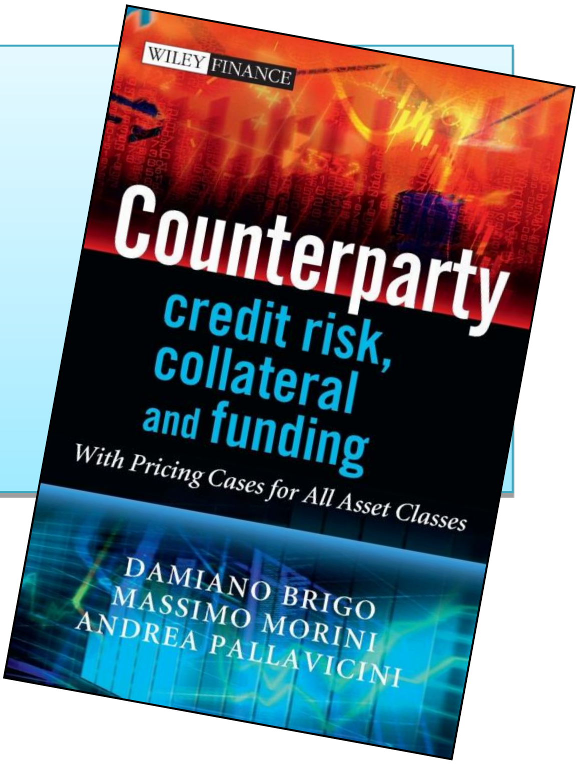
Counterparty Credit Risk, Collateral and Funding

With Pricing Cases for all Asset Classes

Damiano Brigo, Massimo Morini and Andrea Pallavicini

Order now, and save!!



The book's content is focused on rigorous and advanced quantitative methods for the pricing and hedging of counterparty credit and funding risk. The new general theory that is required for this methodology is developed from scratch, leading to a consistent and comprehensive framework for counterparty credit and funding risk, inclusive of collateral, netting rules, possible debit valuation adjustments, re-hypothecation and closeout rules. The book however also looks at quite practical problems, linking particular models to particular 'concrete' financial situations across asset classes, including interest rates, FX, commodities, equity and credit itself.

The authors also aim to help quantitative analysts, traders, and anyone else needing to frame and price counterparty credit and funding risk, to develop a 'feel' for applying sophisticated mathematics and stochastic calculus to solve practical problems.

The main models are illustrated from theoretical formulation to final implementation with calibration to market data, always keeping in mind the concrete questions being dealt with. The authors stress that each model is suited to different situations and products, pointing out that there does not exist a single model which is uniformly better than all the others, although the problems originated by counterparty credit and funding risk point in the direction of global valuation. Finally, proposals for restructuring counterparty credit risk, ranging from contingent credit default swaps to margin lending, are considered.

