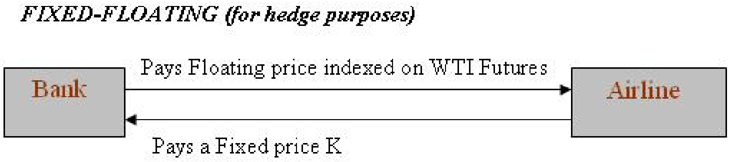
The correlation between interest rates dr_t (LIBOR, OIS) and credit intensities $d\lambda_t$, if measured historically, if often quite small in absolute value. Hence interest rates are a case where including correlation is good for stress tests and conservative hedging of CVA, but a number of market participant think that CVA can be computed by assuming zero correlations.

Whether one agrees or not, there are other asset classes on which CVA can be computed and where there is agreement on the necessity of including correlation in CVA pricing. We provide an example: Oil swaps traded with an airline.

It's natural to think that the future credit quality of the airline will be correlated with prices of oil.

Commodities: Futures, Forwards and Swaps

- **Forward**: OTC contract to buy a commodity to be delivered at a maturity date T at a price specified today. The cash/commodity exchange happens at time T.
- **Future**: Listed Contract to buy a commodity to be delivered at a maturity date T. Each day between today and T margins are called and there are payments to adjust the position.
- **Commodity Swap: Oil Example**:



Commodities: Modeling Approach

Schwartz-Smith Model

$$
\begin{aligned}
ln(S_t) &= x_t + l_t + \varphi(t) \\
dx_t &= -kx_t dt + \sigma_x dW_x \\
dl_t &= \mu dt + \sigma_l dW_l \\
dW_x \, dW_l &= \rho_{x,l} dt
\end{aligned}
$$

Variables

S_t: Spot oil price;
x_t, l_t: short and long term components of S_t;
This can be re-cast in a classic convenience yield model

Correlation with credit

$$
\begin{aligned}
dW_x \, dW_y &= \rho_{x,y} dt, \\
dW_l \, dW_y &= \rho_{l,y} dt
\end{aligned}
$$

Calibration

φ: defined to exactly fit the oil forward curve.
Dynamic parameters k, μ, σ, ρ are calibrated to At the money implied volatilities on Futures options.

Commodities

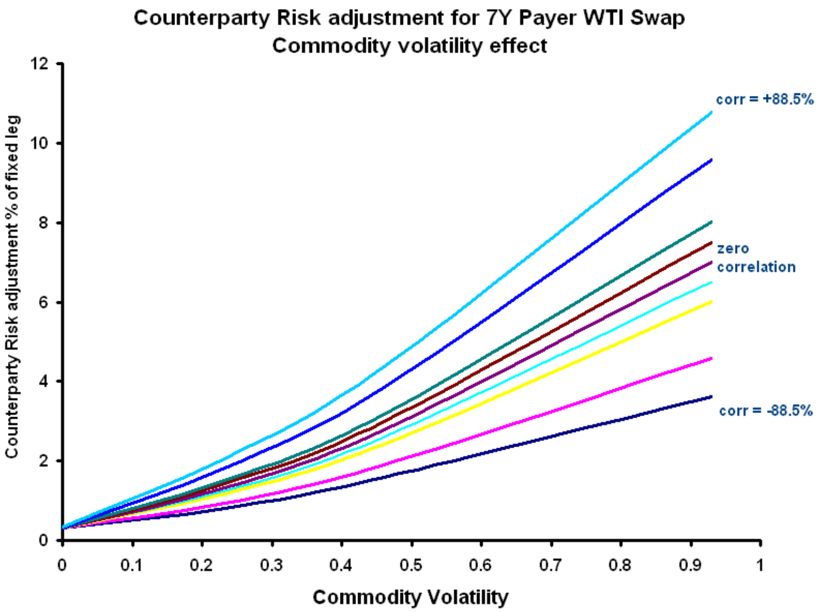
Total correlation Commodities - Counterparty default

$$\bar{\rho} = \mathrm{corr}(d\lambda_t,\ dS_t) = \frac{\sigma_X \rho_{X,y} + \sigma_L \rho_{L,y}}{\sqrt{\sigma_X^2 + \sigma_L^2 + 2\rho_{X,L}\sigma_X\sigma_L}}$$

We assumed no jumps in the intensity

We show the counterparty risk CVA computed by the AIRLINE on the BANK. This is because after 2008 a number of bank's credit quality deteriorated and an airline might have checked CVA on the bank with whom the swap was negotiated.

Commodities: Commodity Volatility Effect



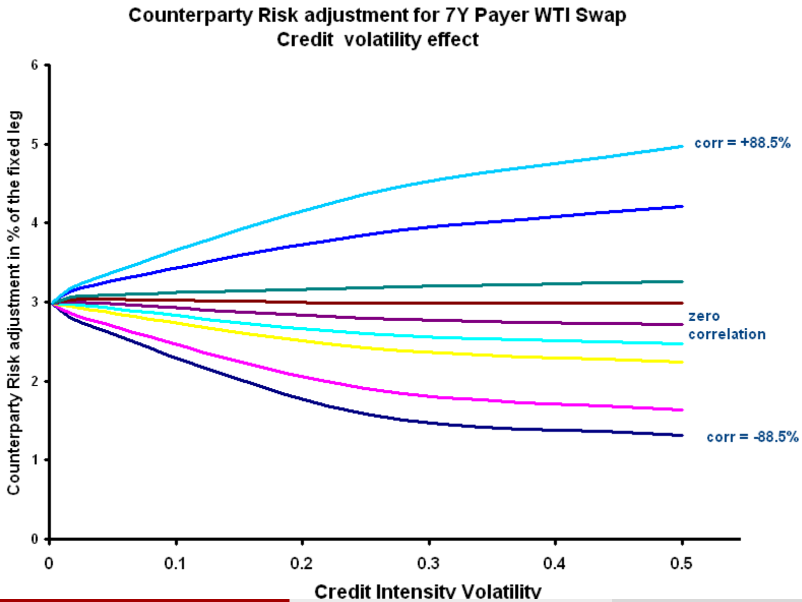
Commodities: Commodity Volatility Effect

Notice: In this example where CVA is calculated by the AIRLINE, positive correlation implies larger CVA.

This is natural: if the Bank credit spread widens, and the bank default becomes more likely, with positive correlation also OIL goes up.

Now CVA computed by the airline is an option, with maturity the default of the bank=counterparty, on the residual value of a Payer swap. As the price of OIL will go up at default due to the positive correlation above, the *payer* oil-swap will move in-the-money and the OIL option embedded in CVA will become more in-the-money, so that CVA will increase.

Commodities: Credit Volatility Effect



Commodities[1] : Credit volatility effect

$\bar{\rho}$	intensity volatility ν_R	0.025	0.25	0.50
-88.5	Payer adj	2.742	1.584	1.307
	Receiver adj	1.878	2.546	3.066
-63.2	Payer adj	2.813	1.902	1.63
	Receiver adj	1.858	2.282	2.632
-25.3	Payer adj	2.92	2.419	2.238
	Receiver adj	1.813	1.911	2.0242
-12.6	Payer adj	2.96	2.602	2.471
	Receiver adj	1.802	1.792	1.863
0	Payer adj	2.999	2.79	2.719
	Receiver adj	1.79	1.676	1.691
+12.6	Payer adj	3.036	2.985	2.981
	Receiver adj	1.775	1.562	1.527
+25.3	Payer adj	3.071	3.184	3.258
	Receiver adj	1.758	1.45	1.371
+63.2	Payer adj	3.184	3.852	4.205
	Receiver adj	1.717	1.154	0.977
+88.5	Payer adj	3.229	4.368	4.973
	Receiver adj	1.664	0.988	0.798

Fixed Leg Price maturity 7Y: 7345.39 USD for a notional of 1 Barrel per Month

[1] adjusment expressed as % of the fixed leg price

Commodities[2] : Commodity volatility effect

$\bar{\rho}$	Commodity spot volatility σ_S	0.0005	0.232	0.46	0.93
-88.5	Payer adj	0.322	0.795	1.584	3.607
	Receiver adj	0	1.268	2.546	4.495
-63.2	Payer adj	0.322	0.94	1.902	4.577
	Receiver adj	0	1.165	2.282	4.137
-25.3	Payer adj	0.323	1.164	2.419	6.015
	Receiver adj	0	0.977	1.911	3.527
-12.6	Payer adj	0.323	1.246	2.602	6.508
	Receiver adj	0	0.917	1.792	3.325
0	Payer adj	0.324	1.332	2.79	6.999
	Receiver adj	0	0.857	1.676	3.115
+12.6	Payer adj	0.324	1.422	2.985	7.501
	Receiver adj	0	0.799	1.562	2.907
+25.3	Payer adj	0.324	1.516	3.184	8.011
	Receiver adj	0	0.742	1.45	2.702
+63.2	Payer adj	0.325	1.818	3.8525	9.581
	Receiver adj	0	0.573	1.154	2.107
+88.5	Payer adj	0.326	2.05	4.368	10.771
	Receiver adj	0	0.457	0.988	1.715

Fixed Leg Price maturity 7Y: 7345.39 USD for a notional of 1 Barrel per Month

[2] adjusment expressed as % of the fixed leg price

Wrong Way Risk?

Basel 2, under the "Internal Model Method", models wrong way risk by means of a 1.4 multiplying factor to be applied to the zero correlation case, even if banks have the option to compute their own estimate of the multiplier, which can never go below 1.2 anyway.
What did we get in our cases? Two examples:

$$(4.973 - 2.719)/2.719 = 82\% >> 40\%$$

$$(1.878 - 1.79)/1.79 \approx 5\% << 20\%$$

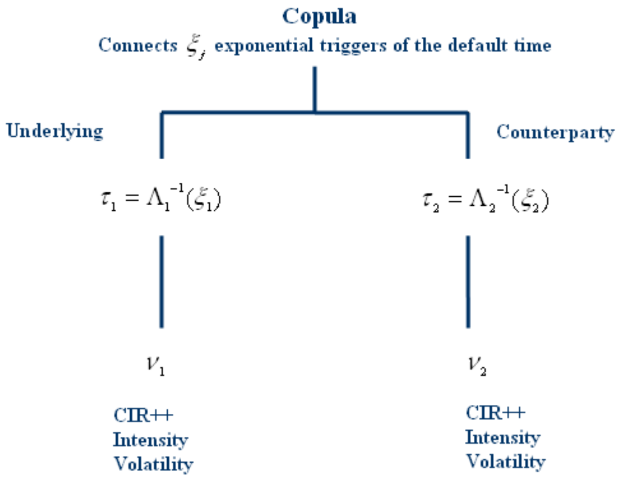
Credit (CDS)

- Model equations: ("1" = CDS underlying, "2" = counterparty)

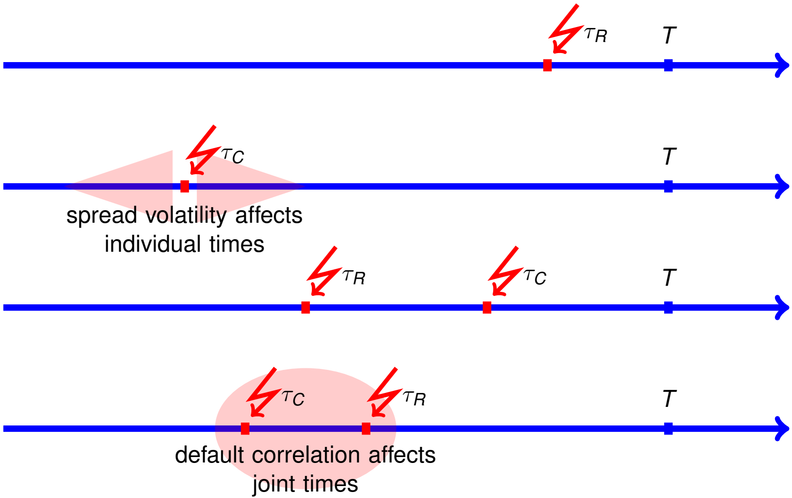
$$d\lambda_j(t) = k_j(\mu_j - \lambda_j(t))dt + \nu_j\sqrt{\lambda_j(t)}dZ_j(t), \quad j = 1,2$$

- Cumulative intensities are defined as : $\Lambda(t) = \int_0^t \lambda(s)ds$.
- Default times are $\tau_j = \Lambda_j^{-1}(\xi_j)$. Exponential triggers ξ_1 and ξ_2 are connected through a gaussian copula with correlation parameter ρ.
- In our approach, we take into account default correlation between default times τ_1 and τ_C **and** credit spreads volatility $\nu_j, j = 1,2$.
- Important: volatility can amplify default time uncertainty, while high correlation reduces conditional default time uncertainty. Taking into account ρ and $\nu \Longrightarrow$ better representation of market information and behavior, especially for wrong way risk.

