Credit Models Pre- and In-Crisis: Extreme Scenarios and Systemic Risk in Valuation

Inaugural Meeting of the Scottish Risk Academy

Damiano Brigo
Gilbart Professor of Mathematical Finance
Dept. of Mathematics
King's College, London[1]

www.damianobrigo.it

Edinburgh, Nov 4, 2010

[1] This presentation reflects the author opinion and not the views of the companies and institutions where the author is working or has worked in the past

Agenda

1. Mathematical Models and the Crisis
2. The case of CDOs
3. Real modeling problems with synthetic CDO
4. CDO models in the press: out of proportion
5. CDO models warning pre-crisis
6. More consistent and dynamic models pre-crisis: GPL
7. GPL model in-crisis
7. Exploding spreads: Credit Index Options
8. CVA and strange wrong way risk profiles with copulas
9. The big picture and Conclusions

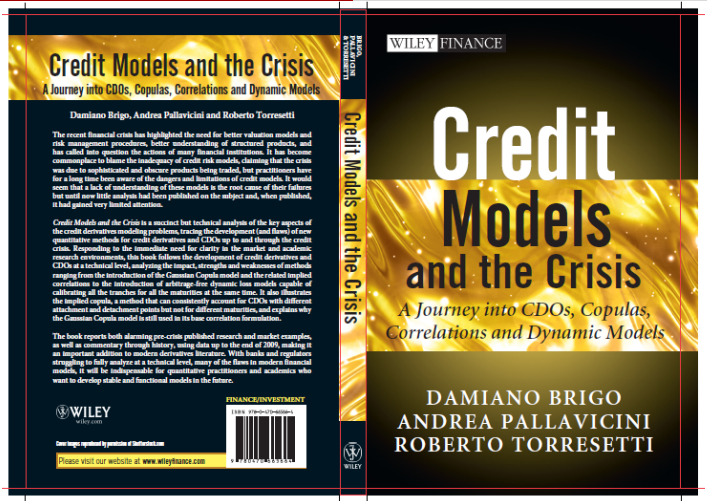
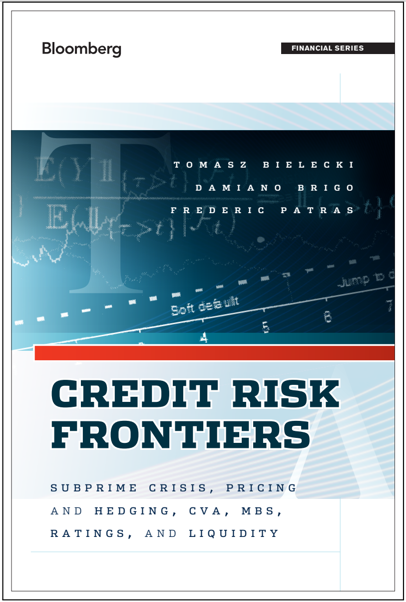


Figure: See also "Credit Models and the crisis or: How I learned to stop worrying and love the CDOs". Available at arXiv.org, ssrn.com, defaultrisk.com. Papers in *Mathematical Finance, Risk Magazine, IJTAF*

Forthcoming by Wiley / Bloomberg Press. 2011
Credit Risk Frontiers: subprime crisis, pricing and hedging , cva, mbs, ratings, and liquidity



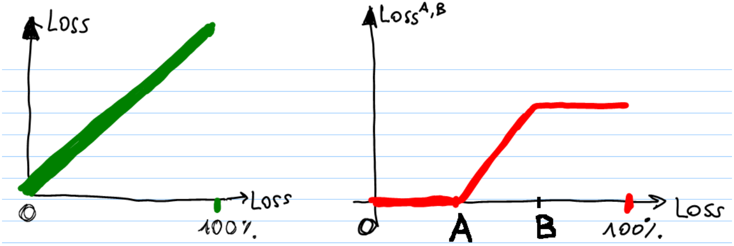
Mathematics and the Crisis

- In the past years criticism from governments, press etc against mathematics and quantitative models.
- The core criticism focuses on the use of Mathematics for Credit Risk and Valuation of CDOs.
- Quants have been blamed for blindly believing ungranted assumptions, not being aware of the models limitations and providing the market with a false sense of security.
- Mathematics is accused at the same time of being obscurely sophisticated and naively simplistic
- Quants were aware of limitations and have been proposing improved methodology before and across the crisis.
- **Inclusion of systemic risk and extreme events crucial**
- CDO, Credit Index Options and CVA Wrong Way Risk are good illustrations of this.

CDOs: The standard synthetic case I

- Portfolio of names, say 125. Names may default, generating losses.
- A tranche is a portion of the loss between two percentages. The $3\% - 6\%$ tranche focuses on the losses between 3% (attachment point) and 6% (detachment point).
- The CDO protection seller agrees to pay to the buyer all notional default losses (minus the recoveries) in the portfolio whenever they occur due to one or more defaults, within 3% and 6% of the total pool loss.
- In exchange for this, the buyer pays the seller a periodic fee on the notional given by the portion of the tranche that is still "alive" in each relevant period.
- Valuation problem: What is the fair price of this "insurance"?

CDOs: The standard synthetic case II



- Pricing (marking to market) a tranche: taking expectation of the future tranche losses under the pricing measure.
- From nonlinearity, the tranche expectation will depend on the loss distribution: marginal distributions of the single names defaults **and** dependency among different names' defaults. Dependency is commonly called "correlation".
- Abuse of language: correlation is a complete description of dependence for jointly Gaussians, but more generally it is not.

Copulas

The complete description is either the whole multivariate distribution or the so-called “copula function” (marginal distributions have been standardized to uniform distributions).

CDO Valuation: The culprit.

One-factor Gaussian copula

$$\boxed{\int_{-\infty}^{+\infty} \prod_{i=1}^{125} \Phi\left(\frac{\Phi^{-1}(1-\exp(-\Lambda_i(T))) - \sqrt{\rho_i}m}{\sqrt{1-\rho_i}}\right)\varphi(m)dm.}$$

“MEA COPULA!” From Nobel award to universal scapegoat

Introduced in Credit Risk modeling by David X. Li. Commentators went from suggesting a Nobel award to blaming Li for the whole Crisis.

The scapegoat

David Li, 2005, Wall Street Journal

[...] "The most dangerous part," Mr. Li himself says of the model, "is when people believe everything coming out of it." Investors who put too much trust in it or don't understand all its subtleties may think they've eliminated their risks when they haven't.

Indeed, these models are static. they ignore Credit Spread Volatilities, that in Credit can be 100%; this has further paradoxical consequences in copula models for wrong way risk, as we will see later on.

Tranches and Correlations

The dependence of the tranche on "correlation" is crucial. The market assumes a Gaussian Copula connecting the defaults of the 125 names, parametrized by a correlation matrix with 125*124/2 = 7750 entries. However, when looking at a tranche:

$$7750 \text{ parameters} \longrightarrow 1 \text{ parameter.}$$

The unique parameter is reverse-engineered to reproduce the price of the liquid tranche under examination. "Implied correlation". Once obtained it is used to value related products.