Pre-order now from www.wiley.com

978-0-470-74846-6 • Hardback • 500+ pages • March 2013

WILEY Global Finance
WHERE DATA FINDS DIRECTION

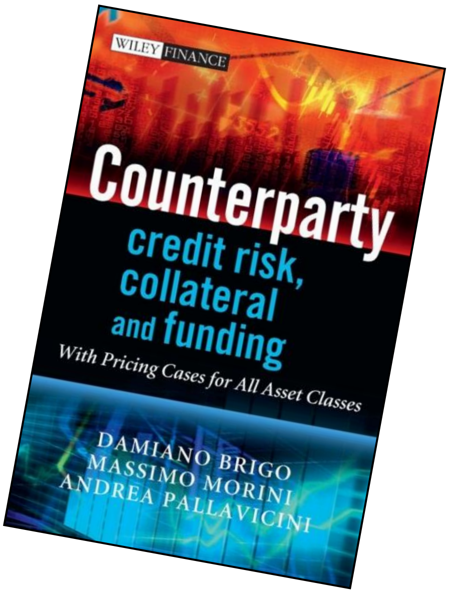
About the Authors

Damiano Brigo (London, UK) is a Professor (Chair) of Mathematical Finance at Imperial College, London, where he co-heads the Mathematical Finance research group. Previously, he held the Gilbart Chair of Financial Mathematics at King's College and he held positions as Managing Director and Global Head of the Quantitative Innovation team at London-based Fitch Solutions and Head of Credit Models at Banca IMI. He has a Ph.D. in stochastic filtering with differential geometry from the Free University of Amsterdam, following a BSc in Mathematics from the University of Padua. He is author of the field reference book "Interest Rate Models: Theory and Practice" for Springer-Verlag and of two volumes on credit modelling for Wiley. He teaches regularly at post-university and Master courses in Milan and for professional training companies in London. He has been included in scientific committees of international conferences occurring at MIT and other academic and professional institutions. Damiano has been listed as the most cited author in Risk Magazine in 2006 and 2010 and is Managing Editor of the International Journal of Theoretical and Applied Finance. His current professional interests include default and credit modelling, counterparty risk, interest-rate and smile modelling, commodities and hybrids models and risk measurement, stochastic nonlinear filtering and information geometry.

Massimo Morini (Milan, Italy) is Head of Interest Rate and Credit Models and Coordinator of Model Research at IMI Bank of Intesa San Paolo. Massimo is also Professor of Fixed Income at Bocconi University and was a Research Fellow at Cass Business School, City University London. He regularly delivers advanced training in London, New York and worldwide. He has led workshops on credit risk and the financial crisis at major international conferences. He has published papers in journals including Risk Magazine, Mathematical Finance, and the Journal of Derivatives, and is the author of "Understanding and Managing Model Risk: A Practical Guide for Quants, Traders and Validators". Massimo holds a PhD in Mathematics and an MSc in Economics.

Andrea Pallavicini (Milan, Italy) is Head of Equity, FX and Commodity Models at Banca IMI, where he has the responsibility of numerical algorithm's design, financial modelling and research activity. Previously, he held positions as Head of Financial Models at Mediobanca and Head of Financial Engineering at Banca Leonardo, and he worked also in aerospace industries and financial institutions. He has a Degree in Astrophysics and a Ph.D. in Theoretical and Mathematical Physics from the University of Pavia for his reasearch activity at CERN laboratory in Genève.
Over the years he published several academic and practitioner-oriented articles in financial modelling, theoretical physics and astrophysics on major peer-reviewed journals. He is author of the book "Credit Models and the Crisis: a Journey into CDOs, Copulas, Correlations and Dynamic Models" for Wiley. He teaches regularly at professional training courses and at Master and Ph.D. courses in finance at different Universities and private institutions. Main contributions in finance concern interest-rate and credit modelling, counterparty credit risk, and hybrid derivative pricing.

Table of contents


WILEY FINANCE
Counterparty credit risk, collateral and funding
With Pricing Cases for All Asset Classes
DAMIANO BRIGO
MASSIMO MORINI
ANDREA PALLAVICINI

Ignition 9

Abbreviations and Notation 17

PART I Counterparty Credit Risk, Collateral and Funding 25

1 Introduction 27

1.1 A Dialogue on CVA 27
1.2 Risk Measurement: Credit VaR 27
1.3 Exposure, CE, PFE, EPE, EE, EAD 31
1.4 Exposure and Credit VaR 33
1.5 Interlude: P and Q 33
1.6 Basel 35
1.7 CVA and Model Dependence 36
1.8 Input and Data issues on CVA 37
1.9 Emerging asset classes: Longevity Risk 39
1.10 CVA and Wrong Way Risk 40
1.11 Basel III: VaR of CVA and Wrong Way Risk 41
1.12 Discrepancies in CVA valuation: Model risk and Payoff Risk 43
1.13 Bilateral Counterparty Risk: CVA and DVA 44
1.14 First to Default in CVA and DVA 47