Credit (CDS) : Overview



Credit (CDS) Correlation and Volatility Effects

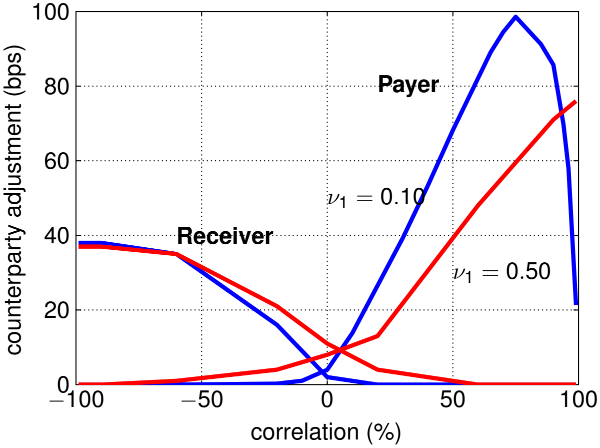


Moderate counterparty spread $\nu_2 = 0.10$

ρ	Vol parameter ν_1	0.01	0.10	0.20	0.30	0.40	0.50
	CDS Implied vol	1.5%	15%	28%	37%	42%	42%
-99	Payer adj	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)
	Receiver adj	40(2)	38(2)	39(2)	38(2)	36(1)	37(1)
-90	Payer adj	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)
	Receiver adj	39(2)	38(2)	38(2)	38(2)	35(1)	37(2)
-60	Payer adj	0(0)	0(0)	0(0)	0(0)	0(0)	1(0)
	Receiver adj	36(1)	35(1)	36(1)	36(1)	32(1)	35(1)
-20	Payer adj	0(0)	0(0)	1(0)	2(0)	3(0)	4(1)
	Receiver adj	16(1)	16(1)	17(1)	19(1)	18(1)	21(1)
0	Payer adj	3(0)	4(0)	5(0)	7(1)	7(1)	8(1)
	Receiver adj	0(0)	2(0)	5(0)	8(0)	10(0)	11(1)
+20	Payer adj	27(1)	25(1)	23(1)	20(1)	16(2)	13(1)
	Receiver adj	0(0)	0(0)	1(0)	2(0)	2(0)	4(0)
+60	Payer adj	80(4)	82(4)	67(4)	64(4)	55(3)	48(3)
	Receiver adj	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)
+90	Payer adj	87(6)	86(6)	88(6)	78(5)	80(5)	71(4)
	Receiver adj	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)
+99	Payer adj	10(2)	21(3)	52(5)	68(5)	73(5)	76(5)
	Receiver adj	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)

Large counterparty spread $\nu_2 = 0.20$

ρ	Vol parameter ν_1	0.01	0.10	0.20	0.30	0.40	0.50
	CDS Implied vol	1.5%	15%	28%	37%	42%	42%
-99	Payer adj	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)
	Receiver adj	41(2)	40(2)	39(2)	40(2)	40(2)	40(2)
-90	Payer adj	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)
	Receiver adj	41(2)	39(2)	39(2)	41(2)	40(2)	40(2)
-60	Payer adj	0(0)	0(0)	0(0)	0(0)	1(0)	1(0)
	Receiver adj	39(1)	37(1)	37(1)	37(1)	36(1)	35(1)
-20	Payer adj	0(0)	0(0)	2(0)	3(0)	3(0)	4(1)
	Receiver adj	17(1)	17(1)	17(1)	19(1)	21(1)	20(1)
0	Payer adj	3(0)	5(0)	6(0)	7(1)	6(1)	6(1)
	Receiver adj	0(0)	2(0)	4(0)	7(0)	10(0)	12(1)
+20	Payer adj	25(1)	24(1)	23(1)	20(1)	17(1)	15(1)
	Receiver adj	0(0)	0(0)	1(0)	2(0)	2(0)	4(0)
+60	Payer adj	74(4)	74(4)	69(4)	59(3)	54(3)	52(3)
	Receiver adj	0(0)	0(0)	0(0)	0(0)	0(0)	1(0)
+90	Payer adj	91(6)	90(6)	88(5)	80(5)	81(5)	81(5)
	Receiver adj	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)
+99	Payer adj	43(4)	56(5)	57(5)	72(5)	74(5)	78(5)
	Receiver adj	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)



counterparty adjustment (bps)
100
80
60
40
20
0
−100
−50
0
50
100
correlation (%)
Payer
Receiver
$\nu_1 = 0.10$
$\nu_1 = 0.50$

Credit Spread Volatility as a Smoothing Parameter

The dropping blue correlation pattern is due to a feature inherent in the copula notion (any copula).
Take for example the case with constant deterministic (zero volatility) intensities for simplicity. Push dependence to co-monotonicity ($\rho = 1$ in the Gaussian case), so that

$$\tau_1 = \frac{\boxed{\xi}}{\lambda_1} \quad \tau_C = \frac{\boxed{\xi}}{\lambda_C} \quad (*)$$

Usually $\lambda_1 > \lambda_C$ because one does not buy default protection for name 1 from an entity C that is riskier than 1.
Then $\tau_1 < \tau_C$ in all scenarios.
Then whenever τ_C hits, the CDS has already defaulted and there is no loss faced by B. This is why CVA drops to zero when $\rho \to 1$.

Credit Spread Volatility as a Smoothing Parameter

$$\tau_1 = \frac{\boxed{\xi}}{\lambda_1} \quad \tau_C = \frac{\boxed{\xi}}{\lambda_C} \quad (*)$$

However, if we increase Credit Volatility ν to values that are realistic (Brigo 2005 on CDS options) the uncertainty in (*) comes back in the "denominator" and the pattern goes back to be increasing.

The fundamental role of Credit Volatility

Credit Vol is a fundamental risk factor and should be taken into account. Current models for multiname credit derivatives (CDO, Default Baskets) ignore credit volatility assuming it is zero. This can lead to very funny results when the correlation becomes very high (unrealistic representation of systemic risk)

CDS: Bilateral

- Need to add one Model equations: ("0" = CDS investor)

$$d\lambda_j(t) = k_j(\mu_j - \lambda_j(t))dt + \nu_j\sqrt{\lambda_j(t)}dZ_j(t), \quad j = 0, 1, 2$$

- Cumulative intensities are defined as : $\Lambda(t) = \int_0^t \lambda(s)ds$.
- Default times are $\tau_j = \Lambda_j^{-1}(\xi_j)$. Exponential triggers ξ_0, ξ_1 and ξ_2 are connected through a gaussian copula with correlation parameters r_{01}, r_{02} and r_{12}.
- In our approach, we take into account default correlation between default times τ_B, τ_1 and τ_C **and** credit spreads volatility $\nu_j, j = 0, 1, 2$.
- Important: volatility can amplify default time uncertainty, while high correlation reduces conditional default time uncertainty.
  Taking into account vols and correlations $\Longrightarrow$ better representation of market information and behavior, especially for wrong way risk.

CDS: Bilateral Adjustment to be subtracted

CDS on reference entity "1" traded between investor "0" (protection seller) and counterparty "2" (protection buyer). $\tau = \min(\tau_B, \tau_C)$.

$$
\begin{aligned}
&\mathrm{CVA}_t - \mathrm{DVA}_t = \\
&= LGD_2 \cdot \mathbb{E}_t \left\{ \mathbf{1}_{\tau=\tau_C \le T} \cdot D(t,\tau_C) \cdot \left[\mathbf{1}_{\tau_1 > \tau_C} \overline{\mathrm{CDS}}_{a,b}(\tau_C, S, LGD_1) \right]^+ \right\} \\
&- LGD_0 \cdot \mathbb{E}_t \left\{ \mathbf{1}_{\tau=\tau_B \le T} \cdot D(t,\tau_B) \cdot \left[-\mathbf{1}_{\tau_1 > \tau_B} \overline{\mathrm{CDS}}_{a,b}(\tau_B, S, LGD_1) \right]^+ \right\}
\end{aligned}
$$

CDS: Bilateral Adjustment to be subtracted

where

$$\overline{\text{CDS}}_{a,b}(T_j, S_1, LGD_1) =$$

$$:= \Bigg\{ S_1 \Bigg[-\int_{\max\{T_a,T_j\}}^{T_b} D(T_j,t)(t - T_{\gamma(t)-1}) d\mathbb{Q}(\tau_1 > t|\mathcal{G}_{T_j})$$

$$+ \sum_{i=\max\{a,j\}+1}^{b} \alpha_i D(T_j, T_i)\mathbb{Q}(\tau_1 > T_i|\mathcal{G}_{T_j}) \Bigg]$$

$$+ \mathrm{L}_{\mathrm{GD}1} \Bigg[\int_{\max\{T_a,T_j\}}^{T_b} D(T_j,t) d\mathbb{Q}(\tau_1 > t|\mathcal{G}_{T_j}) \Bigg] \Bigg\}$$

Key quantities are CONDITIONAL default probabilities. Conditioning makes their calculations quite complicated, bringing in a number of non-tractable copula terms. We used Fourier transforms techniques combined with copula computations.

CDS Bilateral Adjustment: A market case with Lehman, Shell and BA

Maturity	Royal Dutch Shell	Lehman Brothers	British Airways
1y	4/24	6.8/203	10/151
2y	5.8/24.6	10.2/188.5	23.2/230
3y	7.8/26.4	14.4/166.75	50.6/275
4y	10.1/28.5	18.7/152.25	80.2/305
5y	11.7/30	23.2/145	110/335
6y	15.8/32.1	27.3.3/136.3	129.5/342
7y	19.4/33.6	30.5/130	142.8/347
8y	20.5/35.1	33.7/125.8	153.6/350.6
9y	21/36.3	36.5/122.6	162.1/353.3
10y	21.4/37.2	38.6/120	168.8/355.5

Market spread quotes in basis points for Royal Dutch Shell, Lehman Brothers and British Airways on January 5, 2006 and May 1, 2008. The notation a/c indicates that a is the CDS spread on $T_a =$Jan 5, 2006, while c is the CDS spread on $T_c =$ May 1, 2008.

CDS Bilateral Adjustment: volatility dynamics

2006/2008	$y(0)$	κ	μ	ν
Lehm "0"	0.0001/ 0.6611	0.036/ 7.8788	0.0432/ 0.0208	0.0553/ 0.5722
Shell "1"	0.0001/0.003	0.0394/0.1835	0.0219/0.0089	0.0192/ 0.0057
BA "2"	0.00002/0.00001	0.0266/0.6773	0.2582/0.0782	0.0003/ 0.2242

The CIR parameters of Lehman Brothers, Royal Dutch Shell and British Airways calibrated to the market quotes of CDS on January 5, 2006, and May 1, 2008. The notation a/c indicates that a is the CIR parameter on $T_a =$Jan 5 2006, while c is the CIR parameter on $T_c =$May 1, 2008.

CDS Bilateral Adjustment: MTM pre- / in- crisis

We calculate the mtm of the CDS contract as follows:

$$MTM_{a,c}(S_1, LGD_{012}) = CDS^D_{c,d}(T_c, S_1, LGD) - \frac{CDS^D_{a,b}(T_a, S_1, LGD)}{D(T_a, T_c)}$$

CDS Bilateral Adjustment: MTM pre- / in- crisis

r_{01} r_{02} r_{12}	**Vol. ν_1** **CDS IV**	**0.01** **1.5%**	**0.10** **15%**	**0.20** **28%**	**0.30** **37%**	**0.40** **42%**	**0.50** **42%**
-.3, -.3, .6	L Pay, BA R	39.1(2.1)	44.7(2.0)	51.1(1.9)	58.4(1.4)	60.3(1.7)	63.8(1.1)
	BA P, L Rec	-84.2(0.0)	-83.8(0.1)	-83.5(0.1)	-83.8(0.1)	-83.8(0.2)	-83.8(0.2)
-.3, -.3, .8	L P, BA R	13.6(3.6)	22.6(3.2)	35.2(2.6)	43.7(2.0)	45.3(2.4)	52.0(1.4)
	BA P, L R	-84.2(0.0)	-83.9(0.1)	-83.6(0.1)	-83.9(0.1)	-83.9(0.2)	-83.8(0.2)
.6, -.3, -.2	L P, BA R	83.1(0.0)	81.9(0.2)	81.6(0.3)	82.4(0.3)	82.6(0.3)	82.8(0.4)
	BA P, L R	-55.6(1.8)	-58.7(1.7)	-66.1(1.4)	-71.3(1.1)	-73.2(1.0)	-74.1(0.9)
.8, -.3, -.3	L P, BA R	83.9(0.0)	82.9(0.1)	82.3(0.3)	82.9(0.2)	82.9(0.3)	83.0(0.3)
	BA P, L R	-36.4(3.3)	-41.9(3.0)	-55.9(2.2)	-63.4(1.6)	-65.8(1.5)	-66.4(1.5)
0, 0, .5	L P, BA R	50.6(1.5)	54.3(1.5)	59.2(1.5)	64.4(1.1)	65.5(1.3)	68.8(0.8)
	BA P, L R	-80.9(0.2)	-80.5(0.3)	-80.9(0.4)	-82.3(0.3)	-82.6(0.3)	-82.8(0.3)
0, 0, .8	L P, BA R	12.3(3.5)	21.0(3.0)	34.9(2.5)	41.3(2.1)	44.6(1.9)	50.6(1.4)
	BA P, L R	-80.9(0.2)	-81.5(0.2)	-81.9(0.3)	-81.9(0.4)	-82.1(0.4)	-82.7(0.3)
0, 0, 0	L P, BA R	78.1(0.2)	77.9(0.3)	79.5(0.5)	79.5(0.5)	80.1(0.6)	82.1(0.4)
	BA P, L R	-81.6(0.2)	-81.9(0.2)	-82.3(0.3)	-82.2(0.4)	-82.7(0.3)	-83.2(0.3)
0, .7, 0	L P, BA R	77.3(0.3)	77.3(0.4)	78.5(0.5)	79.2(0.5)	79.7(0.6)	81.5(0.4)
	BA P, L R	-81.2(0.2)	-81.8(0.2)	-81.9(0.3)	-80.8(1.3)	-82.4(0.3)	-82.6(0.3)
.3, .2, .6	L P, BA R	54.1(1.4)	56.7(1.3)	62.5(1.1)	63.6(1.1)	66.4(0.9)	69.7(0.6)
	BA P, L R	-81.3(0.2)	-81.7(0.2)	-81.4(0.4)	-81.3(0.5)	-81.6(0.4)	-82.1(0.4)
.3, .3, .8	L P, BA R	22.8(4.2)	28.8(3.5)	38.6(2.9)	42.6(2.9)	45.9(2.5)	52.0(2.2)
	BA P, L R	-83.0(0.2)	-83.2(0.2)	-82.8(0.3)	-82.4(0.4)	-82.5(0.4)	-82.9(0.4)
.5, .5, .5	L P, BA R	62.8(0.8)	64.5(0.8)	67.7(0.8)	68.5(0.9)	71.3(0.7)	73.2(0.6)
	BA P, L R	-67.4(1.1)	-70.4(0.9)	-72.9(0.9)	-74.4(0.9)	-75.8(0.8)	-76.7(0.7)
.7, 0, 0	L P, BA R	77.4(0.2)	77.3(0.3)	78.9(0.5)	79.1(0.5)	79.9(0.5)	81.4(0.4)
	BA P, L R	-47.3(2.2)	-55.0(1.9)	-61.6(1.6)	-65.0(1.5)	-67.5(1.3)	-69.6(1.1)

CDS marked to market by Lehman Brothers on May 1, 2008. The mark-to-market value of the CDS without risk adjustment when Lehman Brothers is respectively payer (receiver) is 84.2(-84.2) bps, due to the widening of Shell spreads.