Problem with this implied "compound correlation"

If at a given time the $3\% - 6\%$ tranche for a five year maturity has a given implied correlation, the $6\% - 9\%$ tranche for the same maturity will have a different one. The two tranches on the *same pool* are priced (and hedged!!!) with two inconsistent loss distributions

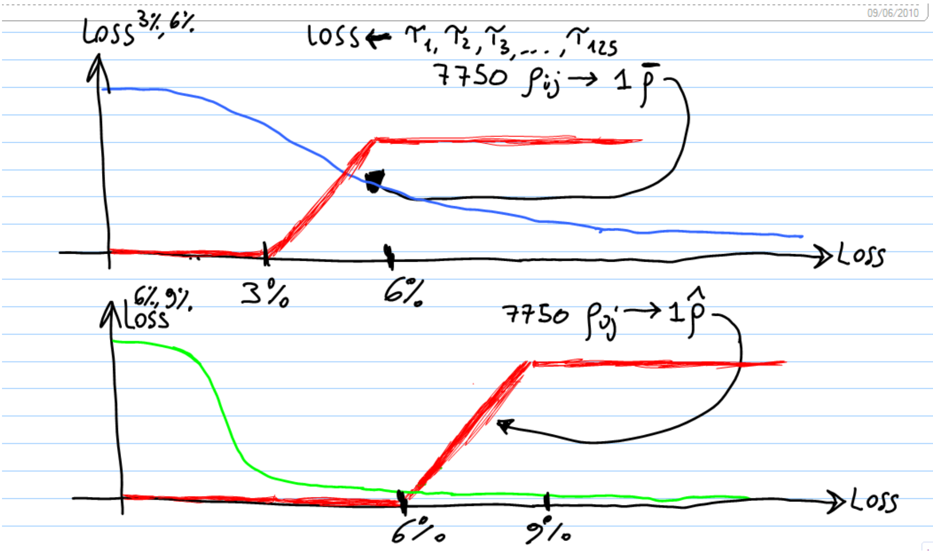


Figure: Compound correlation inconsistency

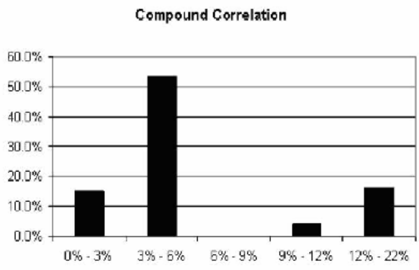


Figure: (After Edvard Munch's The Scream; Compound correlation DJ-iTraxx S5, 10y on 3 Aug 2005)

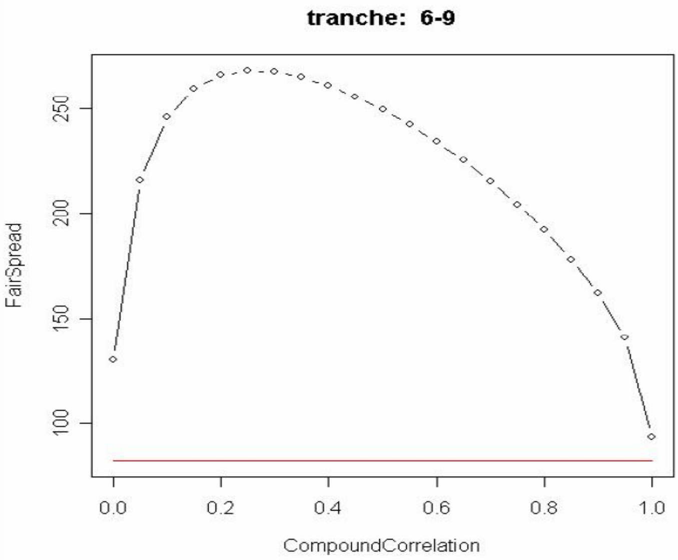


Figure: Non-invertibility compound correl DJ-iTraxx S5, 10y on 3 Aug 2005

Base correlation I

As a possible remedy for non-invertibility of compound correlation and other matters, the market introduced Base Correlation, which is still prevailing in the market.

Problems with base correlation

Base correlation is easier to interpolate but is inconsistent even at single tranche level, in that it prices the $3\% - 6\%$ tranche by decomposing it into the $0\% - 3\%$ tranche and $0\% - 6\%$ tranche and using two different correlations (and hence distributions) for those. This inconsistency shows up occasionally in negative losses (i.e. in defaulted names resurrecting).

[in the graph we use put-call parity to simplify]

Base correlation II

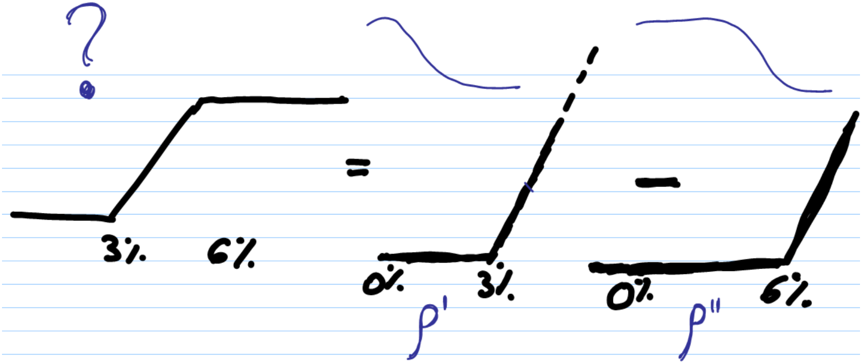
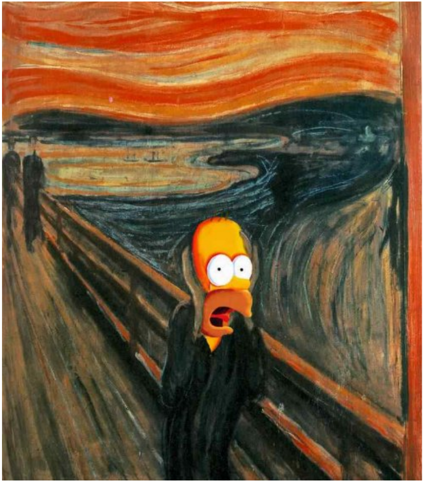


Figure: Base correlation inconsistency

Base correlation III



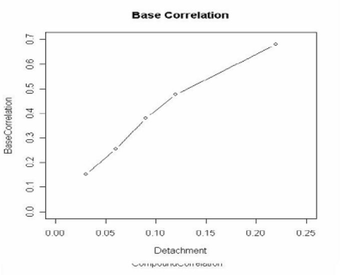


Figure: (Base correl DJ-iTraxx S5, 10y on 3 Aug 2005)

Base correlation

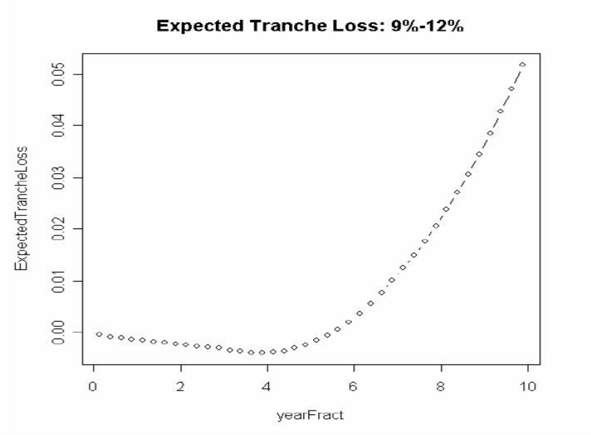


Figure: Expected tranche loss coming from Base correlation calibration, 3d August 2005, First published in 2006.