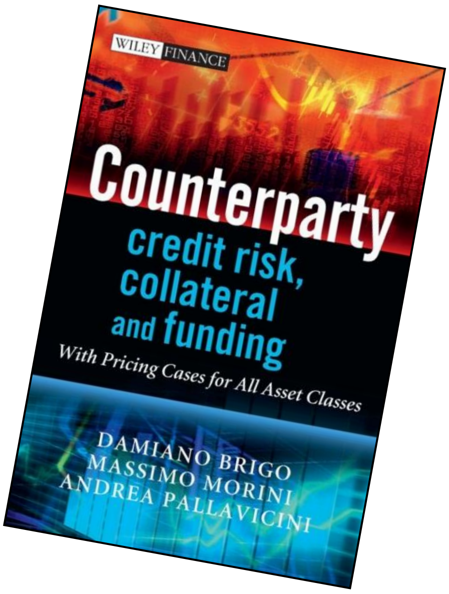
1.15 DVA mark to market and DVA hedging 48
1.16 Impact of Closeout in CVA and DVA 49
1.17 Closeout Contagion 51
1.18 Collateral Modeling in CVA and DVA 52
1.19 Re-hypothecation 53
1.20 Netting 54
1.21 Funding 54
1.22 Hedging Counterparty Risk: CCDS 57
1.23 Restructuring Counterparty Risk: CVA-CDOs and Margin Lending 59

2 **Context** 65

2.1 Definition of Default: Six basic cases 65
2.2 Definition of Exposures 67
2.3 Definition of Credit Valuation Adjustment (CVA) 70
2.4 Counterparty Risk Mitigants: netting 72
2.5 Counterparty Risk Mitigants: collateral 73
  2.5.1 The Credit Support Annex (CSA) 74
  2.5.2 The ISDA Proposal for a New Standard CSA 75
  2.5.3 Collateral Effectiveness as a Mitigant 76
2.6 Funding 77
  2.6.1 A First Attack on Funding Cost Modeling 78
  2.6.2 The General Funding Theory and its Recursive Nature 78
2.7 Value at Risk (VaR) and Expected Shortfall (ES) of CVA 79
2.8 The Dilemma of Regulators and Basel III 81

3 **Modeling the Counterparty Default** 83

3.1 Firm Value (or Structural) Models 83
  3.1.1 The Geometric Brownian Motion assumption 84
  3.1.2 Merton's Model 84
  3.1.3 Black and Cox's (1976) Model 87
  3.1.4 Credit Default Swaps and default probabilities 89
  3.1.5 Black and Cox (B\&C) model calibration to CDS: Problems 94
  3.1.6 The AT1P Model 95
  3.1.7 A Case Study with AT1P: Lehman Brothers default history 96
    Lehman Brothers CDS Calibration: July 10th, 2007 98
    Lehman Brothers CDS Calibration: June 12th, 2008 98
    Lehman Brothers CDS Calibration: September 12th, 2008 99
  3.1.8 Comments 99
  3.1.9 SBTV Model 100
  3.1.10 A Case Study with SBTV: Lehman Brothers default history 101
    Lehman Brothers CDS Calibration: July 10th, 2007 102
    Lehman Brothers CDS Calibration: June 12th, 2008 103
    Lehman Brothers CDS Calibration: September 12th, 2008 103
  3.1.11 Comments 104
3.2 Firm Value Models: Hints at the multiname picture 104
3.3 Reduced form (Intensity) models 105
  3.3.1 CDS calibration and Intensity Models 107
  3.3.2 A simpler formula for calibrating intensity to a single CDS 111
  3.3.3 Stochastic Intensity: The CIR family 113
  3.3.4 The Cox-Ingersoll-Ross model (CIR) short-rate model for r 113
  3.3.5 Time-inhomogeneous case: CIR++ Model 115
  3.3.6 Stochastic diffusion intensity is not enough: Adding jumps. The JCIR(++) Model 116
  3.3.7 The jump-diffusion CIR model (JCIR) 117
    Bond (or Survival Probability) Formula. 118
    Exact calibration of CDS: The JCIR++ model 118
    Simulating the JCIR++ model 119
  3.3.8 Market incompleteness and default unpredictability 120
  3.3.9 Further Models 120
3.4 Intensity models: The Multi-name picture 120
  3.4.1 Choice of variables for the dependence structure 120
  3.4.2 Firm value models? 122