CDS Bilateral Adjustment: MTM pre- / in- crisis

Negative or null BA“2”/Shell“1” dependence

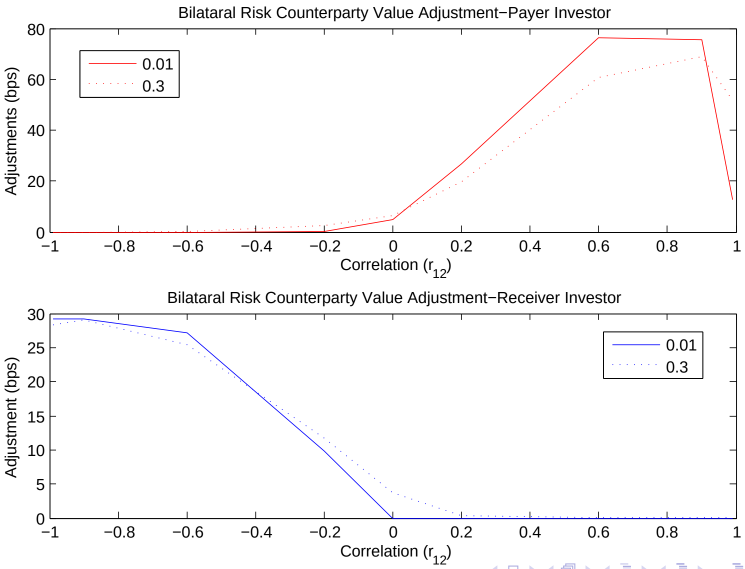
- If BA is negatively correlated or uncorrelated to Shell, see triples $(0.6, -0.3, -0.2)$, $(0.8, -0.3, -0.3)$, and $(0.7, 0, 0)$, then the MTM appears to be the largest for Lehman
- This happens whether Lehman is the CDS payer or the CDS receiver.

CDS Bilateral Adjustment: MTM pre- / in- crisis

Shell"1" credit spread volatility

- Increases in credit spreads volatility of Shell increase the MTM when Lehman is the CDS payer and decrease the contract valuation when Lehman is the CDS receiver.
- Conversely, if Lehman is receiver, this implies smaller CDS contract valuations for Lehman.

CDS Bilateral on a different portfolio: Wrong way risk



Equity: Intensity vs Firm value models

If we have equity S_t of a name '1' as contract underlying and we have the default of the counterparty

$$\tau_C = \Lambda_C^{-1}(\xi_C)$$

it's hard to correlate τ_C and S_1 enough, given that the exponential random variable ξ_C and any Brownian motion W_1 driving S_1 will necessarily be independent.

Underlying Equity/ Counterparty Default correlation

The only hope to create correlation is to put a stochastic λ_C and correlate it with W_1 driving S_1. However, since most of the randomness of τ_C comes from ξ_C, this does not create enough correlation.

With equity we change family of credit models, and resort to Firm Value (or structural) models for the default of the counterparty.

Equity: Intensity vs Firm Value models

Intensity VS Firm Value models

$$\tau_C = \Lambda_C^{-1}(\xi_C) \text{ vs } \tau_C = \inf\{t : V(t) \leq H(t)\}$$

Default of the counterparty is the first time when the counterparty firm value V hits a default barrier H.

Equity/Credit Correlation with Firm Value Models

Now if the underlying equity S_1 is driven by a brownian motion W_1,

$$dS_1(t) = (r - y_1)S_1(t)dt + \sigma_1(t)S_1(t)dW_1(t)$$

and the counterparty $V = V_C$ is also driven by a brownian motion W_C,

$$dV(t) = (r - q)V(t)dt + \sigma(t)V(t)dW_C(t)$$

then an effective way to create correlation is $dW_1 dW_C = \rho_{1C} dt$

Equity: Firm Value models for the counterparty default

AT1P model

Let the risk neutral firm value V dynamics and the default barrier $\widehat{H}(t)$ of the counterparty ‘C‘ be

$$dV(t) = V(t)(r(t) - q(t))dt + V(t)\sigma(t)dW_C(t)$$

$$H(t) = \frac{H}{V_0}\mathbb{E}\left[V_t\right]\ e^{\left(-B\int_0^t \sigma_s^2 ds\right)}$$

and let the default time τ be **the 1st time** V_C **hits** $H(t)$ **from above**, starting from $V_0 > H$. Here $H > 0$ and B are free parameters we may use to shape the barrier.

Then the survival probability is given analytically in close form by a barrier option type formula (see Brigo and Tarenghi (2005) and Brigo, Morini and Tarenghi (2011)).

Firm Value model Calibration to CDS data

It is possible to fit exactly the CDS spreads for the counterparty through the firm value volatility $\sigma(t)$ using a bootstrapping procedure.

$$\left.\begin{array}{c} S^{\text{MktCDS}}_{0,1y} \\ S^{\text{MktCDS}}_{0,2y} \\ \vdots \\ S^{\text{MktCDS}}_{0,10y} \end{array}\right\} \longleftrightarrow \left\{ \begin{array}{c} dV(t) = (r-q)V(t)dt + \sigma_V(t)V(t)dW(t) \\ H(t) \\ \text{model parameters: } \sigma_V(t) \end{array}\right.$$

This ensures that the firm value model is consistent with liquid credit data of the counterparty.
In the papers we give examples based on Lehman and Parmalat.

Counterparty risk in equity return swap (ERS)

$$
\begin{array}{lcccc}
\text{Initial Time 0:} & & \text{NO FLOWS, or} & & \\
\boxed{B} & \longrightarrow & KS_0 \text{ cash} & \longrightarrow & \boxed{C} \\
& \longleftarrow & K \text{ equity} & \longleftarrow & \\
& & \cdots\cdot & & \\
\text{Time } T_i: & \longrightarrow & \text{equity dividends} & \longrightarrow & \\
\boxed{B} & \longleftarrow & \text{Libor + Spread} & \longleftarrow & \boxed{C} \\
& & \cdots\cdot & & \\
\text{Final Time } T_b: & \longrightarrow & K \text{ equity or } KS_{T_b} \text{ cash} & \longrightarrow & \\
\boxed{B} & \longleftarrow & KS_0 \text{ cash} & \longleftarrow & \boxed{C}
\end{array} \quad (3)
$$

Counterparty risk in equity return swap (ERS)

- We are a default-free company (bank) “B” entering a contract with counterparty “C” (corporate). The reference underlying equity is “1”.
- “B” and “C” agree on an amount K of stocks of “1” (with price S) to be taken as nominal ($N = K\, S_0$). The contract starts in $T_a = 0$ and has final maturity $T_b = T$.
- At $t = 0$ there is no exchange of cash (alternatively, we can think that “C” delivers to “B” an amount K of “1” stock and receives a cash amount equal to KS_0).
- At intermediate times “B” pays to “C” the dividend flows of the stocks (if any) in exchange for a periodic risk free rate plus a spread X.
- At final maturity $T = T_b$, “B” pays KS_T to “C” (or gives back the amount K of stocks) and receives a payment KS_0.

The (fair) spread X is chosen in order to obtain a contract whose value at inception is zero.

Counterparty risk in equity return swap (ERS)

$S_0 = 20$, volatility $\sigma = 20\%$ and constant dividend yield $y = 0.80\%$. The simulation date is September 16th, 2009. The contract has maturity $T = 5y$ and the settlement of the risk free rate has a semi-annual frequency. Finally, we included a recovery rate $\text{R}_{\text{EC}} = 40\%$ for the counterparty default.

T_i	$S_i^{BID,CDS}$ (bps)	$S_i^{ASK,CDS}$ (bps)
1y	25	31
3y	34	39
5y	42	47
7y	46	51
10y	50	55

Table: CDS spreads used for the counterparty "B" credit quality in the valuation of the equity return swap.

Counterparty risk in equity return swap (ERS)

Fair spread X is driven by CVA

We compute the unilateral CVA adjustment by simulation in the model above. We search for the spread X such that the total value of the ERS INLCUDING THE CVA ADJUSTMENT is zero. In fact, it can be proven that without counterparty credit risk the theoretical fair spread X would be 0. We see that the spread X is due entirely to counterparty risk.

ρ	X (AT1P)
-1	0.0
-0.2	3.0
0	5.5
0.5	14.7
1	24.9

Table: Fair spread X (in basis points) of the Equity Return Swap in five different correlation cases for AT1P.

Compare with "Basel 2" deduced adjustments

Basel 2, under the "Internal Model Method", models wrong way risk by means of a 1.4 multiplying factor to be applied to the zero correlation case, even if banks have the option to compute their own estimate of the multiplier, which can never go below 1.2 anyway.

Is this confirmed by our model?

$$(24.9 - 5.5)/5.5 \approx 353\% >> 40\%$$

Collateral Management and Gap Risk I

Collateral (CSA) is considered to be the solution to counterparty risk.

Periodically, the position is re-valued (”marked to market”) and a quantity related to the change in value is posted on the collateral account from the party who is penalized by the change in value.

This way, the collateral account, at the periodic dates, contains an amount that is close to the actual value of the portfolio and if one counterparty were to default, the amount would be used by the surviving party as a guarantee (and viceversa).
Gap Risk is the residual risk that is left due to the fact that the realingment is only periodical. If the market were to move a lot between two realigning (”margining”) dates, a significant loss would still be faced.

Folklore: Collateral completely kills CVA and gap risk is negligible.

Collateral Management and Gap Risk I

Folklore: Collateral completely kills CVA and gap risk is negligible.

We are going to show that there are cases where this is not the case at all (B. Capponi and Pallavicini 2012, Mathematical Finance)

- Risk-neutral evaluation of counterparty risk in presence of collateral management can be a difficult task, due to the complexity of clauses.
- Only few papers in the literature deal with it. Among them we cite Cherubini (2005), Alavian *et al.* (2008), Yi (2009), Assefa *et al.* (2009), Brigo et al (2011) and citations therein.
- Example: Collateralized bilateral CVA for a netted portfolio of IRS with 10y maturity and 1y coupon tenor for different default-time correlations with (and without) collateral re-hypothecation. See B, Capponi, Pallavicini and Papatheodorou (2011)

Collateral Management and Gap Risk II

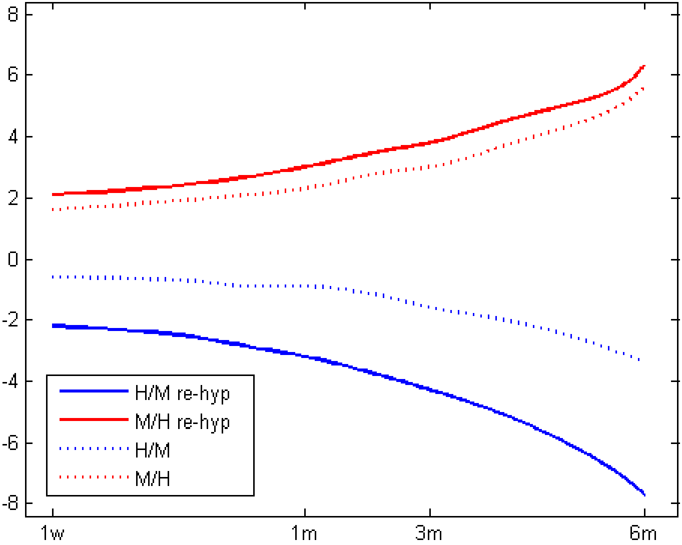


Figure explanation

Bilateral valuation adjustment, margining and rehypotecation

The figure shows the BVA(DVA-CVA) for a ten-year IRS under collateralization through margining as a function of the update frequency δ with zero correlation between rates and counterparty spread, zero correlation between rates and investor spread, and zero correlation between the counterparty and the investor defaults. The model allows for nonzero correlations as well.

Continuous lines represent the re-hypothecation case, while **dotted lines** represent the opposite case. The *red line* represents an investor riskier than the counterparty, while the *blue line* represents an investor less risky than the counterparty. All values are in basis points.

See the full paper by Brigo, Capponi, Pallavicini and Papatheodorou ‘Collateral Margining in Arbitrage-Free Counterparty Valuation Adjustment including Re-Hypotecation and Netting” available at `http://arxiv.org/abs/1101.3926`

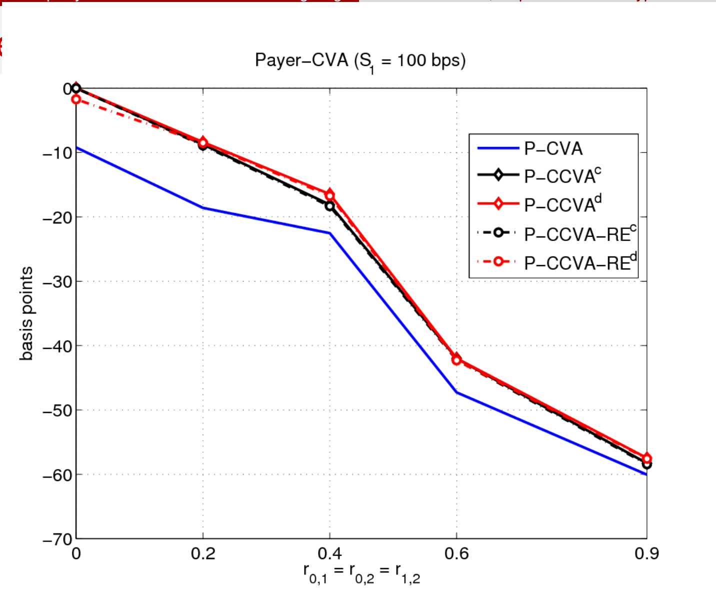
Figure explanation

From the fig, we see that the case of an investor riskier than the counterparty (M/H) leads to positive value for DVA-CVA, while the case of an investor less risky than the counterparty has the opposite behaviour. If we inspect the DVA and CVA terms as in the paper we see that when the investor is riskier the DVA part of the correction dominates, while when the investor is less risky the counterparty has the opposite behaviour.

Re-hypothecation enhances the absolute size of the correction, a reasonable behaviour, since, in such case, each party has a greater risk because of being unsecured on the collateral amount posted to the other party in case of default.

Let us now look at a case with more contagion: a CDS.

Collate


Payer–CVA (S_1 = 100 bps)
basis points
$r_{0,1} = r_{0,2} = r_{1,2}$
0
−10
−20
−30
−40
−50
−60
−70
0
0.2
0.4
0.6
0.9
P–CVA
P–CCVA^c
P–CCVA^d
P–CCVA–RE^c
P–CCVA–RE^d

Collateral Management and Gap Risk II

The figure refers to a payer CDS contract as underlying. See the full paper by Brigo, Capponi and Pallavicini (2011) for more cases.

If the investor holds a payer CDS, he is buying protection from the counterparty, i.e. he is a protection buyer.

We assume that the spread in the fixed leg of the CDS is 100 while the initial equilibrium spread is about 250.

Given that the payer CDS will be positive in most scenarios, when the investor defaults it is quite unlikely that the net present value be in favor of the counterparty.

We then expect the CVA term to be relevant, given that the related option will be mostly in the money. This is confirmed by our outputs.

Collateral Management and Gap Risk III

We see in the figure a relevant CVA component (part of the bilateral DVA - CVA) starting at 10 and ending up at 60 bps when under high correlation.

We also see that, for zero correlation, collateralization succeeds in completely removing CVA, which goes from 10 to 0 basis points.

However, collateralization seems to become less effective as default dependence grows, in that collateralized and uncollateralized CVA become closer and closer, and for high correlations we still get 60 basis points of CVA, even under collateralization.