CDOs in the press: the blame-game

Popular accounts on CDO resort to quite colorful expressions:

- “the formula that killed Wall Street”[2]
- “Of couples and copulas: the formula that felled Wall St”[3]
- “Wall Street’s Math Wizards Forgot a Few Variables”[4]
- “Misplaced reliance on sophisticated maths”[5]
- etc etc

[2] Recipe for disaster: the Formula that killed Wall Street. Wired Magazine, 17.03.

[3] The Financial Times, Jones, S. (2009). April 24 2009.

[4] Lohr (2009), New York Times of September 12.

[5] Turner, J.A. (2009). Section title in The Turner Review. March 2009. Financial Services Authority, UK.
www.fsa.gov.uk/pubs/other/turner_review.pdf.

Context or Contest?

The culprit model

$$\boxed{\int_{-\infty}^{+\infty} \prod_{i=1}^{125} \Phi\left(\frac{\Phi^{-1}(1-\exp(-\Lambda_i(T))) - \sqrt{\rho_i}m}{\sqrt{1-\rho_i}}\right)\varphi(m)dm.}$$

VS

The Crisis

US Home Polices, New Bank - Originate to Distribute system fragility, Volatile Monetary Policies, Myopic Compensation System, Regulatory oversight, Liquidity risk underestimation, NINJAs, Lack of Data, Madoff... (Szegö, 2009-2010).

Mathematical models Reloaded I

The impact of model limitations has been blown out of proportions

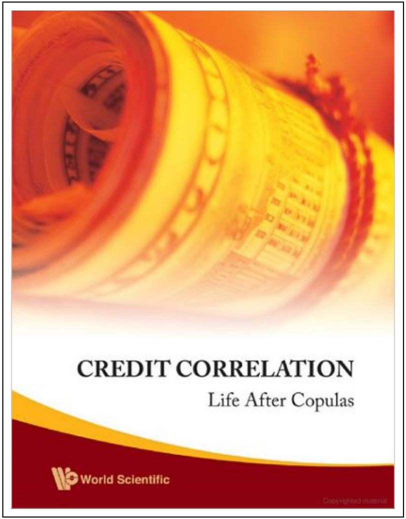
This overall hostility and blaming attitude towards mathematics and mathematicians, whether in the industry or in academia, is the reason why we feel it is important to point out the following.

Were Quantitative Analysts and Academics really unaware?

The notion that even more mathematically oriented quants have not been aware of the Gaussian Copula/implied correlation model limitations is simply false, as we are going to show. Furthermore, more advanced models had been put forward well before the crisis.

Mathematical models Reloaded II

Proceedings of a Practitioners Conference held in London, 2006, organized by Lipton and Rennie, Merrill Lynch.
I was there (as a speaker)



Not so long ago...

And what about the earlier 2005 mini credit-correlation crisis when implied correlation went crazy?

September 12, 2005. Wall Street Journal

How a Formula [Base correlation + Gaussian Copula] Ignited Market That Burned Some Big Investors.

There are several publications that appeared pre-crisis (also stimulated by the 2005 mini-crisis) and that questioned the Gaussian Copula and implied correlation. For example

Implied Correlation: A paradigm to be handled with care, 2006, SSRN

`http://ssrn.com/abstract=946755`

Beyond copulas: GPL and GPCL Models (2006-on)

We model the total number of defaults in the pool by t as

$$Z_t := \sum_{j=1}^{n} \delta_j Z_j(t)$$

(for integers δ_j) where Z_j are independent Poissons. This is consistent with the Common Poisson Shock framework, where defaults are linked by a Marshall Olkin copula (Lindskog and McNeil).

$$\text{Example}: \quad n = 125, \;\; Z_t = 1\, Z_1(t) + 2\, Z_2(t) + \ldots + 125\, Z_{125}(t).$$

If Z_1 jumps there is just one default (idiosyncratic), if Z_{125} jumps there are 125 ones and the whole pool defaults one shot (total systemic risk), otherwise for other Z_i's we have intermediate situations (sectors).

The GPL and GPCL Models: Default clusters?

- Thrifts in the early 90s at the height of the loan and deposit crisis.
- Airliners after 2001.
- Autos and financials more recently. From the September, 7 2008 to the October, 8 2008, we witnessed seven credit events: Fannie Mae, Freddie Mac, Lehman Brothers, Washington Mutual, Landsbanki, Glitnir, Kaupthing.

S&P ratings and default clusters

Moreover, S&P issued a request for comments related to changes in the rating criteria of corporate CDO. Tranches rated 'AAA' should be able to withstand the default of the largest single industry in the pool with zero recoveries. Stressed but plausible scenario that a cluster of defaults in the objective measure exists.

The GPL and GPCL Models

Problem: infinite defaults. Solution 1: **GPL:** Modify the aggregated pool default counting process so that this does not exceed the number of names, by simply capping Z_t to n, regardless of cluster structures:

$$C_t := \min(Z_t, n)$$

Solution 2: **GPCL.** Force clusters to jump only once and deduce single names defaults consistently.
The first choice is ok at top level but it does not really go down towards single names. The second choice is a real top down model, but combinatorially more complex.