3.4.3 Copula functions 122
3.4.4 Copula Calibration, CDOs and Criticism of Copula Functions 129

PART II Pricing Counterparty Risk: Unilateral CVA 131

4 **Unilateral CVA and Netting for Interest-Rates Products** 133

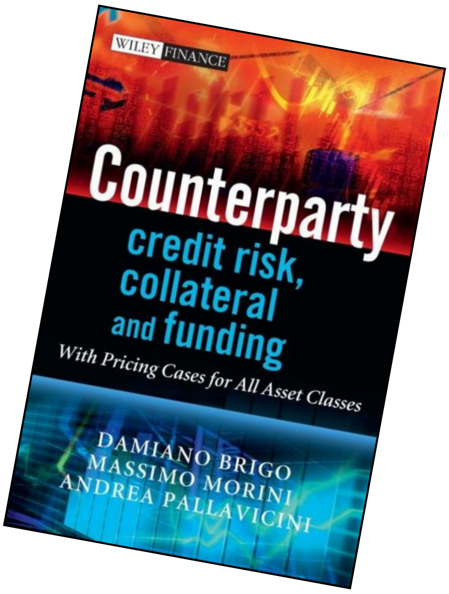
4.1 First Steps towards a CVA Pricing Formula 134
 4.1.1 Symmetry vs. Asymmetry 134
 4.1.2 Modeling the Counterparty Default Process 136
4.2 The Probabilistic Framework 137
4.3 The General Pricing Formula for Unilateral Counterparty Risk 139
4.4 Interest-Rate Swap (IRS) Portfolios 142
 4.4.1 Counterparty Risk in Single IRS 143
 4.4.2 Counterparty Risk in a Portfolio of IRS with Netting 146
 4.4.3 The Drift Freezing Approximation 149
 4.4.4 The Three Moments Matching Technique 151
4.5 Numerical Tests 153
 4.5.1 Case A: IRS with co-terminal payment dates 154
 4.5.2 Case B: IRS with co-starting resetting date 154
 4.5.3 Case C: IRS with first positive, then negative flows 155
 4.5.4 Case D: IRS with first negative, then positive flows 156
 4.5.5 Case E: IRS with first alternate flows 157
4.6 Conclusions 158

5 Wrong-Way Risk (WWR) for Interest-Rates 169

5.1 Modeling Assumptions 170
 5.1.1 G2++ Interest-Rate Model 171
 5.1.2 CIR++ Stochastic-Intensity Model 171
 5.1.3 CIR++ Model: CDS calibration 172
 5.1.4 Interest-Rate / Credit-Spread Correlation 175
 5.1.5 Adding Jumps to the Credit Spread 175
5.2 Numerical Methods 176
 5.2.1 Discretization Scheme 177
 5.2.2 Simulating Intensity Jumps 177
 5.2.3 "American Monte Carlo" (Pallavicini 2006 \cite BrigoPalla06) 177
 5.2.4 Callable Payoffs 178
5.3 Results and Discussion 178
 5.3.1 WWR in Single IRS 178
 5.3.2 WWR in a Portfolio of IRS with Netting 178
 5.3.3 WWR in European Swaptions 179
 5.3.4 WWR in Bermudan Swaptions 180
 5.3.5 WWR in CMS Spread Options 180
5.4 Contingent CDS (CCDS) 181
5.5 Results Interpretation and Conclusions 182