The reason for this is the instantaneous default contagion that, under positive dependency, pushes up the intensity of the survived entities, as soon as there is a default of the counterparty.

Collateral Management and Gap Risk IV

Indeed, the term structure of the on-default survival probabilities (see paper) lies significantly below the one of the pre-default survival probabilities conditioned on $\mathcal{G}_{\tau-}$, especially for large default correlation.

The result is that the default leg of the CDS will increase in value due to contagion, and instantaneously the Payer CDS will be worth more. This will instantly increase the loss to the investor, and most of the CVA value will come from this jump.

Given the instantaneous nature of the jump, the value at default will be quite different from the value at the last date of collateral posting, before the jump, and this explains the limited effectiveness of collateral under significantly positive default dependence.

Monitoring Counterparty Credit Risk

- When we monitor a (symmetric) risk in a bilateral agreement, we should introduce a “metric” which is shared by both parties.
  - $\rightarrow$ The ISDA Master Agreement defines the term *exposure* to be the netted mid-market mark-to-market value of the transaction.
- We name the exposure priced at time t, either by the investor or by the counterparty, with ε_t.
- Notice that the ISDA Master Agreement allows the calculation agent to be a third party.
- Since counterparty risk can be sized in term of exposure, we can operate to mitigate the risk by reducing such quantity.

Mitigating Counterparty Credit Risk – I

- The ISDA Master Agreement lists two different tools to reduce exposure:
  - $\rightarrow$ close-out netting rules, which state that if a default occurs, multiple obligations between two parties are consolidated into a single net obligation; and
  - $\rightarrow$ collateralization, namely the right of recourse to some asset of value that can be sold or the value of which can be applied in the event of default on the transaction.
- We consider that assets used as collaterals are posted on a Collateral Account held by a Collateral Taker, and we name its value at time t with C_t.
- Notice that if at time t the investor posts some collateral we consider that $dC_t < 0$, the other way round if the counterparty is posting.

Mitigating Counterparty Credit Risk – II

- In the following we assume that close-out netting rules are always active, so that we consider the transaction $\Pi(t, T)$ and the collateral account C_t together when calculating the CVA.
- Thus, under close-out netting rules we get

$$\mathrm{C_{VA}}(t, T; C) := \mathbb{E}_t\left[\,\bar{\Pi}(t, T; C) - \Pi(t, T) - C_T D(t, T)\,\right]$$

where the expectation is taken under risk-neutral measure, and $\bar{\Pi}(t, T; C)$ will be analyzed in the following slides.

- Furthermore, we assume that mid-market exposure ε_t can be calculated from the risk-free $\Pi(t, T)$ as

$$\varepsilon_t \doteq \mathbb{E}_t[\,\Pi(t, T)\,]$$

Re-hypothecation Liquidity Risk – I

- At transaction maturity or after applying close-out netting, the originating party expects to get back the remaining collateral.
- Yet, prevailing legislations may give to the Collateral Taker some rights on the collateral itself.
- For instance, on an early termination date a counterparty to an English CSA will find itself as an unsecured creditor, thus entitled to only a fraction of the value of the collateral it transferred.
- With a New York CSA transferred cash collateral or re-hypothecated collateral are both likely to leave the collateral provider in the same position as an unsecured creditor, but, in this case, the parties may agree on amending the provisions of the CSA which make re-hypothecation possible.

Re-hypothecation Liquidity Risk – II

- In case of collateral re-hypothecation the surviving party must consider the possibility to recover only a fraction of his collateral.
  - $\rightarrow$ We name such recovery rate $\text{R}_{\text{EC}}{}'_I$, if the investor is the Collateral Taker, or $\text{R}_{\text{EC}}{}'_C$ in the other case (we often use $\text{L}_{\text{GD}}{}'_I := 1 - \text{R}_{\text{EC}}{}'_I$ and $\text{L}_{\text{GD}}{}'_C := 1 - \text{R}_{\text{EC}}{}'_C$).
- In the worst case the surviving party has no precedence on other creditors to get back his collateral. In such case the recovery rate of collateral is the one of the transaction. Thus, we get

$$\text{R}_{\text{EC}}{}_I \leq \text{R}_{\text{EC}}{}'_I \leq 1 \;, \quad \text{R}_{\text{EC}}{}_C \leq \text{R}_{\text{EC}}{}'_C \leq 1$$

- If the Collateral Taker is a risk-free third-party we can assume that $\text{R}_{\text{EC}}{}'_I = \text{R}_{\text{EC}}{}'_C = 1$.

Collateral Choice

Ideally, firms would like an asset of stable and predictable value, an asset that is not linked to the value of the transaction in any way and an asset that can be sold quickly and easily if the need arises. [ISDA, Coll. Review, 1.1]

- Thus, in order to achieve an effective collateralization of the transaction, we require that
  - → collaterals hedge investor's exposure on counterparty's default event,
  - → they are liquid assets,
  - → they are not related to the deal's underlying assets or to the counterparty.
- In practice, when collaterals do not match such requirements, their value is reduced by means of corrective factors named haircuts.

Margining Practice – I

- In general, margining practice consists in a pre-fixed set of dates during the life of a deal when both parties post or withdraw collaterals, according to their current exposure, to or from an account held by the Collateral Taker.
- The Collateral Taker may be a third party or the party of the transaction who is not posting collateral.
- Notice that in legal documents where a pledge or a security interest is in act the Collateral Taker is named the Secured Party, while the other party is the Pledgor.
- We do not consider legal issues which may change collateral arrangement (pledge vs. title transfer) but for re-hypothecation issues.

Margining Practice – II

- The Collateral Taker remunerates the account (usually) at over-night rate.
  - $\rightarrow$ In the following we consider that the collaterals are risk-free and their account is a cash account accruing at risk-free rate.
- At deal termination date the parties are not forced to close the collateral account, but they may agree to use it for a new deal.
  - $\rightarrow$ We consider that the collateral account is opened anew for each new deal and it is closed upon a default event occurs or maturity is reached.
- If the account is closed any collateral held by the Collateral Taker would be required to be returned to the originating party.
  - $\rightarrow$ We have $C_u = 0$ for all $u \leq t$ or $u \geq T$.
- We do not consider haircuts in the following.

Margining Practice – III

- A realistic margining practice should allow for collateral posting only on a fixed time-grid ($t_0 = t, \dots, t_N = T$), and for the presence of independent amounts (A), minimum transfer amounts (M), and thresholds (H), with $H \geq M$.
- Independent amounts represent a further insurance on the transaction and they are often posted as an upfront protection, but they may be updated according to exposure changes. We do not consider them in the following.
- Thresholds represent the amount of permitted unsecured risk, so that they may depend on the credit quality of the counterparties.
- Moving thresholds depending on a deterioration of the credit quality of the counterparties (downgrade triggers) have been a source of liquidity strain during the market crisis.

Margining Practice – IV

- At each collateral posting date t_i, the collateral account is updated according to changes in exposure, otherwise producing an unsecured risk.

1. First, we consider how much collateral the investor should post to or withdraw from the collateral account:

$$1_{\{|(\varepsilon_{t_i}+H_I)^- - C^-_{t_i^-}|>M\}}((\varepsilon_{t_i} + H_I)^- - C^-_{t_i^-})$$

2. Then, we consider how much collateral the counterparty should post to or withdraw from the collateral account:

$$1_{\{|(\varepsilon_{t_i}-H_C)^+ - C^+_{t_i^-}|>M\}}((\varepsilon_{t_i} - H_C)^+ - C^+_{t_i^-})$$

Margining Practice – V

- By adding the two terms we get how the collateral account is updated during the life of the transaction

$$C_{t_0} := 0 \,, \quad C_{t_N^+} := 0 \,, \quad C_{u^-} := \frac{C_{\beta(u)^+}}{D(\beta(u),u)}$$

$$\begin{aligned} C_{t_i^+} := C_{t_i^-} & \\ & + 1_{\{|(\varepsilon_{t_i}+H_I)^- - C^-_{t_i^-}|>M\}}((\varepsilon_{t_i}+H_I)^- - C^-_{t_i^-}) \\ & + 1_{\{|(\varepsilon_{t_i}-H_C)^+ - C^+_{t_i^-}|>M\}}((\varepsilon_{t_i}-H_C)^+ - C^+_{t_i^-}) \end{aligned}$$

where $\beta(u)$ is the last update time before u, and $t_0 < u \leq t_N$.

- In case of no thresholds ($H_I = H_C = 0$) and no minimum transfer amount ($M = 0$), we obtain a simpler rule

$$C_{t_0} = C_{t_N^+} = 0 \,, \quad C_{t^-} = \frac{\varepsilon_{\beta(u)}}{D(\beta(u),u)} \,, \quad C_{t_i^+} = \varepsilon_{t_i}$$

Close-Out Netting Rules – I

The effect of close-out netting is to provide for a single net payment requirement in respect of all the transactions that are being terminated, rather than multiple payments between the parties. Under the applicable accounting rules and capital requirements of many jurisdictions, the availability of close-out netting allows parties to an ISDA Master Agreement to account for transactions thereunder on a net basis. [ISDA, Coll. Review, 2.1.1]

- The occurrence of an event of default gives the parties the right to terminate all transactions that are concluded under the relevant ISDA Master Agreement.
- The ISDA Master Agreement provides for the mechanism of close-out netting to be enforced.

Close-Out Netting Rules – II

The Secured Party will transfer to the Pledgor any proceeds and posted credit support remaining after liquidation and/or set-off after satisfaction in full of all amounts payable by the Pledgor with respect to any obligations; the Pledgor in all events will remain liable for any amounts remaining unpaid after any liquidation and/or set-off. [ISDA, CSA Annex, 8]

- In case of default of one party, the surviving party should evaluate the transactions just terminated, due to the default event occurrence, to claim for a reimbursement after the application of netting rules to consolidate the transactions, inclusive of collateral accounts.
  - → The ISDA Master Agreement defines the term *close-out amount* to be the amount of the losses or costs of the surviving party would incur in replacing or in providing for an economic equivalent.

Close-Out Netting Rules – III

- Notice that the close-out amount is not a symmetric quantity w.r.t. the exchange of the role of two parties, since it is valued by one party after the default of the other one.
- Instead of the close-out amount we introduce the ”on-default exposure”, namely the price of the replacing transaction or of its economic equivalent.
- We name the on-default exposure priced at time t by the investor on counterparty's default with $\varepsilon_{I,t}$ (and $\varepsilon_{C,t}$ in the other case, namely when the investor is defaulting). Notice that we always consider all prices from the point of view of the investor. Thus,
  - $\rightarrow$ a positive value for $\varepsilon_{I,t}$ means the investor is a creditor of the counterparty, while
  - $\rightarrow$ a negative value for $\varepsilon_{C,t}$ means the counterparty is a creditor of the investor.

Cash Flows on Counterparty Default Event – I

- We start by listing all the situations may arise on counterparty default event. The case of the investor's default event will be derived accordingly.
- Our goal is to calculate the present value of all cash flows involved by the contract by taking into account:
  - $\rightarrow$ collateral margining operations, and
  - $\rightarrow$ close-out netting rules in case of default.
- Notice that we can safely aggregate the cash flows of the contract with the ones of the collateral account, since on contract termination all the posted collateral are returned to the originating party.
- We introduce the (first) default time $\tau := \min\{\tau_C, \tau_I\}$.

Cash Flows on Counterparty Default Event – II

1. The investor measures a positive (on-default) exposure on counterparty default ($\varepsilon_{I,\tau_C} > 0$), and some collateral posted by the counterparty is available ($C_{\tau_C} > 0$).
   - $\rightarrow$ Then, the exposure is reduced by netting, and the remaining collateral (if any) is returned to the counterparty. If the collateral is not enough, the investor suffers a loss for the remaining exposure.

$$1_{\{\tau=\tau_C<T\}} 1_{\{\varepsilon_{I,\tau}>0\}} 1_{\{C_\tau>0\}} (\mathrm{Rec}_C (\varepsilon_{I,\tau} - C_\tau)^+ + (\varepsilon_{I,\tau} - C_\tau)^-)$$

Cash Flows on Counterparty Default Event – III

2. The investor measures a positive (on-default) exposure on counterparty default ($\varepsilon_{I,\tau_C} > 0$), and some collateral posted by the investor is available ($C_{\tau_C} < 0$).
   - $\rightarrow$ Then, the investor suffers a loss for the whole exposure. All the collateral (if any) is returned to the investor if it is not re-hypothecated, otherwise an unsecured claim is needed.

$$1_{\{\tau=\tau_C<T\}} 1_{\{\varepsilon_{I,\tau}>0\}} 1_{\{C_\tau<0\}} (\text{Rec}_C \varepsilon_{I,\tau} - \text{Rec}'_C C_\tau)$$

Cash Flows on Counterparty Default Event – IV

3. The investor measures a negative (on-default) exposure on counterparty default ($\varepsilon_{I,\tau_C} < 0$), and some collateral posted by the counterparty is available ($C_{\tau_C} > 0$).
   - $\rightarrow$ Then, the exposure is paid to the counterparty, and the counterparty gets back its collateral in full.

$$1_{\{\tau=\tau_C<T\}}1_{\{\varepsilon_{I,\tau}<0\}}1_{\{C_\tau>0\}}(\varepsilon_{I,\tau} - C_\tau)$$

Cash Flows on Counterparty Default Event – V

4. The investor measures a negative (on-default) exposure on counterparty default ($\varepsilon_{I,\tau_C} < 0$), and some collateral posted by the investor is available ($C_{\tau_C} < 0$).
   - $\rightarrow$ Then, the exposure is reduced by netting and paid to the counterparty. The investor gets back its remaining collateral (if any) in full if it is not re-hypothecated, otherwise an unsecured claim is needed for the part of collateral exceeding the exposure.

$$1_{\{\tau=\tau_C<T\}} 1_{\{\varepsilon_{I,\tau}<0\}} 1_{\{C_\tau<0\}} \left((\varepsilon_{I,\tau} - C_\tau)^- + \mathrm{Rec}'_C (\varepsilon_{I,\tau} - C_\tau)^+ \right)$$

Aggregating Cash Flows – I

- Now, we can aggregate all these cash flows, along with cash flows coming from the default of the investor and the ones due in case of non-default, inclusive of the cash-flows of the collateral account.
- We obtain the cash flows coming from the default of the investor simply by reformulating the previous line of reasoning from the point of view of the counterparty.
- In the following equations we use the risk-free discount factor $D(t, T)$, which is implicitly used also in the definitions of the risk-free discounted payoff $\Pi(t, T)$, and in the accumulation curve used for the collateral account C_t.