Calibration

The GPL model is calibrated to the market quotes observed on March 1 and 6, 2006. Deterministic discount rates are listed in Brigo, Pallavicini and Torresetti (2006). Tranche data and DJi-TRAXX fixings, along with bid-ask spreads, are (I=index,T=Tranche,TI=Tranchelet)

	Att-Det	**March, 1 2006**		**March, 6 2006**		
		5y	7y	3y	5y	7y
I		35(1)	48(1)	20(1)	35(1)	48(1)
T	0-3	2600(50)	4788(50)	500(20)	2655(25)	4825(25)
	3-6	71.00(2.00)	210.00(5.00)	7.50(2.50)	67.50(1.00)	225.50(2.50)
	6-9	22.00(2.00)	49.00(2.00)	1.25(0.75)	22.00(1.00)	51.00(1.00)
	9-12	10.00(2.00)	29.00(2.00)	0.50(0.25)	10.50(1.00)	28.50(1.00)
	12-22	4.25(1.00)	11.00(1.00)	0.15(0.05)	4.50(0.50)	10.25(0.50)
TI	0-1	6100(200)	7400(300)			
	1-2	1085(70)	5025(300)			
	2-3	393(45)	850(60)			

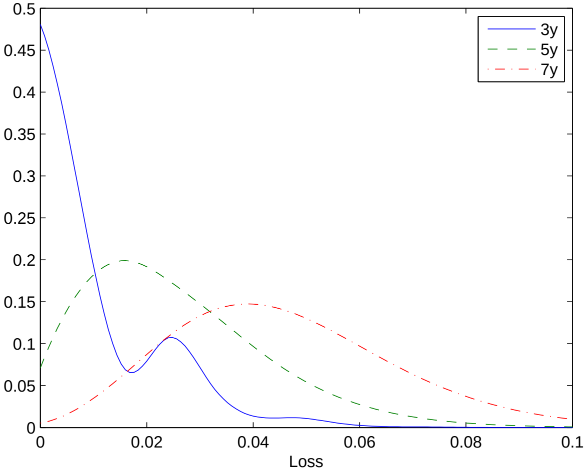
Calibration: All standard tranches up to seven years

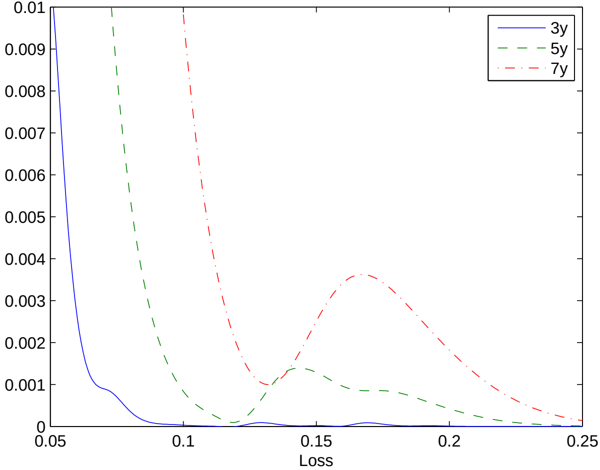
As a first calibration example we consider standard DJi-TRAXX tranches up to a maturity of 7y with constant recovery rate of 40%. The calibration procedure selects five Poisson processes. The 18 market quotes used by the calibration procedure are almost perfectly recovered. In particular all instruments are calibrated within the bid-ask spread (we show the ratio calibration error / bid ask spread).

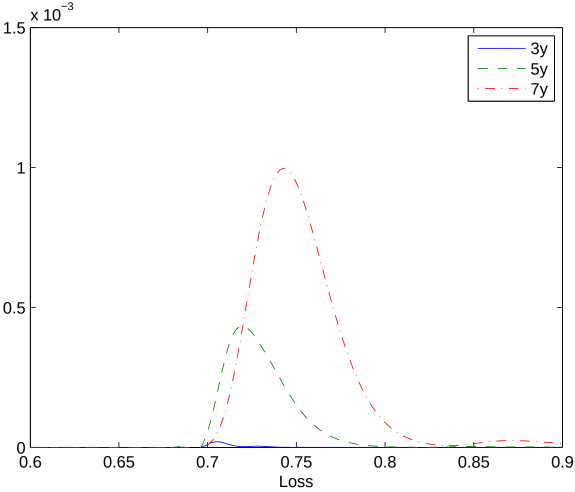
	Att-Det	**Maturities**		
		3y	5y	7y
Index		-0.4	-0.2	-0.9
Tranche	0-3	0.1	0.0	-0.7
	3-6	0.0	0.0	0.7
	6-9	0.0	0.0	-0.2
	9-12	0.0	0.0	0.0
	12-22	0.0	0.0	0.2

δ	$\Lambda(T)$		
	3y	5y	7y
1	0.535	2.366	4.930
3	0.197	0.266	0.267
16	0.000	0.007	0.024
21	0.000	0.003	0.003
88	0.000	0.002	0.007

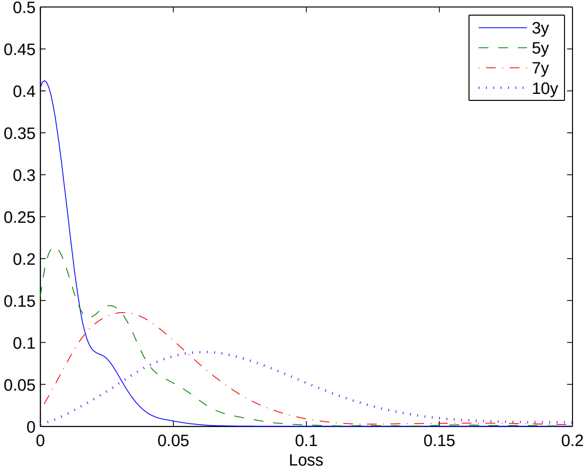
Loss distribution of the calibrated GPL model at different times



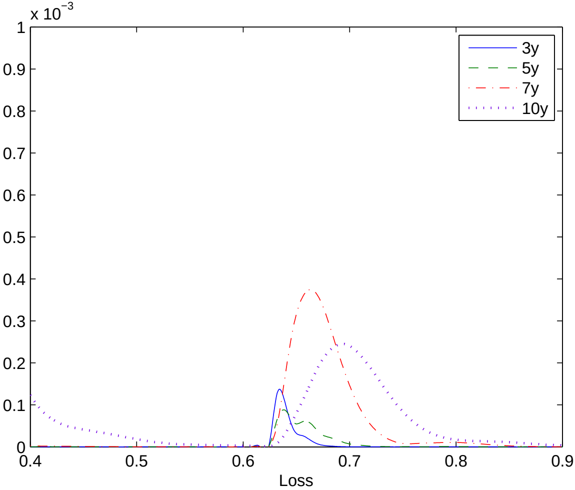

0.01
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
0.05
0.1
0.15
0.2
0.25
Loss
3y
5y
7y


x 10^-3
1.5
1
0.5
0
0.6
0.65
0.7
0.75
0.8
0.85
0.9
Loss
3y
5y
7y

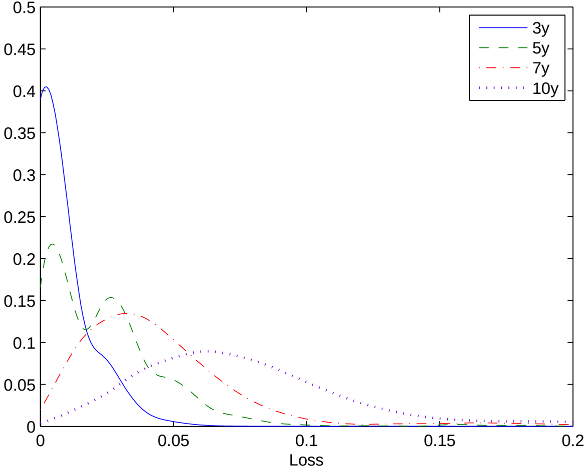
October 2 2006, GPL, Calibration up to 10y