6 Unilateral CVA for Commodities with WWR 185

6.1 Oil Swaps and Counterparty Risk 186
6.2 Modelling Assumptions 188
 6.2.1 Commodity Model 188
 6.2.2 CIR++ Stochastic-Intensity Model 190
6.3 Forward vs. Futures Prices 190
 6.3.1 CVA for Commodity's Forwards without WWR 192
 6.3.2 CVA for Commodity's Forwards with WWR 192
6.4 Swaps and Counterparty Risk 193
6.5 UCVA for Commodity Swaps 194
 6.5.1 Counterparty Risk from the Payer Perspective: the Airline computes counterparty risk 196
 6.5.2 Counterparty Risk from the Receiver Perspective: the Bank computes counterparty risk 197
6.6 Inadequacy of Basel's WWR Multipliers 198
6.7 Conclusions 199



WILEY FINANCE

Counterparty credit risk, collateral and funding

With Pricing Cases for All Asset Classes

DAMIANO BRIGO
MASSIMO MORINI
ANDREA PALLAVICINI

7 Unilateral CVA for Credit with WWR 205

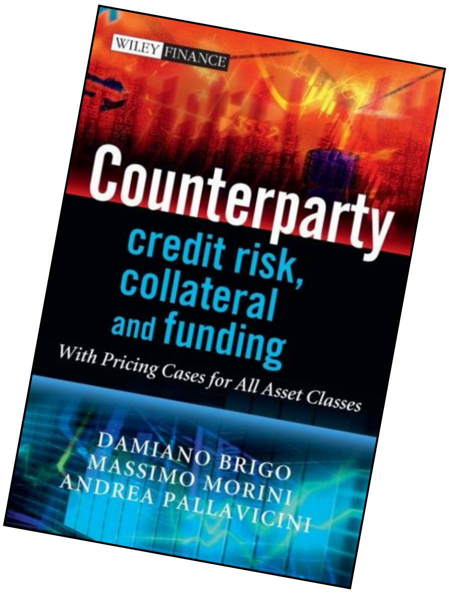
7.1 Introduction to CDS with Counterparty Risk 205
 7.1.1 Structure of the chapter 207
7.2 Modelling Assumptions 208
 7.2.1 CIR++ Stochastic-Intensity Model 209
 7.2.2 CIR++ Model: CDS calibration 210
7.3 CDS Options Embedded in CVA Pricing 212
7.4 UCVA for Credit Default Swaps: A case study 213
 7.4.1 Changing the Copula Parameters 214
 7.4.2 Changing the Market Parameters 217
7.5 Conclusions 219

8 **Unilateral CVA for Equity with WWR** 221

8.1 Counterparty Risk for Equity without a Full Hybrid Model 222
 8.1.1 Calibrating AT1P to the Counterparty's CDS Data 222
 8.1.2 Counterparty Risk in Equity Return Swaps (ERS) 223
8.2 Counterparty Risk with a Hybrid Credit-Equity Structural Model 227
 8.2.1 The Credit Model 228
 8.2.2 The Equity Model 230
 8.2.3 From Barrier Options to Equity Pricing 232
  Pricing Formulas for a Barrier Option 232
  Adapting the Barrier Option to the First Passage Model 233
 8.2.4 Equity and Equity Options 235
8.3 Model Calibration and Empirical Results 237
 8.3.1 BP and FIAT in 2009 238
  BP on April 6, 2009 238
  FIAT on April 6, 2009 240
  The Impact of Recovery Rates 242
 8.3.2 Uncertainty in Market Expectations 244
  BP on April 6, 2009 244
  FIAT on April 6, 2009 245
  Results Discussion 245
 8.3.3 Further Results: FIAT in 2008 and BP in 2010 247
  FIAT on March 11, 2008 -- before Lehman's default event 247
  BP on June 17, 2010 -- after Deepwater Horizon's accident 248
8.4 Counterparty Risk and Wrong Way Risk 250
 8.4.1 Deterministic Default Barrier 252
 8.4.2 Uncertainty on the Default Barrier 257
  The Model 258
  Counterparty Risk in Equity Options under Uncertainty 260