Aggregating Cash Flows – II

- We obtain by summing all the contributions

$$
\begin{aligned}
&\bar{\Pi}(t,T;C) = \\
&\quad 1_{\{\tau>T\}}\Pi(t,T) \\
&\quad + 1_{\{\tau<T\}}(\Pi(t,\tau) + D(t,\tau)C_\tau) \\
&\quad + 1_{\{\tau=\tau_C<T\}}D(t,\tau)1_{\{\varepsilon_{I,\tau}<0\}}1_{\{C_\tau>0\}}(\varepsilon_{I,\tau} - C_\tau) \\
&\quad + 1_{\{\tau=\tau_C<T\}}D(t,\tau)1_{\{\varepsilon_{I,\tau}<0\}}1_{\{C_\tau<0\}}((\varepsilon_{I,\tau} - C_\tau)^- + \mathrm{Rec}'_C(\varepsilon_{I,\tau} - C_\tau)^+) \\
&\quad + 1_{\{\tau=\tau_C<T\}}D(t,\tau)1_{\{\varepsilon_{I,\tau}>0\}}1_{\{C_\tau>0\}}((\varepsilon_{I,\tau} - C_\tau)^- + \mathrm{Rec}_C(\varepsilon_{I,\tau} - C_\tau)^+) \\
&\quad + 1_{\{\tau=\tau_C<T\}}D(t,\tau)1_{\{\varepsilon_{I,\tau}>0\}}1_{\{C_\tau<0\}}(\mathrm{Rec}_C\varepsilon_{I,\tau} - \mathrm{Rec}'_C C_\tau) \\
&\quad + 1_{\{\tau=\tau_I<T\}}D(t,\tau)1_{\{\varepsilon_{C,\tau}>0\}}1_{\{C_\tau<0\}}(\varepsilon_{C,\tau} - C_\tau) \\
&\quad + 1_{\{\tau=\tau_I<T\}}D(t,\tau)1_{\{\varepsilon_{C,\tau}>0\}}1_{\{C_\tau>0\}}((\varepsilon_{C,\tau} - C_\tau)^+ + \mathrm{Rec}'_I(\varepsilon_{C,\tau} - C_\tau)^-) \\
&\quad + 1_{\{\tau=\tau_I<T\}}D(t,\tau)1_{\{\varepsilon_{C,\tau}<0\}}1_{\{C_\tau<0\}}((\varepsilon_{C,\tau} - C_\tau)^+ + \mathrm{Rec}_I(\varepsilon_{C,\tau} - C_\tau)^-) \\
&\quad + 1_{\{\tau=\tau_I<T\}}D(t,\tau)1_{\{\varepsilon_{C,\tau}<0\}}1_{\{C_\tau>0\}}(\mathrm{Rec}_I\varepsilon_{C,\tau} - \mathrm{Rec}'_I C_\tau)
\end{aligned}
$$

Aggregating Cash Flows – III

- Hence, by a straightforward calculation we get

$$
\begin{aligned}
\bar{\Pi}(t, T; C) = \Pi(t, T) \\
& - 1_{\{\tau<T\}} D(t,\tau)\left(\Pi(\tau, T) - 1_{\{\tau=\tau_C\}}\varepsilon_{I,\tau} - 1_{\{\tau=\tau_I\}}\varepsilon_{C,\tau}\right) \\
& - 1_{\{\tau=\tau_C<T\}} D(t,\tau)(1-\mathrm{Rec}_C)(\varepsilon^+_{I,\tau} - C^+_\tau)^+ \\
& - 1_{\{\tau=\tau_C<T\}} D(t,\tau)(1-\mathrm{Rec}'_C)(\varepsilon^-_{I,\tau} - C^-_\tau)^+ \\
& - 1_{\{\tau=\tau_I<T\}} D(t,\tau)(1-\mathrm{Rec}_I)(\varepsilon^-_{C,\tau} - C^-_\tau)^- \\
& - 1_{\{\tau=\tau_I<T\}} D(t,\tau)(1-\mathrm{Rec}'_I)(\varepsilon^+_{C,\tau} - C^+_\tau)^-
\end{aligned}
$$

- Notice that the collateral account enters only as a term reducing the exposure of each party upon default of the other one, keeping into account which is the party who posted the collateral.

Collateralized Bilateral CVA

- Now, by taking risk-neutral expectation of both sides of the above equation, and by plugging in the definition of mid-market exposure, we obtain the general expression for collateralized bilateral CVA.

$$
\begin{aligned}
\mathrm{CVA}(t,T;C) = &-\mathbb{E}_t\left[1_{\{\tau<T\}} D(t,\tau)\left(\varepsilon_\tau - 1_{\{\tau=\tau_C\}}\varepsilon_{I,\tau} - 1_{\{\tau=\tau_I\}}\varepsilon_{C,\tau}\right)\right] \\
&-\mathbb{E}_t\left[1_{\{\tau=\tau_C<T\}} D(t,\tau)\mathrm{LGD}_C(\varepsilon^+_{I,\tau} - C^+_\tau)^+ \right] \\
&-\mathbb{E}_t\left[1_{\{\tau=\tau_C<T\}} D(t,\tau)\mathrm{LGD}'_C(\varepsilon^-_{I,\tau} - C^-_\tau)^+ \right] \\
&-\mathbb{E}_t\left[1_{\{\tau=\tau_I<T\}} D(t,\tau)\mathrm{LGD}_I(\varepsilon^-_{C,\tau} - C^-_\tau)^- \right] \\
&-\mathbb{E}_t\left[1_{\{\tau=\tau_I<T\}} D(t,\tau)\mathrm{LGD}'_I(\varepsilon^+_{C,\tau} - C^+_\tau)^- \right]
\end{aligned}
$$

- Now, we need a recipe to calculate on-default exposures ε_{I,τ_C} and ε_{C,τ_I}, that, in the practice, are approximated from today exposure corrected for haircuts or add-ons.

Formulae for Collateralized Bilateral CVA – I

- We consider all the exposures being evaluated at mid-market, namely we consider:

$$\varepsilon_{I,t} \doteq \varepsilon_{C,t} \doteq \varepsilon_t$$

- Thus, in such case we obtain for collateralized bilateral CVA

$$\begin{aligned}
\text{CVA}(t,T;C) = &-\mathbb{E}_t\left[\,1_{\{\tau=\tau_C<T\}} D(t,\tau)\text{LGD}_C(\varepsilon_\tau^+ - C_\tau^+)^+\,\right] \\
&- \mathbb{E}_t\left[\,1_{\{\tau=\tau_C<T\}} D(t,\tau)\text{LGD}'_C(\varepsilon_\tau^- - C_\tau^-)^+\,\right] \\
&- \mathbb{E}_t\left[\,1_{\{\tau=\tau_I<T\}} D(t,\tau)\text{LGD}_I(\varepsilon_\tau^- - C_\tau^-)^-\,\right] \\
&- \mathbb{E}_t\left[\,1_{\{\tau=\tau_I<T\}} D(t,\tau)\text{LGD}'_I(\varepsilon_\tau^+ - C_\tau^+)^-\,\right]
\end{aligned}$$

- After this section we show a possible way to relax such approximation.

Formulae for Collateralized Bilateral CVA – II

- If collateral re-hypothecation is not allowed ($\text{LGD}'_C \doteq \text{LGD}'_I \doteq 0$) the above formula simplifies to

$$\begin{aligned}\text{CVA}(t,T;C) = &-\mathbb{E}_t\left[\, 1_{\{\tau=\tau_C<T\}} D(t,\tau)\text{LGD}_C(\varepsilon_\tau^+ - C_\tau^+)^+ \,\right] \\ &- \mathbb{E}_t\left[\, 1_{\{\tau=\tau_I<T\}} D(t,\tau)\text{LGD}_I(\varepsilon_\tau^- - C_\tau^-)^- \,\right]\end{aligned} \tag{4}$$

- On the other hand, if re-hypothecation is allowed and the surviving party always faces the worst case ($\text{LGD}'_C \doteq \text{LGD}_C$ and $\text{LGD}'_I \doteq \text{LGD}_I$), we get

$$\begin{aligned}\text{CVA}(t,T;C) = &-\mathbb{E}_t\left[\, 1_{\{\tau=\tau_C<T\}} D(t,\tau)\text{LGD}_C(\varepsilon_\tau - C_\tau)^+ \,\right] \\ &- \mathbb{E}_t\left[\, 1_{\{\tau=\tau_I<T\}} D(t,\tau)\text{LGD}_I(\varepsilon_\tau - C_\tau)^- \,\right]\end{aligned} \tag{5}$$

Formulae for Collateralized Bilateral CVA – III

- If we remove collateralization ($C_t = 0$), we recover the result of Brigo and Capponi (2008), and used in Brigo, Pallavicini and Papatheodorou (2009).

$$\begin{aligned}\mathrm{CVA}^{\mathrm{BC}}(t,T) = &-\mathbb{E}_t\left[\,1_{\{\tau=\tau_C<T\}} D(t,\tau)\mathrm{LGD}_C \varepsilon_\tau^+\,\right] \\ &-\mathbb{E}_t\left[\,1_{\{\tau=\tau_I<T\}} D(t,\tau)\mathrm{LGD}_I \varepsilon_\tau^-\,\right]\end{aligned} \tag{6}$$

- If we remove collateralization ($C_t = 0$) and we consider a risk-free investor ($\tau_I \to \infty$), we recover the result of Brigo and Pallavicini (2007), but see also Canabarro and Duffie (2004).

$$\mathrm{CVA}^{\mathrm{BP}}(t,T) = -\mathbb{E}_t\left[\,1_{\{\tau_C<T\}} D(t,\tau_C)\mathrm{LGD}_C \varepsilon_{\tau_C}^+\,\right] \tag{7}$$

An Example: Perfect Collateralization

- We consider, for this example, updating the collateral account continuously. We obtain the following (perfect) collateralization rule.

$$C_t^{\text{perfect}} := \varepsilon_t$$

- Thus, if we plug it into the collateralized bilateral CVA equation (with all exposure at mid-market), we get that all terms drop, as expected, leading to

$$\mathrm{C}_{\mathrm{VA}}(t, T; C^{\text{perfect}}) = 0$$

$$\mathbb{E}_t\left[\,\bar{\Pi}(t, T; C)\,\right] = \mathbb{E}_t[\,\Pi(t, T)\,] = \varepsilon_t = C_t^{\text{perfect}}$$

- Thus, the proper discount curve for pricing the deal is the collateral accrual curve (see also Fujii et al. (2010) or Piterbarg (2010)).

Inclusion of Funding Cost

When managing a trading position, one needs to obtain cash in order to do a number of operations:

- hedging the position,
- posting collateral,
- paying coupons or notionals
- set reserves in place

and so on. Where are such founds obtained from?

- Obtain cash from her Treasury department or in the market.
- receive cash as a consequence of being in the position:
  - a coupon or notional reimbursement,
  - a positive mark to market move,
  - getting some collateral or interest on posted collateral,
  - a closeout payment.

All such flows need to be remunerated:

- if one is ”borrowing”, this will have a cost,
- and if one is ”lending”, this will provide revenues.

Inclusion of Funding Cost

Funding is not just different discounting

- CVA and DVA are not obtained just by adding a spread to the discount factor of assets cash flows
- Similarly, a hypothetical FVA is not simply applying spreads to borrowing and lending cash flows.

One has to carefully and properly analyze and price the real cash flows rather than add an artificial spread. The simple spread may emerge for very simple deals and under simplifying assumptions (no correlations, uni-directional cash flows, etc)

Inclusion of Funding Cost: literature

- Crepey (2011) is one of the most comprehensive treatments so far. The only limitation is that it does not allow for underlying credit instruments in the portfolio, and has possible issues with FX.
- A related framework that is more general, is in Pallavicini, Perini and B. (2011). Earlier works are partial.
- Piterbarg (2010) considers an initial analysis of the problem of replication of derivative transactions under collateralization but in a standard Black Scholes framework without default risk. Burgard and Kjaer (2011) are more general but do not consider collateral subtelties and resort to a PDE approach.

Funding: The self financing condition

- A number of papers (and Hull's book) have a mistake in the self-financing condition, in that they assume the risky asset position is self-financing on its own. They assume the replicating portfolio to be $P_t = \Delta_t S_t + \gamma_t$, and the self financing condition

$$dP_t = \Delta_t dS_t + d\gamma_t \Rightarrow d(\Delta_t S_t) = \Delta_t dS_t \text{ (wrong)}.$$

  The funding account γ is NOT properly defined that way.
- Going back to literature, Morini and Prampolini (2011) focus on simple products (zero coupon bonds or loans) in order to highlight some essential features of funding costs and their relationship with DVA.
- Fujii and Takahashi (2010) analyzes implications of currency risk for collateral modeling.

Funding Valuation Adjustment? Can FVA be additive?

A fundamental point is including funding consistently with counterparty risk. Industry wishes for a “Funding Valuation Adjustment”, or FVA, that would be additive:

TOTAL PRICE =

= RISK FREE PRICE + DVA - CVA + FVA

Since I need to pay the funding costs to my treasury desk or to the market party that is funding me, or perhaps since I am receiving interest on collateral I posted, the real value of the deal is affected.

But is the effect just additive and decomposable with CVA and DVA?

It is not so simple

Funding, credit and market risk interact in a nonlinear and recursive way and they cannot be decomposed additively.

Funding and DVA

DVA a component of FVA?

DVA is related to funding costs when the payout is uni-directional, eg shorting/issuing a bond, borrowing in a loan, or going short a call option.

Indeed, if we are short simple products that are uni-directional, we are basically borrowing.

As we shorted a bond or a call option, for example, we received cash V_0 in the beginning, and we will have to pay the product payout in the end.

This cash can be used by us to fund other activities, and allows us to spare the costs of fuding this cash V_0 from our treasury.

Funding and DVA

Our treasury usually funds in the market, and the market charges our treasury a cost of funding that is related to the borrowed amount V_0, to the period T and to our own bank credit risk $\tau_B < T$.

In this sense the funding cost we are sparing when we avoid borrowing looks similar to DVA: it is related to the price of the object we are shorting and to our own credit risk.

However quite a number of assumptions is needed to identify DVA with a pure funding benefit, as we will see below.