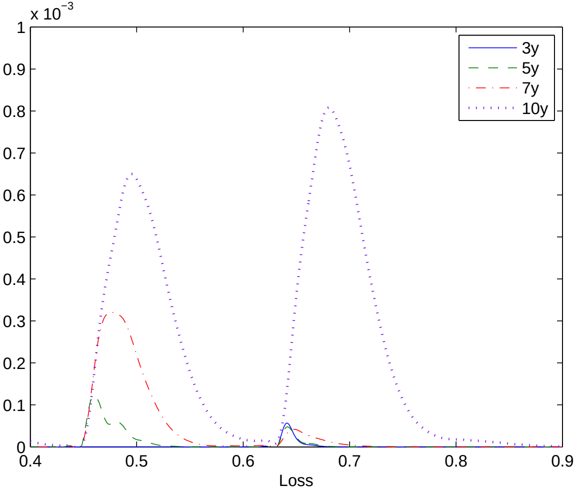
October 2 2006, GPL tail



October 2 2006, GPCL, Calibration up to 10y



October 2 2006, GPCL tail



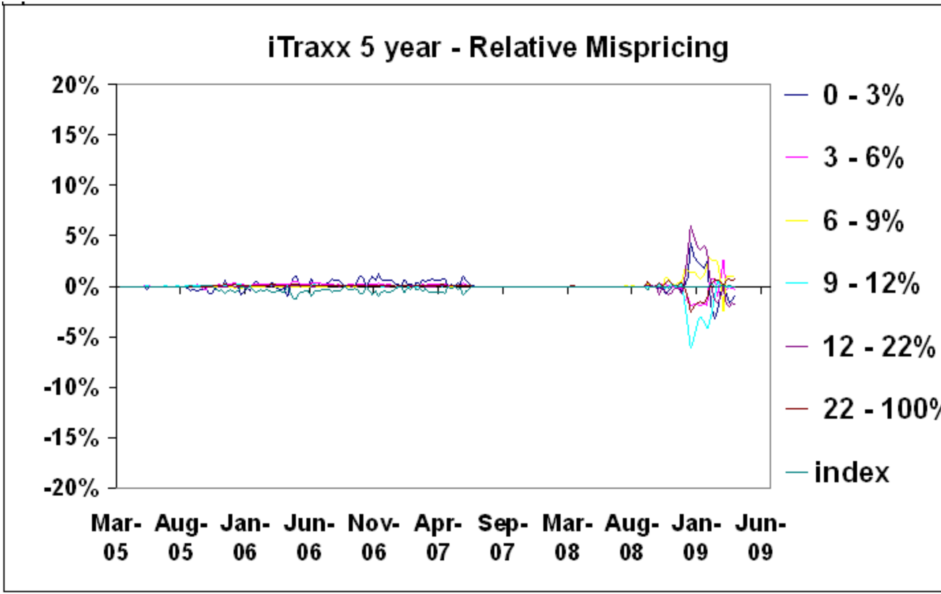
Calibration comments I

Sector / systemic calibration:

Notice the large portion of mass concentrated near the origin, the subsequent modes (default clusters) when moving along the loss distribution for increasing values, and the bumps in the far tail. Modes in the tail represent risk of default for large sectors. This is systemic risk as perceived by the dynamical model from the CDO quotes. With the crisis these probabilities have become larger, but they were already observable pre-crisis. Difficult to get this with parametric copula models.

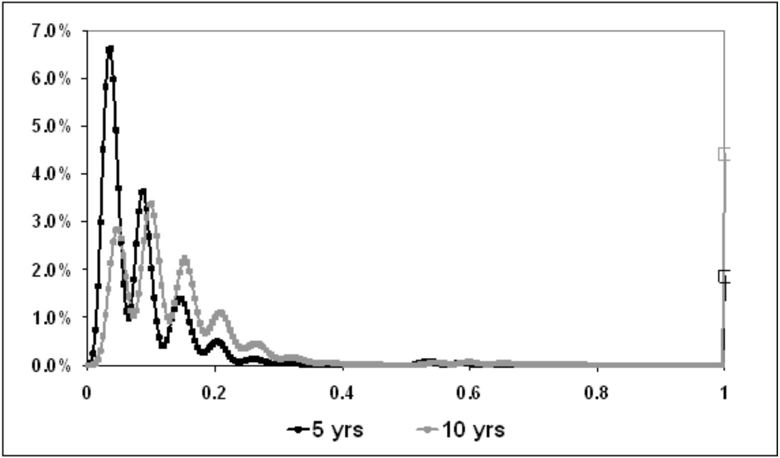
History of calibration in-crisis with a different parametrization (α's fixed a priori):

Calibration comments II

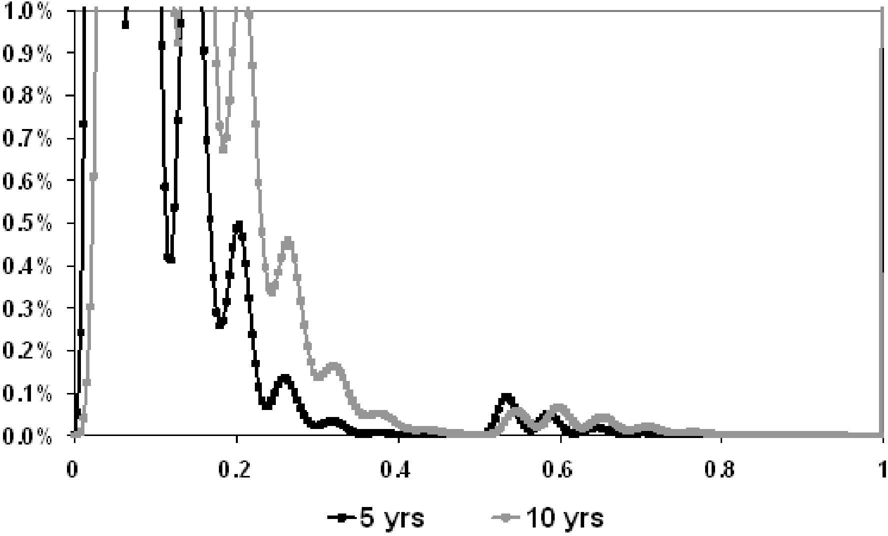


Calibration in-crisis

Default Rate Distribution (CDX_3-oct-08)