9 **Unilateral CVA for FX** 265

9.1 Pricing with Two Currencies: foundations 267
9.2 Unilateral CVA for a Fixed-Fixed CCS 271
 9.2.1 Approximating the Volatility of Cross-Currency Swap Rates 277
 9.2.2 Parameterizing the FX Correlation 279
9.3 Unilateral CVA for Cross-Currency Swaps with Floating Legs 286
9.4 Why a Cross-Currency Basis? 288
 9.4.1 The Approach of Fujii, Shimada and Takahashi (2010) 289
 9.4.2 Collateral Rates vs. Risk-Free Rates 291
 9.4.3 Consequences of Perfect Collateralization 292
9.5 CVA for CCS in Practice 294
 9.5.1 Changing the CCS Moneyness 298
 9.5.2 Changing the Volatility 301
 9.5.3 Changing the FX Correlations 302
9.6 Novations and the Cost of Liquidity 304
 9.6.1 A Synthetic Contingent CDS: the novation 304
 9.6.2 Extending the Approach to the Valuation of Liquidity 307
9.7 Conclusions 310



PART III Advanced Credit and Funding Risk Pricing 313

10 **New Generation Counterparty and Funding Risk Pricing** 315

10.1 Introducing the Advanced Part of the Book 315
10.2 What We Have Seen Before: unilateral CVA 317
  10.2.1 Approximation: default bucketing and independence 319
10.3 Unilateral Debit Valuation Adjustment (UDVA) 320
10.4 Bilateral Risk and DVA 321
10.5 Undesirable Features of DVA 323
  10.5.1 Profiting From Own Deteriorating Credit Quality 323
  10.5.2 DVA Hedging? 324
  10.5.3 DVA: accounting vs. capital requirements 324
    Yes DVA: FAS 157 324
    No DVA: Basel III 325
  10.5.4 DVA: Summary and debate on realism 325
10.6 Close-Out: risk-free or replacement? 326
10.7 Can We Neglect the First-to-Default Time? 328
  10.7.1 A Simplified Formula without First-to-Default:\\the case of an equity forward 329
10.8 Payoff Risk 329
10.9 Collateralization, Gap Risk and Re-Hypothecation 331
10.10 Funding Costs 334
10.11 Restructuring Counterparty Risk 335
  10.11.1 CVA Volatility: the wrong way 336
  10.11.2 Floating Margin Lending 336
  10.11.3 Global Valuation 338
10.12 Conclusions 339

11 **A First Attack on Funding Cost Modeling** 343

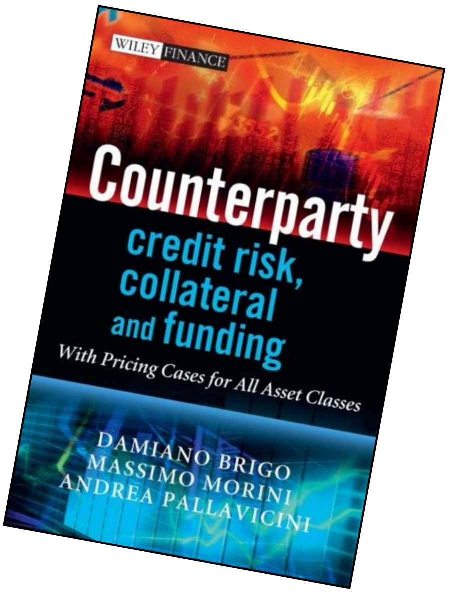
11.1 The Problem 344
11.2 A Closer Look at Funding and Discounting 345
11.3 The Approach Proposed by Morini and Prampolini (2010) 347
  11.3.1 The Borrower's Case 347
  11.3.2 The Lender's Case 349
  11.3.3 The Controversial Role of DVA: the borrower 350
  11.3.4 The Controversial Role of DVA: the lender 351
  11.3.5 Discussion 352
11.4 What Next on Funding? 353