Basic Payout plus Credit and Collateral: Cash Flows I

- We calculate prices by discounting cash-flows under the pricing measure. Collateral and funding are modeled as additional cashflows (as for CVA and DVA)
- We start from derivative's cash flows.

$$\bar{V}_t(C;F) \quad := \quad \mathbb{E}_t[\,\Pi(t, T \wedge \tau) + \ldots\,]$$

where

$\longrightarrow$ $\tau := \tau_C \wedge \tau_I$ is the first default time, and

$\longrightarrow$ $\Pi(t,u)$ is the sum of all discounted payoff terms up from t to u,

Basic Payout plus Credit and Collateral: Cash Flows II

- As second contribution we consider the collateralization procedure and we add its cash flows.

$$
\begin{aligned}
\bar{V}_t(C;F) \quad &:= \quad \mathbb{E}_t[\,\Pi(t, T \wedge \tau)\,] \\
&+ \quad \mathbb{E}_t\Big[\,\gamma(t, T \wedge \tau; C) + 1_{\{\tau < T\}} D(t,\tau) C_{\tau^-} + \dots\Big]
\end{aligned}
$$

where

$\longrightarrow$ C_t is the collateral account defined by the CSA,

$\longrightarrow$ C_{τ^-} is the pre-default value of the collateral account, and

$\longrightarrow$ $\gamma(t, u; C)$ are the collateral margining costs up to time u.

- Notice that when applying close-out netting rules, first we will net the exposure against C_{τ^-}, then we will treat any remaining collateral as an unsecured claim.

Basic Payout plus Credit and Collateral: Cash Flows III

$$\gamma(t,u;C) := \sum_{k=1}^{n-1} 1_{\{t \le t_k < u\}} D(t,t_k) C_{t_k} \left(1 - P_{t_k}(t_{k+1})(1 + \alpha_k \tilde{c}_{t_k}(t_{k+1}))\right)$$

where $\alpha_k = t_{k+1} - t_k$ and the collateral accrual rates are given by

$$\tilde{c}_t := c_t^+ 1_{\{C_t > 0\}} + c_t^- 1_{\{C_t < 0\}}$$

- Then, according to CSA, we introduce the pre-default value of the collateral account C_{τ^-} as

$$C_{\tau^-} := \sum_{k=1}^{n-1} 1_{\{t_k < \tau < t_{k+1}\}} C_{t_k} P_\tau(t_{k+1})(1 + \alpha_k \tilde{c}_{t_k}(t_{k+1}))$$

Close-Out: Trading-CVA/DVA under Collateral – I

- As third contribution we consider the cash flow happening at 1st default, and we have

$$\begin{aligned}\bar{V}_t(C;F) \quad &:= \quad \mathbb{E}_t[\,\Pi(t, T\wedge\tau)\,] \\ &+ \quad \mathbb{E}_t\left[\,\gamma(t, T\wedge\tau; C) + 1_{\{\tau<T\}} D(t,\tau) C_{\tau^-}\,\right] \\ &+ \quad \mathbb{E}_t\left[\,1_{\{\tau<T\}} D(t,\tau)\,(\theta_\tau(C,\varepsilon) - C_{\tau^-}\,) + \ldots\,\right]\end{aligned}$$

  where

  $\longrightarrow$ ε_τ is the amount of losses or costs the surviving party would incur on default event (close-out amount), and

  $\longrightarrow$ $\theta_\tau(C,\varepsilon)$ is the on-default cash flow.

- θ_τ will contain collateral adjusted CVA and DVA payouts for the instument cash flows
- We define θ_τ including the pre-default value of the collateral account since it is used by the close-out netting rule to reduce exposure

Close-Out: Trading-CVA/DVA under Collateral – II

- The close-out amount is not a symmetric quantity w.r.t. the exchange of the role of two parties, since it is valued by one party after the default of the other one.

$$\varepsilon_\tau := 1_{\{\tau=\tau_C\}}\varepsilon_{I,\tau} + 1_{\{\tau=\tau_I\}}\varepsilon_{C,\tau}$$

- Without entering into the detail of close-out valuation we can assume a close-out amount equal to the risk-free price of remaining cash flows inclusive of collateralization and funding costs. More details in the examples.
  - $\longrightarrow$ See ISDA document "Market Review of OTC Derivative Bilateral Collateralization Practices" (2010).
  - $\longrightarrow$ See, for detailed examples, Parker and McGarry (2009) or Weeber and Robson (2009)
  - $\longrightarrow$ See, for a review, Brigo, Morini, Pallavicini (2013).

Close-Out: Trading-CVA/DVA under Collateral – III

- At transaction maturity, or after applying close-out netting, the originating party expects to get back the remaining collateral.
- Yet, prevailing legislation's may give to the Collateral Taker some rights on the collateral itself.
  - $\longrightarrow$ In presence of re-hypothecation the collateral account may be used for funding, so that cash requirements are reduced, but counterparty risk may increase.
  - $\longrightarrow$ See Brigo, Capponi, Pallavicini and Papatheodorou (2011).
- In case of collateral re-hypothecation the surviving party must consider the possibility to recover only a fraction of his collateral.
  - $\longrightarrow$ We name such recovery rate $\text{R}_{\text{EC}}{}'_I$, if the investor is the Collateral Taker, or $\text{R}_{\text{EC}}{}'_C$ in the other case.
  - $\longrightarrow$ In the worst case the surviving party has no precedence on other creditors to get back his collateral, so that

$$\text{R}_{\text{EC}}{}_I \leq \text{R}_{\text{EC}}{}'_I \leq 1 \;, \quad \text{R}_{\text{EC}}{}_C \leq \text{R}_{\text{EC}}{}'_C \leq 1$$

Close-Out: Trading-CVA/DVA under Collateral – IV

- The on-default cash flow $\theta_\tau(C,\varepsilon)$ can be calculated by following ISDA documentation. We obtain

$$
\begin{aligned}
\theta_\tau(C,\varepsilon) \quad &:= \quad 1_{\{\tau=\tau_C<\tau_I\}} \left(\varepsilon_{I,\tau} - \mathrm{L{\scriptstyle GD}}_C (\varepsilon^+_{I,\tau} - C^+_{\tau-})^+ - \mathrm{L{\scriptstyle GD}}'_C (\varepsilon^-_{I,\tau} - C^-_{\tau-})^+ \right) \\
&+ \quad 1_{\{\tau=\tau_I<\tau_C\}} \left(\varepsilon_{C,\tau} - \mathrm{L{\scriptstyle GD}}_I (\varepsilon^-_{C,\tau} - C^-_{\tau-})^- - \mathrm{L{\scriptstyle GD}}'_I (\varepsilon^+_{C,\tau} - C^+_{\tau-})^- \right)
\end{aligned}
$$

  where loss-given-defaults are defined as $\mathrm{L{\scriptstyle GD}}_C := 1 - \mathrm{R{\scriptstyle EC}}_C$, and so on.

- If both parties agree on exposure, namely $\varepsilon_{I,\tau} = \varepsilon_{C,\tau} = \varepsilon_\tau$ then

$$
\begin{aligned}
\theta_\tau(C,\varepsilon) \quad &:= \quad \varepsilon_\tau - 1_{\{\tau=\tau_C<\tau_I\}} \Pi_{\mathrm{CVAcoll}} + 1_{\{\tau=\tau_I<\tau_C\}} \Pi_{\mathrm{DVAcoll}} \\
\Pi_{\mathrm{CVAcoll}} \quad &= \quad \mathrm{L{\scriptstyle GD}}_C (\varepsilon^+_\tau - C^+_{\tau-})^+ + \mathrm{L{\scriptstyle GD}}'_C (\varepsilon^-_\tau - C^-_{\tau-})^+ \\
\Pi_{\mathrm{DVAcoll}} \quad &= \quad \mathrm{L{\scriptstyle GD}}_I ((-\varepsilon_\tau)^+ - (-C_{\tau-})^+)^+ + \mathrm{L{\scriptstyle GD}}'_I (C^+_{\tau-} - \varepsilon^+_\tau)^+
\end{aligned}
$$

Close-Out: Trading-CVA/DVA under Collateral – V

- In case of re-hypothecation, when $\text{LGD}_C = \text{LGD}'_C$ and $\text{LGD}_I = \text{LGD}'_I$, we obtain a simpler relationship

$$
\begin{aligned}
\theta_\tau(C,\varepsilon) \quad := & \quad \varepsilon_\tau \\
- & \quad 1_{\{\tau=\tau_C<\tau_I\}} \text{LGD}_C(\varepsilon_{I,\tau} - C_{\tau^-})^+ \\
- & \quad 1_{\{\tau=\tau_I<\tau_C\}} \text{LGD}_I(\varepsilon_{C,\tau} - C_{\tau^-})^-
\end{aligned}
$$

Funding and Hedging – I

- As fourth and last contribution we consider the funding and hedging procedures and we add their cash flows.

$$
\begin{aligned}
\bar{V}_t(C;F) \quad &:= \quad \mathbb{E}_t[\,\Pi(t, T \wedge \tau)\,] \\
&+ \quad \mathbb{E}_t\left[\,\gamma(t, T \wedge \tau; C) + 1_{\{\tau < T\}} D(t,\tau)\theta_\tau(C,\varepsilon)\,\right] \\
&+ \quad \mathbb{E}_t[\,\varphi(t, T \wedge \tau; F, H)\,]
\end{aligned}
$$

  where

  $\longrightarrow$ F_t is the cash account needed for trading,

  $\longrightarrow$ H_t is the risky-asset account implementing the hedging strategy, and

  $\longrightarrow$ $\varphi(t, T; F, H)$ are the cash F and hedging H funding costs up to u.

- In classical Black Scholes on Equity, for a call option (no credit risk, no collateral, no funding costs),

$$
\bar{V}_t^{\text{Call}} = \Delta_t S_t + \eta_t B_t =: H_t + F_t, \quad \tau = +\infty, \gamma = 0, \varphi = 0.
$$

Funding and Hedging – II

- The cash flows due to the funding and hedging strategy on the time grid $\{t_j\}$ are equal to

$$\begin{aligned}\varphi(t,u;F,H) \quad &:= \quad \sum_{j=1}^{m-1} 1_{\{t\le t_j<u\}} D(t,t_j)(F_{t_j}+H_{t_j})\left(1-P_{t_j}(t_{j+1})(1+\alpha_j\tilde{f}_{t_j}(t_{j+1}))\right) \\ &- \quad \sum_{j=1}^{m-1} 1_{\{t\le t_j<u\}} D(t,t_j)H_{t_j}\left(1-P_{t_j}(t_{j+1})(1+\alpha_j\tilde{h}_{t_j}(t_{j+1}))\right)\end{aligned}$$

where the funding/borrowing and investing/lending rates for F and H are given by

$$\tilde{f}_t := f_t^+ 1_{\{F_t>0\}} + f_t^- 1_{\{F_t<0\}}\ , \quad \tilde{h}_t := h_t^+ 1_{\{H_t>0\}} + h_t^- 1_{\{H_t<0\}}$$

Funding and Hedging – III

Cash is borrowed $F > 0$ from the treasury at an interest f^+ (cost) or is lent $F < 0$ at a rate f^- (revenue)

Risky Hedge asset is worth H. Cash needed to buy $H > 0$ ie the risky hedge is borrowed at an interest f^+ from the treasury (cost); in this case H can be used for asset lending (Repo for example) at a rate h^+ (revenue);

On the other hand if risky hedge is worth $H < 0$, we may borrow from the repo market by posting the asset H as guarantee (rate h^-, cost), and lend the obtained cash to the treasury to be remunerated at a rate f^- (revenue).

It is possible to include the risk of default of the funder and funded, leading to CVA and DVA adjustments for the funding position, see PPB.

The Recursive Nature of Pricing Equations – I

$$(*) \quad \bar{V}_t(C; F) = \mathbb{E}_t\big[\,\Pi(t, T\wedge\tau) + \gamma(t, T\wedge\tau) + 1_{\{\tau<T\}} D(t,\tau)\theta_\tau(C,\varepsilon)\,\big] \\ + \mathbb{E}_t[\,\varphi(t, T\wedge\tau; F, H)\,]$$

where we recall that $\varphi(t, T\wedge\tau; F) =$ sum of all the Investor funding borrowing and lending positions costs/revenues to hedge its trading position, up to the 1st default.

Recursive pricing algorithm (see full PPB (2011) paper for details)

We obtain a *recursive* equation: the product price $\bar{V}_t(C, F)$ in $(*)$ depends on the funding strategy $F((t, T])$ after t via φ, and the funding $F = \bar{V} - (C-)H$ after t depends on the future product price $\bar{V}((t, T])$.

- This recursive equation can be solved iteratively via LS MC techniques as in standard CVA calculations $\rightarrow$ See PPB (2011)

The Recursive Nature of Pricing Equations – II

$\longrightarrow$ Numerical solutions based on BSDE techniques are required to solve the general problem.
$\longrightarrow$ See Pallavicini, Perini, Brigo (2011) for a discrete time solution.
$\longrightarrow$ See Crépey et al. (2012a) for further examples.

- The recursive feature of pricing equations is hidden in simplified approaches starting either from spreading the discount curve, or from adding simplistic extra pricing terms (FVA?).
- A different approach, leading to similar results, is followed by Crépey et al. (2011) or Burgard and Kjaer (2010,2011) where the usual risk-neutral evaluation framework is extended to include many cash accounts accruing at different rates.

Explain Funding Rates: Trading vs. Funding DVA – I

- The funding rate $\tilde{f}_t$ is determined by the party managing the funding account for the investor, usually the bank's treasury according to its liquidity policy:
  - $\longrightarrow$ trading positions may be netted before searching for funds on the market;
  - $\longrightarrow$ a Funds Transfer Pricing (FTP) process may be implemented to gauge the performances of different business units;
  - $\longrightarrow$ a maturity transformation rule can be used to link portfolios to effective maturity dates;
  - $\longrightarrow$ many source of funding can be mixed to obtain the internal funding curve; etc. . .
- In the literature the role of the treasury is usually neglected, leading to some controversial results particularly when the funding positions are not distinguished from the trading positions.

Explain Funding Rates: Trading vs. Funding DVA – II

- In particular, the false claim "funding costs are the DVA", or even "there are no funding costs at all", are often cited in the practitioners' literature.

$\longrightarrow$ See the querelle following Hull and White (2012), "FVA =0" (???)

DealPrice $=$ RiskFreePrice - CVA + DVA $\pm$ FVA?

Can we simply add a new term called FVA to account for funding costs, "funding valuation adjustment"?

We have seen that when including funding we obtain a recursive nonlinear problem on a specific portfolio (netting set? Aggregation level? Treasury decision?).

Not additive with CVA and DVA as these cash flows feed each other in a nonlinear and overlapping way. These risk interact and we can only compute a total adjustment.

Funding structures inside a bank?

Funding implications on a Bank structure

Including funding costs into valuation, even via a simplistic FVA, involves methodological, organisational, and structural challenges.

Many difficulties are similar to CVA's and DVA's, so Funding can be integrated in the CVA effort typically.

- Reboot IT functions, analytics, methodology, by adopting a consistent global methodology including a consistent credit-debit-collateral-funding adjustment
- Very strong investment, discontinuity, and against the ”internal competition” culture
- OR include separate and inconsistent CVA and FVA adjustments, accepting simplifications and double counting.
- It can be important to analyze the global funding implications of the whole trading activity of the bank.