Calibration in-crisis



GPL in-Crisis: Fix the α's

Fix the independent Poisson jump amplitudes to the levels just above each tranche detachment, when considering a 40% recovery.
For the DJi-Traxx, for example, this would be realized through jump amplitudes $a_i = \alpha_i/125$ where

$$\alpha_{5,6,7} = \text{roundup}\left(\frac{125 \cdot \{0.03, 0.06, 0.09\}}{(1 - \mathrm{R_{EC}})}\right),$$

$$\alpha_8 = \text{roundup}\left(\frac{125 \cdot 0.12}{(1 - \mathrm{R_{EC}})}\right), \ \alpha_9 = \text{roundup}\left(\frac{125 \cdot 0.22}{(1 - \mathrm{R_{EC}})}\right),$$

$$\alpha_{10} = 125$$

and, in order to have more granularity, we add the sizes 1,2,3,4:

$$\alpha_1 = 1, \ \alpha_2 = 2, \ \alpha_3 = 3, \ \alpha_4 = 4.$$

GPL in-Crisis

In total we have $n = 10$ jump amplitudes. We then modify slightly the obtained sizes in order to account also for CDX attachments that are slightly different.

$$\alpha_i \equiv 125 \cdot a_i \in \{1, 2, 3, 4, 7, 13, 19, 25, 46, 125\}$$

Given these amplitudes, we obtain the default counting process fraction as

$$\bar{C}_t = 1_{\{N_n(t)=0\}}\bar{c}_t + 1_{\{N_n(t)>0\}} \ , \quad \bar{c}_t := \min\left(\sum_{i=1}^{n-1} a_i N_i(t), 1\right) \ .$$

GPL in-Crisis

Now let the random time $\hat{\tau}$ be defined as the first time where $\sum_{i=1}^{n} a_i N_i(t)$ reaches or exceeds the relative pool size of 1.

$$\hat{\tau} = \inf\{t : \sum_{i=1}^{n} a_i N_i(t) \geq 1\} .$$

We define the loss fraction as

$$\begin{aligned}\bar{L}_t &:= 1_{\{\hat{\tau}>t\}}(1 - \mathrm{R{\scriptsize EC}})1_{\{N_n(t)=0\}}\bar{c}_t + 1_{\{\hat{\tau}\leq t\}}\left[(1 - \mathrm{R{\scriptsize EC}})1_{\{N_n(\hat{\tau})=0\}}\right. \\ &\qquad \left. + 1_{\{N_n(\hat{\tau})>0\}}(1 - \mathrm{R{\scriptsize EC}}\,\bar{c}_{\hat{\tau}})\right] \\ &= 1_{\{\hat{\tau}>t\}}(1 - \mathrm{R{\scriptsize EC}})1_{\{N_n(t)=0\}}\bar{c}_t + 1_{\{\hat{\tau}\leq t\}}(1 - \mathrm{R{\scriptsize EC}}\,\bar{c}_{\hat{\tau}}) .\end{aligned}$$

GPL in-Crisis

- Whenever the armageddon component N_n jumps the first time, the default counting process $\bar{C}_t$ jumps to the entire pool size and no more defaults are possible.
- Whenever armageddon component N_n jumps the first time we will assume that the recovery rate associated to the remaining names defaulting in that instant will be zero.
- The pool loss however will not always jump to 1 as there is the possibility that one or more names already defaulted before N_n jumped, with recovery R_{EC}.

GPL in-Crisis

This way whenever N_n jumps at a time when the pool has not been wiped out yet, we can rest assured that the pool loss will be above $1 - \text{R}\textsc{ec}$.

We do this because the market in 2008 has been quoting CDOs with prices assuming that the super-senior tranche would be impacted to a level impossible to reach with fixed recoveries at 40%.

Supersenior tranche and Systemic risk

For example there was a market for the DJi-Traxx 5 year $60 - 100\%$ tranche on 25-March-2008 quoting a running spread of 24bps bid.

GPL in-Crisis

We know how to calculate the distribution of both $\bar{C}_t$ and $\bar{L}_t$ given that:

- the distribution of $\bar{c}_t = \min\left(\sum_{i=1}^{n-1} a_i N_i(t), 1\right)$ is obtained running a 'reduced' GPL, i.e. a GPL where the jump N_n is excluded.
- N_n is independent from all other processes N_i so that we can factor expectations when calculating the risk neutral discounted payoffs for tranches and indices.

GPL in-Crisis

Concerning recovery issues, in the dynamic loss model recovery can be made a function of the default rate $\bar{C}$ or other solutions are possible, see Brigo Pallavicini and Torresetti (2007) for more discussion.

Here we use the above simple methodology to allow losses of the pool to penetrate beyond $(1 - \mathrm{R_{EC}})$ and thus affect severely even the most senior tranches, in line with market quotations.

Credit Index Options and Armageddon Events

The CDO study above showed the importance of including systemic and sector risk (default clustering) into calibration.
Extreme scenarios important for accurate valuation.

Another credit product where extreme scenarios play a key role, not fully recognized by the current market methodology, are Credit Index Options

Credit Index Options and Armageddon Events I

Credit Index swaps and related options

These positions allow to buy or sell protection on the whole Loss (rather than a tranche) of the pool. This is offered again in exchange for a periodic spread. There are options that allow (but no obligation) to enter into such a swap at a later date at a fixed premium

Since there is no tranching, options prices would depend in principle only on tranche spread levels and volatilities, and would not be explicitly correlation dependent.

Credit Index Options and Armageddon Events II

Problem in the current market formula

B. and Morini (2007, Risk Magazine, and 2010, Mathematical Finance) show that the pricing formula used in the market is ill posed because the numeraire (related to the index DV01) may vanish with positive probability, leading the tranche spread at option exercise to explode toward infinity.

Credit Index Options and Armageddon Events

The fair index spread balancing the premium leg ("insurance premium payment" from the protection buyer) and the loss leg (loss payments at defaults from the protection seller) for protection from initial time T_a to T_b defined at a future time $t > 0$, $t < T_a$ can be written as

$$S_t^{a,b} = \frac{\mathbb{E}_t[\int_{T_a}^{T_b} D(t,u)dL_u + D(t,T_a)L_{T_a}]}{\mathbb{E}_t[\sum_{j=a+1}^{b} D(t,T_j)\alpha_j \sum_{k=1}^{125} 1_{\{\tau^{k-th} > T_j\}}]}$$

The denominator goes to zero in the scenario where the whole pool defaults before t. This scenario has a positive (if small) probability. For example there was a market for the DJi-Traxx 5 year $60 - 100\%$ tranche on 25-March-2008 quoting a running spread of 24bps bid.

Credit Index Options and Armageddon Events

The way to remove this singularity from the spread definition and from the pricing measure associated with the denominator numeraire is to monitor, at any time t, all the defaults but the last one.

$$\tau^{1-th} > t,\ \tau^{2-th} > t,\ \tau^{124-th} > t \text{ but not } \tau^{125-th} > t$$

This is important because, in taking E_t with the full t filtration, we have

$$\mathbb{E}_t[1_{\{\tau^{125} > T_j\}}] = \mathbb{E}_t[1_{\{\tau^{125} > t\}} 1_{\{\tau^{125} > T_j\}}] = 1_{\{\tau^{125} > t\}} \mathbb{E}_t[1_{\{\tau^{125} > T_j\}}]$$

where we could do the last passage because the t filtration sees the last default. If the whole pool defaults this goes to zero and we have a singularity. However, under the restricted t-filtration, we cannot take out the indicator and the above remains a positive (under fairly general assumptions) probability and not a zero-one indicator.