12 **Bilateral CVA-DVA and Interest-Rate Products** 355

12.1 Arbitrage-Free Valuation of Bilateral Counterparty Risk 357
  12.1.1 Symmetry vs. Asymmetry 362
  12.1.2 Worsening of Credit Quality and Positive Mark-to-Market 363
12.2 Modelling Assumptions 363
  12.2.1 G2++ Interest-Rate Model 364
  12.2.2 CIR++ Stochastic-Intensity Model 365
  12.2.3 Realistic Market Data-Set for CDS Options 366
12.3 Numerical Methods 367
12.4 Results and Discussion 370
  12.4.1 Bilateral VA in Single IRS 371
  12.4.2 Bilateral VA in a Portfolio of IRS with Netting 372
  12.4.3 Bilateral VA in Exotic Interest-Rate Products 376
12.5 Conclusions 377

13 **Collateral, Netting, Close-Out and Re-Hypothecation** 385

13.1 Trading under ISDA Master Agreement 387
  13.1.1 Mathematical Setup and BCVA definition 387
  13.1.2 Collateral Delay and Dispute Resolutions 389
  13.1.3 Close-Out Netting Rules 389
  13.1.4 Collateral Re-Hypothecation 390



13.2 Bilateral CVA Formula under Collateralization 391
 13.2.1 Collecting CVA Contributions 391
 13.2.2 CBVA General Formula 394
 13.2.3 CCVA and CDVA Definitions 394
13.3 Close-Out Amount Evaluation 395
13.4 Special Cases of Collateral-inclusive Bilateral credit Valuation Adjustment 397
13.5 Example of Collateralization Schemes 398
 13.5.1 Perfect Collateralization 398
 13.5.2 Collateralization through Margining 398
13.6 Conclusions 399

14 **Close-Out and Contagion with Examples on a Simple Payoff** 401

14.1 Introduction to closeout modeling and earlier work 401
 14.1.1 Closeout modeling: context 402
 14.1.2 Legal documentation on closeout 403
 14.1.3 Literature 403
 14.1.4 Risk-Free vs. Replacement Close-Out: practical consequences 403
14.2 Classical Unilateral and Bilateral Valuation Adjustments 405
14.3 Bilateral Adjustment and Close-Out: risk-free or replacement? 406
14.4 A Quantitative Analysis and a Numerical Example 407
 14.4.1 Contagion issues 410
14.5 Conclusions 412

15 **Bilateral Collateralized CVA and DVA for Rates and Credit** 417

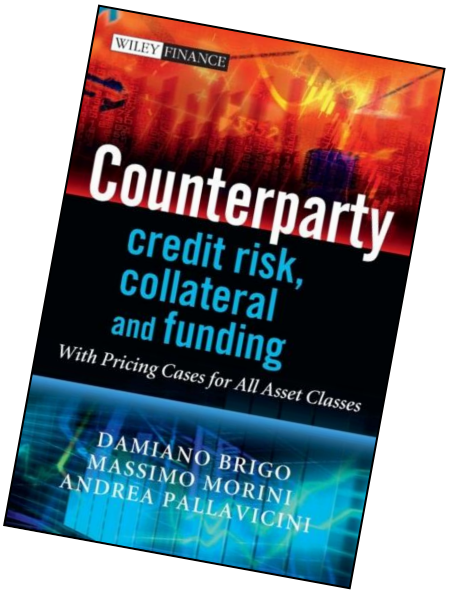
15.1 CBVA for Interest-Rate Swaps 418
 15.1.1 Changing the Margining Frequency 419
 15.1.2 Inspecting the Exposure Profiles 419
 15.1.3 A Case Where Re-Hypothecation is Worse than No Collateral at All. 422
 15.1.4 Changing the Correlation Parameters 423
 15.1.5 Changing the Credit-Spread Volatility 426
15.2 Modelling Credit Contagion 429
 15.2.1 CDS Price Process 429
 15.2.2 Calculation of Survival Probability 430
 15.2.3 Modelling Default-Time Dependence 433
15.3 CBVA for Credit Default Swaps 434
 15.3.1 Changing the Copula Parameters 435
 15.3.2 Inspecting the Contagion Risk 437
 15.3.3 Changing the CDS Moneyness 438
15.4 Conclusions 440