Conclusions on funding

The law of one price

FVA cannot be charged to the counterparty, differently from CVA, and cannot be bilateral, since we do not know the funding policy of our counterparties. So even if DVA was giving us some hope to realign symmetry of prices, funding finally destroys the law of one price and makes prices a matter of perspective. bid ask?

Is the funding inclusive "price" a real price?

Each entity computes a different funding adjusted price for the same product. The funding adjusted "price" is not a price in the conventional term. We may use it to book the deal in our system or to pay our treasury but not to charge a client. It is more a "value" than a "price".

CCPs

Governments pressure

EMIR and Dodd Frank are pushing financial institutions to work through Central Counterparty Clearing Houses (CCPs).

CCPs are commercial entities that, ideally, would interpose themselves between the two parties in a trade.

- Each party will post collateral margins say daily, every time the mark to market goes against that party.
- Collateral is held by the CCP as guarantee for the other party.
- If a party in the deal defaults and the mark to market is in favour of the other party, then the surviving party will obtain the collateral from the CCP and will not be affected (?) by counterparty risk.
- Moreover, there is also an initial margin that is supposed to cover for additional risks like deteriorating quality of collateral, gap risk, wrong way risk, etc.

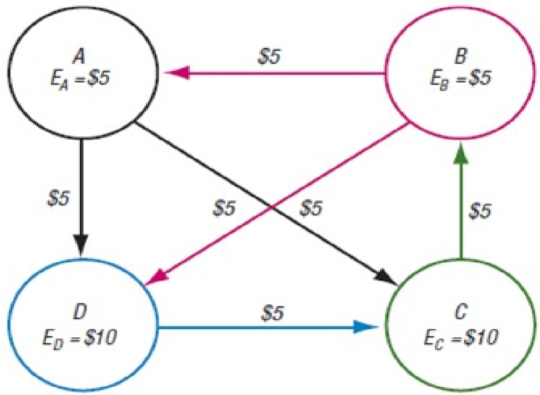


Figure: Bilateral trades and exposures without CCPs. Source: John Kiff.

http://shadowbankers.wordpress.com/2009/05/07/mitigating-counterparty-credit-risk-in-otc-markets-the-basics

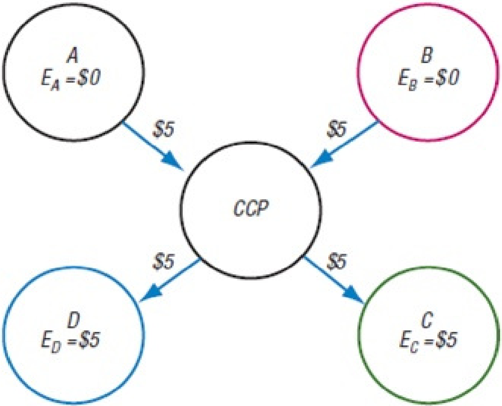


Figure: Bilateral trades and exposures with CCPs. Source: John Kiff.

http://shadowbankers.wordpress.com/2009/05/07/mitigating-counterparty-credit-risk-in-otc-markets-the-basics

CCPs

CCPs as the end of counterparty risk?

A number of market operators believe that CCPs are going to be the end of the counterparty credit and funding risk valuation problems, with CVA and FVA going to zero.

CCP's will reduce risk in many cases but are not a panacea. They also require daily margining, and one may question

- The pricing of the fees they apply
- The appropriateness of the initial margins and of overcollateralization buffers that are supposed to account for wrong way risk and collateral gap risk
- The default risk of CCPs themselves.

CCPs

CVA Analytics may still be necessary even with CCPs

Valuation of the above points requires CVA type analytics, inclusive of collateral gap risk and wrong way risk, similar to those we discuss here. So unless one trusts blindly a specific clearing house, it will be still necessary to access CVA analytics and risk measures.

CCPs

Standing problems with CCPs

The following points are worth keeping in mind:[a]

- CCPs are usually highly capitalised. All clearing members post collateral (asymmetric "CSA"). Initial margin means clearing members are overcollateralised all the time.
- TABB Group says extra collateral could be about 2 $ Trillion.[b]
- CCPs can default and did default. Defaulted ones - 1974: Caisse de Liquidation des Affaires en Marchandises; 1983: Kuala Lumpur Commodity Clearing House; 1987: Hong Kong Futures Exchange. The ones that were close to default- 1987: CME and OCC, US; 1999: BM&F, Brazil.

[a] See for example Piron, B. (2012). Why collateral and CCPs can be bad for your wealth. SunGard's Adaptive White Paper.

[b] Rhode, W. (2011). European Credit and Rates Dealers 2011 – Capital, Clearing and Central Limit Order Books. TABB Group Research Report

CCPs

CCPs and netting

A typical bank may have a quite large number of outstanding trades, making the netting clause quite material. With just one CCP for all asset classes across countries and continents, netting efficiency would certainly improve.

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However, in real life CCPs deal with specific asset classes or geographical areas, and this may even reduce netting efficiency compared to now.

CCPs

CCPs compete with each other

One can be competitive in specific areas but hardly in all of them. Some CCPs will be profitable in specific asset classes and countries. They will deal mostly with standardised transactions.
Even if CCPs could function across countries, bankruptcy laws can make collateral held in one place unusable to cover losses in other places.[a]

[a] Singh, M. (2011) Making OTC Derivatives Safe - A Fresh Look. IMF paper

CCPs

CCPs and EMIR/CDR 4/Basel III/DFA

There is currently "No legal construct to satisfy both Dodd Frank Act and EMIR and allow EU clients to access non-EU CCP's".[a] And there are also other conflicts in this respect. Where will CCPs be located and which countries will they serve? For example, the European Central Bank opposed LCH–Clearnet to work with Euro denominated deals because this CCP is not located in the Eurozone. This lead to a legal battle with LCH invoking the European Court of Justice.

[a] Wayne, H. (2012). Basel 3, Dodd Frank and EMIR. Citigroup Presentation.

CCPs

Competition and conflict of interest

To compete CCPs may lower margin requirements, which would make them riskier (remember the above CCPs defaults). In the US, where the OTC derivatives market is going through slightly more than 10 large dealers and is largely concentrated among 5, we could have a conflict of interest. If CCPs end up incorporating most trades currently occurring OTC bilaterally, then CCPs could become "too big to fail".[a]

[a] Miller, R. S. (2011) Conflicts of interest in derivatives clearing. Congressional Research Service report.

Given that CCPs may default, there is counterparty risk and ideally a CVA towards the CCP.

CCPs

Counterparty Risk with CCPs

The CCP does not post collateral directly to the entities trading with it, as the collateral agreement is not symmetric.
Hence, pricing counterparty risk towards a CCP is like CVA but computed without collateral.
On top of that, one has the overcollateralization cost to lose.
Hopefully, the default probability is low, making CVA small, bar strong contagion, gap risk and WWR

CCPs

CCPs are not the end of CVA and extensions thereof

We need to consider and price/ risk-manage

- Checking initial margin charges across different CCPs to see which ones best reflect actual gap risk and contagion. This requires a strong pricing apparatus
- Computing counterparty risk associated with the default of the CCP itself
- Understanding quantitatively the consequences of the lack of coordination among CCPs across different countries and currencies.

CVA Desks and "Best practices"

How do banks price and trade/hedge CVA?

The idea is to move Counterparty Risk management away from classic asset classes trading desks by creating a specific counterparty risk trading desk, or "CVA desk".

Under a lot of simplifying assumptions, this would allow "classical" traders to work in a counterparty risk-free world in the same way as before the counterparty risk crisis exploded.

CVA Desks and "Best practices"

What lead to CVA desks?

Roughly, CVA followed this historical path:

- Up to 1999/2000 no CVA. Banks manage counterparty risk through rough and static credit limits, based on exposure measurements (related to Credit VaR: Credit Metrics 1997).
- 2000-2007 CVA was introduced to assess the cost of counterparty credit risk. However, it would be charged upfront and would be managed mostly statically, with an insurance based approach.
- 2007 on, banks increasingly manage CVA dynamically. Banks become interested in CVA monitoring, in daily and even intraday CVA calculations, in real time CVA calculations and in more accurate CVA sensitivities, hedging and management.
- CVA explodes after 7[8] financials defaults occur in one month of 2008 (Fannie Mae, Freddie Mac, Washington Mutual, Lehman, [Merrill] and three Icelandic banks).

CVA Desks and "Best practices"

CVA desk location in a bank

In most tier-1 and 2 banks the CVA Desk is in on the capital markets/trading floor division, being a trading desk. Occasionally it may sit on the Treasury department (eg Banca IMI). In a few cases it can be a stand-alone entity outside standard departments classifications.

Trading floor is natural because it is a trading desk.

CVA desk and Classical Trading desks

The CVA desk charges classical trading desks a CVA fee in order to protect their trading activities from counterparty risk through hedging. This may happen also with collateral/CSA in place (Gap Risk, WWR, etc). The cost of implementing this hedge is the CVA fee the CVA desk charges to the classical trading desk.

CVA Desks and "Best practices"

CVA desk in the trasury department

Charging a fee is not easy and can make a lot of P&L sensitive traders nervous. That is one reason why some banks set the CVA desk in the treasury for example. Being outside the trading floor can avoid some "political" issues on P&L charges among traders.

—

Furthermore, given that the treasury often controls collateral flows and funding policies, this would allow to coordinate CVA and FVA calculations and charges after collateral.

CVA Desks and "Best practices"

How the CVA desk helps other trading desks

The CVA desk[a] would free the classical traders from the need to:

- develop advanced credit models to be coupled with classical asset classes models (FX, equity, rates, commodities...);
- know the whole netting sets trading portfolios; traders would have to worry only about their specific deals and asset classes, as the CVA desk takes care of "options on whole portfolios" embedded in counterparty risk pricing and hedging;
- Hedge counterparty credit risk, which is very complicated.

[a] See for example "CVA Desk in the Bank Implementation", *Global Market Solutions* white paper

CVA Desks and "Best practices"

The CVA desk task looks quite difficult

The CVA desk has **little/no control** on inflowing trades, and has to:

- quote quickly to classical trading desks a "incremental CVA" for specific deals, mostly for pre-deal analysis with the client;
- For every classical trade that is done, the CVA desk needs to integrate the position into the existing netting sets and in the global CVA analysis in real time;
- related to pre-deal analysis, after the trade execution CVA desk needs to allocate CVA results for each trade ("marginal CVA")
- Manage the global CVA, and this is the core task: Hedge counterparty credit and classical risks, including credit-classical correlations (WWR), and check with the risk management department the repercussions on capital requirements.

CVA Desks and "Best practices"

CVA Desks effectiveness if often questioned

Of course the idea of being able to relegate all CVA(/DVA/FVA) issues to a single specialized trading desk is a little delusional.

- WWR makes isolating CVA from other activities quite difficult.
- In particular WWR means that the idea of hedging CVA and the pure classical risks separately is not effective.
- CVA calculations may depend on the collateral policy, which does not depend on the CVA desk or even on the trading floor.
- We have seen FVA and CVA interact

In any case a CVA desk can have different levels of sophistication and effectiveness.

CVA Desks and "Best practices"

Classical traders opinions

Clearly, being P&L sensitive, the CVA desk role is rather delicate. There are mixed feelings.

- Because CVA is hard to hedge (especially the jump to default risk and WWR), occasionally classical traders feel that the CVA desk does not really hedge their counterparty risk effectively and question the validity of the CVA fees they pay to the CVA desk.
- Other traders are more optimistic and feel protected by the admittedly approximate hedges implemented by the CVA desk.
- There is also a psychological component of relief in delegating management of counterparty risk elsewhere.

Restructuring Counterparty Credit Risk I

- So far we have looked at the question "What is Counterparty Risk and how do we price it, possibly in a way that is consistent with other risks and arbitrage free?"
- Early attempts to price it have witnessed considerable CVA volatility, resulting in important mark to market losses during the crisis.
- This volatility is related to high volatility of credit spreads (see e.g. Brigo 2005), high volatility of exposure and wrong way risk.
- To deal with CVA mark to market risk, according to regulators, there are mainly two choices: **Collateral/CSA and margins posting** or **CVA VaR and related capital requirements**. Both ways are likely to worsen the liquidity landscape.
- **In other terms, we now look at the question: "What do we do with Counterparty Risk?"**

Restructuring Counterparty Credit Risk II

- The industry has been looking at different possible ways to deal with CVA risks and requirements. We will analyze here ways to **Restructure CVA**.
- Historically, Contingent CDS would be a good hedge for counterparty risk. However, such products are opaque, they are not liquid, can be expensive, and are themselves subject to counterparty risk. As such, their effectiveness has been quite limited.
- A subsequent more recent attempt in the industry has been based on securitization of CVA through traditional cash CDO type structures (e.g. ”Papillon” and ”Score” deals). More on this in a minute.
- Part of the press reported that ”Papillon” failed because the regulators did not recognize capital relief for the bank that was selling it.

Restructuring Counterparty Credit Risk III

- ”Score” had the originating bank more in line with regulators. However, the press reported that the deal failed presumably because, even if regulators approved it for capital relief, no investor has manifested great interest.
- These traditional structures would offer a fixed periodic premium or an upfront as a compensation for the protection being bought. However, as we shall see below, this implies the volatility of CVA to go the wrong way.
- We will therefore look at a different way to restructure CVA, namely a form of securitization based both on Margin Lending and on a floating rate notion of CVA. But first we’ll look at CCDS and at these cash CDO type deals a little more in detail.

Restructuring CVA: Contingent CDS (CCDS)

Definition

When the reference credit defaults at τ, the protection seller pays protection on a notional that is not fixed but given by the NPV of a reference Portfolio Π at that time if positive. This amount is: $(\mathbb{E}_{\tau_C}\Pi(\tau_C, T))^+$, minus a recovery R_{EC} fraction of it.

CCDS default leg payoff = asymmetric counterparty risk adjustm

The payoff of the default leg of a Contingent CDS is exactly

$$(1 - \mathrm{R}_{\mathrm{EC}})\mathbf{1}_{\{(t<\tau_C<T)\}}D(t,\tau_C)(\mathbb{E}_{\tau_C}\Pi(\tau_C, T))^+$$

Precise Valuation? Liquidity?
Counterparty risk of the Protection Seller? Standardization?

General Remarks on CCDS

"[...]Rudimentary and idiosyncratic versions of these so-called CCDS have existed for five years, but they have been rarely traded due to high costs, low liquidity and limited scope. [...] Counterparty risk has become a particular concern in the markets for interest rate, currency, and commodity swaps - because these trades are not always backed by collateral.[...] Many of these institutions - such as hedge funds and companies that do not issue debt - are beyond the scope of cheaper and more liquid hedging tools such as normal CDS. The new CCDS was developed to target these institutions (Financial Times, April 10, 2008)."