Credit Index Options and Armageddon Events

- Use a filtration switching formula (Jeanblanc and Rutkowski developed many such formulas) to connect expectations wrt to the full filtration to expectations wrt to the partial one, and one gets a pricing formula without singularities and arbitrage free.
- Removal of the Armageddon observation and inclusion of front-end protection (losses before T_a) make the index option explicitly correlation dependent.
- The option will require a model for default correlation. Toy (inconsistent) example: Black formula for the spread part plus Gaussian copula for the correlation part. Examples based on market data.

Options on i-Traxx Europe Main - 2007 vs 2008

In the next table we report the market inputs. The bid-offer spread for options in March 08 was in the range 5-8 bps.

	March-09-07	March-11-08
Spot Spread 5y: $S_0^{9m,5y}$	22.50 bp	154.50 bp
Forward Spread Adjusted 9m-5y: $\tilde{S}_0^{9m,5y}$	23.67 bp	163.60 bp
Implied Volatility, $K = \tilde{S}_0^{9m,5y} \times 0.9$	52%	108%
Implied Volatility, $K = \tilde{S}_0^{9m,5y} \times 1.1$	54%	113%
Correlation 22% I-Traxx Main: $\rho_{0.22}^{I}$	0.545	0.912
Correlation 30% CDX IG: $\rho_{0.3}^{C}$	0.701	0.999
"DV01" Annuity 9m-5y:	3.993	3.912

Market Inputs: : March-09-07 (left), March-11-08 (right)

Options on i-Traxx Europe Main - March 2007

Strike (Call)	26	21
Market Formula	**23.289**	**11.619**
No-Arb. Form. $\rho = 0.545$	23.289	11.619
No-Arb. Form. $\rho = 0.597$	23.289	11.619
No-Arb. Form. $\rho = 0.649$	23.289	11.618
No-Arb. Form. $\rho = 0.701$	23.286	11.614
Strike (Put)	26	21
Market Formula	**13.840**	**21.069**
No-Arb. Form. $\rho = 0.545$	13.840	21.069
No-Arb. Form. $\rho = 0.597$	13.840	21.069
No-Arb. Form. $\rho = 0.649$	13.840	21.069
No-Arb. Form. $\rho = 0.701$	13.843	21.071

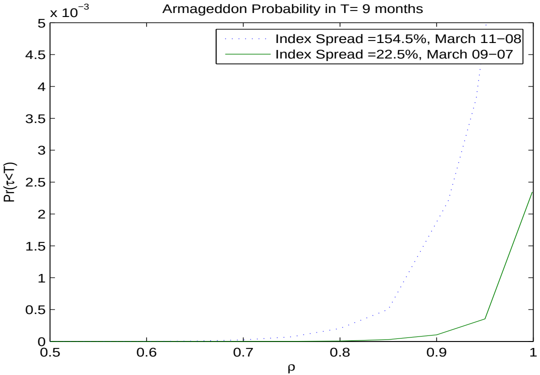
March-09-07 Options on i-Traxx 5y, Maturity 9m

Options on i-Traxx Europe Main - March 2008

Strike (Call)	180	147
Market Formula	**286.241**	**189.076**
No-Arb. Form. $\rho = 0.912$	277.668	179.624
Difference	**8.573**	**9.453**
No-Arb. Form. $\rho = 0.941$	271.460	172.769
Difference	**14.781**	**16.307**
No-Arb. Form. $\rho = 0.970$	258.887	158.862
Difference	**27.354**	**30.215**
No-Arb. Form. $\rho = 0.999$	212.867	107.630
Difference	**73.374**	**81.447**

March-11-08 Options on i-Traxx 5y, Maturity 9m

Armageddon probability



CVA: strange wrong way risk with Copulas

- Copula models used in the intensity framework (like in CDO valuation) are quite misleading for CVA calculations.
- With deterministic credit spreads, boosting the copula correlation to 1 for a total wrong way risk scenario produces counterintuitive results.
- Toy model for CVA on a CDS without collateral. Two names. 1 is the underlying CDS credit, 2 the counterparty.
  Assume $\lambda_1 > \lambda_2$, as is natural.
- Suppose we have the co-monotonic copula connecting the exponential levels ξ in the default times. This means that in all scenarios

$$\tau_1 = \xi/\lambda_1 \; < \; \xi/\lambda_2 = \tau_2$$

CVA: strange wrong way risk with Copulas

$$\tau_1 = \xi/\lambda_1 \; < \; \xi/\lambda_2 = \tau_2$$

Hence whenever τ_2 will default, meaning there is a counterparty default event, the CDS will have defaulted earlier, so that no counterparty risk due to insolvency of the counterparty is present.

However, if the correlation is lower than 1 the two default times could mix and we could get back a strictly positive CVA.

So in a way correlation 1 would be less risky, for wrong way risk pricing, than correlation 0.5.

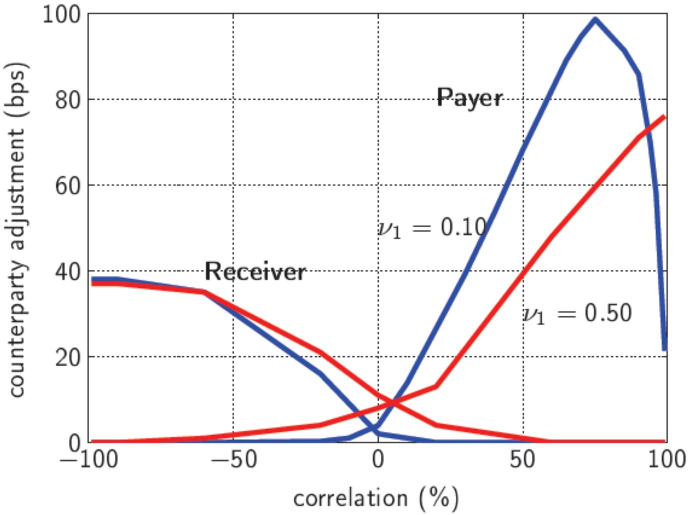
CVA: Poor representation of wrong way risk with Copulas

We can get back an increasing pattern for wrong way risk in the correlation parameter if one puts back relevant credit spread volatility, that in the CDS market reaches easily 50% and beyond (see Brigo 2006 for CDS implied vols)

CIR++ models with single name credit levels and volatility modeling.

Credit spread volatility modeled explicitly.

This is important: Credit spread vol (both implied and historical) is very large, easily in the range 50%-100%


100
80
60
40
20
0
counterparty adjustment (bps)
Payer
$\nu_1 = 0.10$
Receiver
$\nu_1 = 0.50$
−100
−50
0
50
100
correlation (%)

Gaussian Copula/Base Correl. still used for CDOs.