16 **Including Margining Costs in Collateralized Contracts** 443

16.1 Trading under ISDA Master Agreement 444
 16.1.1 Collateral Accrual Rates 445
 16.1.2 Collateral Management and Margining Costs 445
16.2 CBVA General Formula with Margining Costs 448
 16.2.1 Perfect Collateralization 449
 16.2.2 Futures Contracts 450
16.3 Changing the Collateralization Currency 450
 16.3.1 Margining Cost in Foreign Currency 451
 16.3.2 Settlement Liquidity Risk 452
 16.3.3 Gap Risk due to Foreign-Currency Collaterals 452
16.4 Conclusions 452

17 **Funding Valuation Adjustment (FVA)** 455

17.1 Dealing with Costs of Funding 456
 17.1.1 Single-Deal vs. Homogeneous Funding Models 456
 17.1.2 Previous Literature on Funding and Collateral 457
 17.1.3 Including FVA along with CVA and DVA 458
 17.1.4 FVA is not DVA 458
17.2 Bilateral Collateralized Credit and Funding VA Price 459



17.3 Funding Risk and Liquidity Policies 460
 17.3.1 Funding, Hedging and Collateralization 461
 17.3.2 Liquidity Policies 462
  Funding via Bank's Treasury 463
  Funding Directly on the Market 465
17.4 CBVA Pricing Equation with Funding Costs (CFBVA) 466
 17.4.1 Iterative Solution of the CFBVA Pricing Equation 467
 17.4.2 Funding Derivative Contracts in a Diffusion Setting 469
 17.4.3 Implementing Hedging Strategies via Derivative Market 471
17.5 Detailed Examples 472
 17.5.1 Funding with Collateral 473
 17.5.2 Collateralized Contracts Priced by a CCP 474
 17.5.3 Dealing with Own Credit Risk: FVA and DVA 476
 17.5.4 Deriving Earlier Results on FVA and DVA 477
17.6 Conclusions: FVA and beyond 478

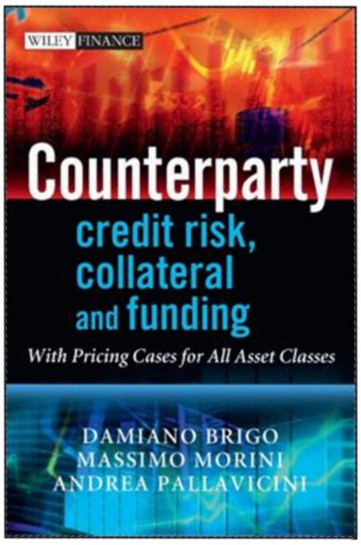
18 **Non-Standard Asset Classes: longevity risk** 479

18.1 Introduction to Longevity Markets 479
 18.1.1 The Longevity Swap Market 479
 18.1.2 Longevity Swaps: collateral and credit risk 480
 18.1.3 Indexed Longevity Swaps 484
 18.1.4 Endogenous Credit Collateral and Funding Inclusive Swap Rates 485
18.2 Longevity Swaps: the payoff 486
18.3 Mark to Market of Longevity Swaps 490
18.4 Counterparty and Own Default Risk, Collateral and Funding 492
18.5 An Example of Modeling Specification from Biffis et al. (2011) 498
18.6 Discussion of the Results in Biffis et al (2011) 502

19 **Conclusions and Further Work** 507

19.1 A Final Dialogue: models, regulations, CVA/DVA, funding and more 507

References


WILEY FINANCE

Counterparty
credit risk,
collateral
and funding

With Pricing Cases for All Asset Classes

DAMIANO BRIGO
MASSIMO MORINI
ANDREA PALLAVICINI

WILEY Global Finance
WHERE DATA FINDS DIRECTION