Being the two payoffs equivalent, UCVA valuation will hold as well for the default leg of a CCDS.

Interest on CCDS has come back in 2011 now that CVA capital charges risk to become punitive.

Basel III and CVA I

When the valuation of a risk is more dangerous than the risk itself

"Under Basel II, the risk of counterparty default and credit migration risk were addressed but mark-to-market losses due to credit valuation adjustments (CVA) were not. During the financial crisis, however, roughly two-thirds of losses attributed to counterparty credit risk were due to CVA losses and only about one-third were due to actual defaults."

Basel Committee on Banking Supervision, BIS (2011). Press release available at *http://www.bis.org/press/p110601.pdf*

Given the above situation, Basel III is imposing very severe capital requirements for CVA.

Basel III and CVA II

This may lead to forms of securitization of CVA such as margin lending on the whole exposure or on tranches of the exposure.

Such ”securitization of CVA” would be very difficult to model and to manage, requiring a global valuation perspective.

Restructuring Counterparty Risk with CDO type structures I

There have been a few deals that tried to restructure away counterparty risk in the hope to get capital relief from regulators. In the Financial Times blog Alphaville Pollack (2012) [78] reports that

Barcap Bistro

”In short, Barclays has taken a pool of loans and securitised them, but retained all but the riskiest piece. On that riskiest Euro 300m, Barclays has bought protection from an outside investor, e.g. hedge fund. That investor will get paid coupons over time for their trouble, but will also be hit with any losses on the loans, up to the total amount of their investment. To ensure that the investor can actually absorb these losses, collateral is posted with Barclays.”

Restructuring Counterparty Risk with CDO type structures II

Looks like a CDO equity tranche backed by collateral. Regulators did not like this and did not concede capital relief. Why?

The trick: Paying the expected loss to free capital

"Collateral: in theory Barclays is not exposed to the counterparty risk of the hedge fund. This is especially important because the hedge fund is outside the normal sphere of regulation, i.e. they aren't required to hold capital against risk-weighted assets in the way banks are." [...] But in some cases **premiums paid over time to the hedge fund are actually equal to or above the expected loss of the transaction**. That the Fed and Basel Committee were concerned enough to issue guidance on this is noteworthy. It'll be down to individual national regulators to prevent "over-engineering", and some regulators are more hands-on than others."

Restructuring Counterparty Risk with CDO type structures III

- Bank buys protection: equity tranche corresponding to the expected loss of the pool from a hedge fund.
- Bank makes sure hedge fund is interested by offering a premium equivalent or even superior to the expected loss itself.
- hedge fund posts collateral as guarantee for protection payment.
- The hedge fund does not need to have capital in place for its potential loss, since it is outside the sphere of regulation.
- The bank has now bought protection at the cost of expected loss, but has obtained that immediately back in form of collateral
- But, what was the bank objective, the bank has now capital relief for the risk on which protection has been bought. This may be a very large effect and the main objective of the bank.

Restructuring Counterparty Risk with CDO type structures IV

Notice this point of transferring risk outside the regulated system. This is a point that is stressed also in the OECD paper [11]. The blog continues:

Restructuring Counterparty Risk with CDO type structures V

RBS SCORE

In Pollack (2012b) [79]. ”RBS had a good go at securitising these exposures, but the deal didn’t quite make it over the line. However, Euroweek reports that banks are still looking into it:

’Royal Bank of Scotlands securitisation of counterparty credit risk, dubbed Score 2011, was pulled earlier this year, but other banks are said to be undeterred by the difficulties of the asset class, and are still looking at the market. However, other hedging options for counterparty risk may have dulled the economics of securitising this risk since the end of last year.’

Difficulties: A CDO of CVA’s. Pricing? Hedging? Risk management?

Restructuring Counterparty Risk with CDO type structures VI

Credit Suisse Bonus policy

Again Pollack (2012c) [80]: ”Last week Credit Suisse announced it had bought protection on the senior slice of its unusual employee compensation plan. The Swiss bank pays some of its senior bankers using a bond referencing counterparty risk, which also involves shifting some counterparty credit risk from the bank to its workers.”

This is like buying protection from your own employees. Interesting concept if you think about it. That way the employee, in theory, is incentivized in improving the risk profile of the company.

Restructuring Counterparty Risk with CDO type structures VII

CVA Volatility the wrong way

The problem with the above solutions, and also with the traditional upfront charge or fixed periodic fee for unilateral CVA is that it leaves CVA volatility with the investor/bank and not with the risky counterparty that generated it. This also affects current attempts to restructure Counterparty Risk ("Papillon/ Barcap, Score/ RBS")

In the unilateral case, the bank charges an upfront for CVA to the counterparty and then implements a hedging strategy. The bank is thus exposed to CVA mark to market volatility in the future.

Alternatively the bank may request collateral from the counterparty, but not all counterparties are able to regularly post collateral, and this can be rather punitive for some corporate counterparties.

Restructuring Counterparty Risk with CDO type structures VIII

See recent example on Lufthansa from Risk magazine:

> *The airline's Cologne-based head of finance, Roland Kern, expects its earnings to become more volatile ' not because of unpredictable passenger numbers, interest rates or jet fuel prices, but because it does not post collateral in its derivatives transactions".*

Floating Margin Lending, based on a floating CVA, is a possible solution this problem with volatility going the right way.

Floating Margin Lending: I

Traditionally, the CVA is typically charged by the structuring bank B (investor) either on an upfront basis or it is built into the structure as a fixed coupon stream.

Floating Margin lending instead is predicated on the notion of floating rate CVA.

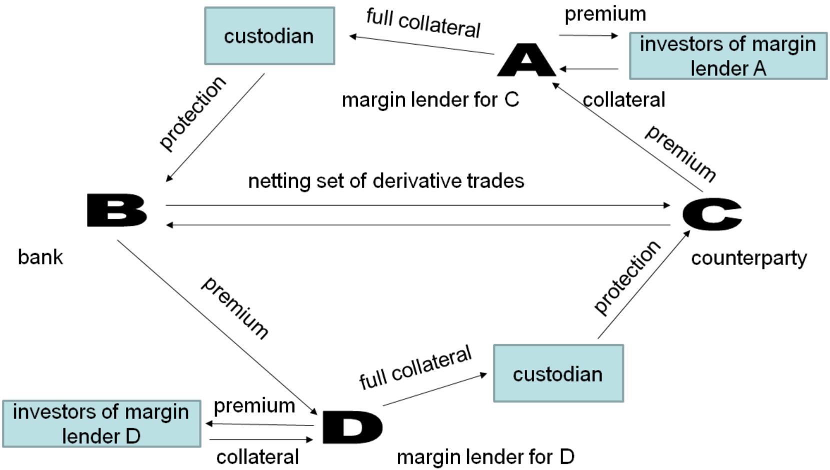
Floating rate CVA in a bi-partite transaction: The bank requires a CVA payment at time 0 for protection on the exposure up to 6 months. Then in 6 months the bank will require a CVA payment for protection for further six months, prevailing at that time, on what will be the exposure then, and on and on, up to the final maturity.

Floating Margin Lending: II

Floating Margin lending is designed in such a way to transfer the conditional credit spread volatility risk and the mark-to-market volatility risk, or in other terms CVA volatility, from the bank to the counterparties.

We may explain this more in detail by following the arrows in the Figure.

Floating Margin Lending: III



Floating Margin Lending: IV

To avoid posting collateral, C enters into a floating margin lending transaction.

C pays periodically (say semi-annually) a floating rate CVA to margin lender A ('premium' arrow connecting C to A), which A pays to investors (premium arrow connecting A to Investors). This latest payment can have a seniority structure similar to that of a cash CDO.

In exchange, for six months the investors provide A with daily collateral posting ('collateral' arrow connecting Investors to A) and A passes the collateral to a custodian ('collateral' arrow connecting A to the custodian).

This collateral need not be cash, but it can be in the form of hypothecs

Floating Margin Lending: V

If C defaults within the semi-annual period, the collateral is paid to B to provide protection ('protection' arrow connecting the custodian to B) and the loss in taken by the Investors who provided the collateral.

At the end of the six months period, the margin lender may decide whether to continue with the deal or to back off.

With this mechanism C is bearing the CVA volatility risk, whereas B is not exposed to CVA volatility risk, which is the opposite of what happens with traditional upfront CVA charges.

In traditional CVA, Albanese, B. and Oertel (2011) argue that whenever an entity's credit worsens, it receives a subsidy from its counterparties in the form of a DVA positive mark to market which can be monetized by the entity's bond holders only upon their own default.

Floating Margin Lending: VI

Whenever an entity's credit improves instead, it is effectively taxed as its DVA depreciates.

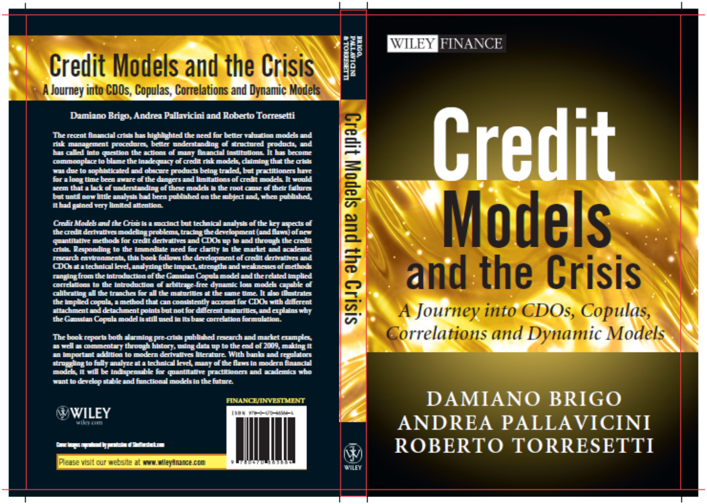
Wealth is thus transferred from firms with improving credit quality to firms with deteriorating credit quality, the transfer being mediated by the traditional CVA/DVA mechanics.

Again, Albanese, B. and Oertel (2011) submit that floating margin lending structures may help reversing this macroeconomic effect.

There are a number of possible **problems** with the above floating margin lending scheme.

First **problem**, proper valuation and hedging of this to the investor who are providing collateral to the lender is going to be tough. There is no satisfactory standard for even simple synthetic CDOs.

Floating Margin Lending: VII



Floating Margin Lending: VIII

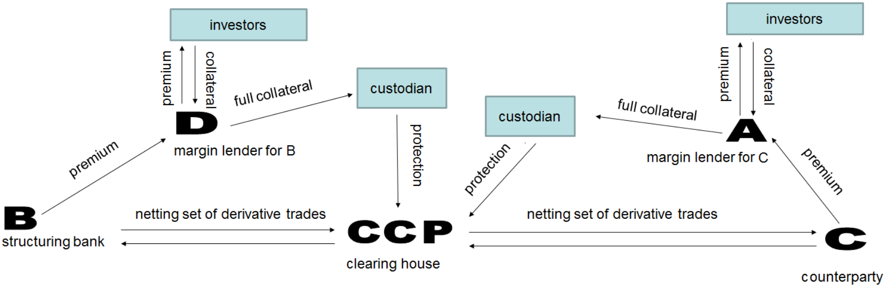
Admittedly this requires and effective global valuation framework, see for example the discussion in Albanese et al (2011).

A second **problem** is: what if all margin lenders pull off at some point due to a systemic crisis?

One may argue that the market is less likely to arrive in such a situation in the first place if the wrong incentives to defaulting firms are stopped and an opposite structure, such as floating margin lending, is implemented.

There is also a penta-partite version including a clearing house.

Floating Margin Lending: IX



CVA Restructuring: Global Valuation? I

A fair valuation and risk management of CVA restructuring through floating margin lending requires a global model, in order to have consistency and sensible greeks

But even when staying with traditional upfront CVA and DVA in large portfolios, as our examples above pointed out, different models are typically used in different asset classes.

This can lead to models that are inconsistent with each other.

For example, our equity example above used a firm value model, whereas in the other asset classes we used reduced form models.

CVA Restructuring: Global Valuation? II

What if one has a portfolio with all asset classes together?

More generally, how does one ensure a consistent modeling framework that is needed to get meaningful prices and especially cross correlation sensitivities?

The problem is rather difficult and involves important computational resources and intelligent systems architecture.

Few papers have appeared in the literature that are attempting a global valuation framework, see for example Albanese et al (2010, 2011).

Delicate points include:

CVA Restructuring: Global Valuation? III

Modeling dependencies across defaults (we do not have even a good standard model for synthetic corporate CDO, base correlation still used there, see for example Brigo Pallavicini and Torresetti (2010))

Modeling dependencies between defaults and each other asset class

Modeling dependencies between different asset classes

Properly including credit volatility with positive credit spreads

Conclusions I

- Counterparty Risk adds one level of optionality.
- Analysis including underlying asset/ counterparty default correlation requires a credit model.
- Highly specialized hybrid modeling framework.
- Accurate scenarios for wrong way risk.
- Outputs vary and can be very different from Basel multipliers
- Outputs are strongly model dependent and involve model risk and model choices
- Bilateral CVA brings in symmetry but also paradoxical statements
- Bilateral CVA requires a choice of closeout (risk free or substitution), and this is relevant.
- The DVA term in bilateral CVA is hard to hedge, especially in the jump-to-default risk component.

Conclusions II

- Approximations ignoring first to default risk (sometimes used in the industry) do not work well.
- Inclusion of Collateral and netting rules is possible
- Gap risk in collateralization remains relevant in presence of strong contagion
- Funding costs can be included consistently but they break the law of one price
- Funding is not just a spread but a complex nonlinear and recursive pricing problem
- Credit Debit and Funding costs can alter the structure of the bank organization and are politically sensitive

Conclusions III

- The creation of a CVA (DVA/FVA) trading desk is an attempt to isolate these advanced effects so that classical trading can work as before, but this is difficult as risks are not isolated from each other and hedging may not be separable
- Basel III will make capital requirements rather severe and values CVA in a simplistic way, especially wrt WWR
- Contingent CDS as hedging instruments have limited effectiveness
- CVA restructuring through floating margin lending and hypothecs is a possible alternative
- Proper valuation and management of CVA and especially CVA restructuring requires a Consistent Global Valuation approach
- This also holds for possible forms of CVA Securitization

Conclusions IV

- CCPs will become more and more central, but they hardly represent the end of CVA problems
- Fair initial margin, CCP default risk, CCP fractioning across geographical areas, lack of coordination, incentive to lower margins to compete and attract clients, too big to fail features...

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