Summing up: Copula-based implied correlations lead to inconsistency, non-invertibility and negative losses for CDOs.

Copula based models lead to misleading wrong way risk profiles in CVA calculations.

Gaussian Copula/Base Correl. still used for CDOs.

- Difficulty of all the loss models, improving the consistency and dynamics issues, in handling single name data and single name sensitivities.
- Alternative models have not been developed and tested enough to become operational on a trading floor or in a large risk management platform.
- Changing the model implies a long path involving a number of issues that have little to do with modeling and more to do with IT problems, integration with other systems, and the likes. Inertia.
- Self-fulfilling prophecy if everyone uses or believes in a “wrong” model
- However, the fact that the modeling effort is unfinished does not mean that the quant community has been unaware of model limitations, as we abundantly document

"How I learned to stop worrying and love the CDOs" I

The big picture? As we have seen, the market has been using simplistic approaches for credit derivatives, but it has also been trying to move beyond those.

Synthetic and Cash CDO

Synthetic Corporate CDOs are the ones we described above. More simple and standardized payouts than other CDOs but typically valued with more sophisticated models, given standardization and availability of market quotes.

"How I learned to stop worrying and love the CDOs" II

CDOs on different asset classes

However, CDOs, especially Cash, are available on other asset classes, such as loans (CLO), residential mortgage portfolios (RMBS), commercial mortgages portfolios (CMBS), and on and on. For many of these CDOs, and especially RMBS, quite related to the asset class that triggered the crisis, the problem is in the data rather than in the models. Even bespoke corporate pools have no data from which to infer default "correlation" and dubious mapping methods are used.

CDOs on different asset classes and Base Correlation

Notice that synthetic CDOs on corporates, where the Implied correlation/copula model has been used massively, are not the ones that lead to the major losses, despite the above sensationalistic articles about killing wall street

"How I learned to stop worrying and love the CDOs" III

Risk of fraud

At times data for valuation in mortgages CDOs (RMBS and CDO of RMBS) can be distorted by fraud (see for example the FBI Mortgage fraud report, 2007, www.fbi.gov/publications/fraud/mortgage_fraud07.htm.

Pricing a CDO on this underlying:

"How I learned to stop worrying and love the CDOs" IV

Figure: The above photos are from condos that were involved in a mortgage fraud. The appraisal described recently renovated condominiums to include Brazilian hardwood, granite countertops, and a value of 275,000 USD

“How I learned to stop worrying and love the CDOs” V

At times it is not even clear what is in the portfolio: From the offering circular of a huge RMBS (more than 300.000 mortgages)

Type of property	% of Total
Detached Bungalow	2.65%
Detached House	16.16%
Flat	13.25%
Maisonette	1.53%
Not Known	$\boxed{2.49}$%
New Property	0.02%
Other	0.21%
Semi Detached Bungalow	1.45%
Semi Detached House	27.46%
Terraced House	34.78%
Total	100.00%

Mathematics or Magic?

All this is before modeling. Models obey a simple rule that is popularly summarized by the acronym GIGO (Garbage In $\rightarrow$ Garbage Out). As Charles Babbage (1791-1871) famously put it:

> *On two occasions I have been asked, "Pray, Mr. Babbage, if you put into the machine wrong figures, will the right answers come out?" I am not able rightly to apprehend the kind of confusion of ideas that could provoke such a question.*

So, in the end, is the crisis due to models inadequacy? Is the crisis due to quantitative analysts and academics pride and unawareness of models limitations?

Conclusions

- Lax lending practices and encouraging home equity extraction
- Lack of data or fraud-corrupted data
- the fragility in the “originate to distribute” system
- poor liquidity and reserves policies
- regulators lack of uniformity
- excessive leverage and concentration in real estate investment,
- accounting rules and excessive reliance on credit rating agencies

Conclusions

The above are factors not to be underestimated. This crisis is a quite complex event that defies witch-hunts, folklore and superstition. Methodology certainly needs to be improved. We presented suggested improvements that had appeared both pre- and in- crisis for

- CDOs
- Credit Index Options
- CVA

Several Quants had been aware of the limitations of the models and had given warnings in talks and publications. Blaming just the models and the quants for the crisis appears, in our opinion, to be the result of a very limited point of view.

References I

- Brigo, D., and Chourdakis, K. (2010). Counterparty Risk for Credit Default Swaps: Impact of spread volatility and default correlation. International Journal of Theoretical and Applied Finance.
- D. Brigo, A. Pallavicini, R. Torresetti (2007). Cluster-based extension of the generalized poisson loss dynamics and consistency with single names. International Journal of Theoretical and Applied Finance, Vol 10, n. 4. Also in: A. Lipton and Rennie (Editors), Credit Correlation - Life After Copulas, World Scientific, 2007.
- D. Brigo, A. Pallavicini, R. Torresetti (2006). CDO calibration with the dynamical Generalized Poisson Loss model. ssrn.com. Published later in Risk Magazine, June 2007 issue.

References II

- Morini, M. and Brigo, D. (2007). No-Armageddon Arbitrage-free Equivalent Measure for Index options in a credit crisis. Forthcoming in *Mathematical Finance*.
- Morini, M. and Brigo, D. (2009). Last option before the Armageddon, *Risk Magazine*, September issue.
- Brigo, Pallavicini, Torresetti (2009). Credit Models and the Crisis or: How I learned to stop worrying and love the CDO's. Available at ssrn.com, arXiv.org, defaultrisk.com
- Torresetti, R., Brigo, D., and Pallavicini, A. (2006a). Implied Expected Tranched Loss Surface from CDO Data. Available at ssrn.com.
- Torresetti, R., Brigo, D., and Pallavicini, A. (2006b). Implied Correlation in CDO Tranches: A Paradigm to be Handled with Care. Available at ssrn.com.

References III

- Torresetti, R., and Pallavicini, A. (2007). Stressing Rating Criteria Allowing for Default Clustering: the CPDO case. Available at ssrn.com.
- Torresetti, R., Brigo, D., and Pallavicini, A. (2006). Risk Neutral Versus Objective Loss Distribution and CDO Tranches Valuation. Available at ssrn.com, updated version appeared in the *Journal of Risk Management in Financial Institutions*, January-March 2009 issue.
- Brigo, D., Pallavicini, A. and Torresetti, R. (2010). Credit Models and the Crisis: A journey into CDOs, Copulas, Correlations and Dynamic Models. Wiley, Chichester.
- Bielecki, T., Brigo, D., and Patras, F. (Editors) (2011). Credit Risk Frontiers: subprime crisis, pricing and hedging , cva, mbs, ratings, and liquidity. Forthcoming, Wiley/Bloomberg Press.

References IV

- Shreve, S. (2008), Don't Blame the Quants, Forbes Commentary
- Donnelly, C., and Embrechts, P. (2009), The devil is in the tails: actuarial mathematics and the subprime mortgage crisis
- Giorgio Szegö (2009), the Crash Sonata in D Major, Journal of Risk Management in Financial Institutions