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Counterparty risk and Contingent CDS valuation under correlation between interest-rates and default

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Abstract

We consider counterparty risk for interest rate payoffs in presence of correlation between the default event and interest rates. The previous analysis of Brigo and Masetti (2006), assuming independence, is further extended to interest rate payoffs different from simple swap portfolios. A stochastic intensity model with possible jumps is adopted for the default event. We find that correlation between interest-rates and default has a relevant impact on the positive adjustment to be subtracted from the default free price to take into account counterparty risk. We analyze the pattern of such impacts as product characteristics and tenor structures change through some fundamental numerical examples. We find the counterparty risk adjustment to decrease with the correlation for receiver payoffs, while the analogous adjustment for payer payoffs increases. The impact of correlation decreases when the default probability increases. Finally, our analysis applies naturally also to Contingent Credit Default Swaps.

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Key words: Counterparty Risk, Contingent Credit Default Swap, Hybrid Products, Interest-rate default Correlation, Risk Neutral Valuation, Default Risk, Interest rate models, Default intensity models.

Contents

1 Introduction

In this paper we consider counterparty risk for interest rate payoffs in presence of correlation between the default event and interest rates.

In particular we analyze in detail counterparty-risk (or default-risk) Interest Rate Swaps (IRS), continuing the work of Sorensen and Bollier (1994), and of Brigo and Masetti (2006), where no correlation is taken into account. We also analyze option payoffs under counterparty risk. In general the reason to introduce counterparty risk when evaluating a contract is linked to the fact that many financial contracts are traded over the counter, so that the credit quality of the counterparty can be relevant. This is particularly appropriated when thinking of the different defaults experienced by some important companies during the last years. Also, regulatory issues related to the IAS 39 framework encourage the inclusion of counterparty risk into valuation. Furthermore, credit hybrid products such as Contingent Credit Default Swaps (Contingent CDS) with interest rate underlying assume exactly the same form as the optional part in the counterparty risk valuation problem for the interest rate payoff. This renders our approach useful also into Contingent CDS valuation.

We are looking at the problem from the viewpoint of a safe (default-free) counterparty entering a financial contract with another counterparty having a positive probability of defaulting before the final maturity. We formalize the general and reasonable fact that the value of a generic claim subject to counterparty risk is always smaller than the value of a similar claim having a null default probability, expressing the discrepancy in precise quantitative terms.

When evaluating default-risky assets one has to introduce the default probabilities in the pricing models. We consider Credit Default Swaps as liquid sources of market default probabilities. Different models can be used to calibrate CDS data and obtain default probabilities: In Brigo and Morini (2006) for example *firm value models (or structural models)* are used, whereas in Brigo and Alfonsi (2005) a stochastic intensity model is used. In this work we adopt the second framework, since this lends itself more naturally to interact with interest rate modeling and allows for a very natural way to correlate the default event to interest rates. We also consider the possible addition of jumps in the intensity model, as in the extensions seen in Brigo and El-Bachir (2006, 2007).

In the paper we find that counterparty risk has a relevant impact on the products prices and that, in turn, correlation between interest-rates and default has a relevant impact on the adjustment due to counterparty risk on an otherwise default-free interest-rate payout. We analyze the pattern of such impacts as products characteristics and tenor structures change through some fundamental numerical examples and find stable and financially reasonable patterns.

In particular, we find the (positive) counterparty risk adjustment to be subtracted from the default free price to decrease with correlation for receiver payoffs. The analogous adjustment for payer payoffs increases with correlation. We analyze products such as standard swaps, swap portfolios, European and Bermudan swaptions, mostly of receiver type. We also consider CMS spread options, which being based on interest rate spreads are out of our "payer/receiver" classification.

In general our results confirm the counterparty risk adjustment to be relevant and the impact of correlation on counterparty risk to be relevant in turn. We comment our findings in more detail in the conclusions.

2 General valuation of counterparty risk

We denote by τ the default time of the counterparty and we assume the investor who is considering a transaction with the counterparty to be default-free. We place ourselves in a probability space $(\Omega, \mathcal{G}, \mathcal{G}_t, \mathbb{Q})$. The filtration $(\mathcal{G}_t)_t$ models the flow of information of the whole market, including credit. $\mathbb{Q}$ is the risk neutral measure. This space is endowed also with a right-continuous and complete sub-filtration $\mathcal{F}_t$ representing all the observable market quantities but the default event (hence $\mathcal{F}_t \subseteq \mathcal{G}_t := \mathcal{F}_t \vee \mathcal{H}_t$ where $\mathcal{H}_t = \sigma(\{\tau \le u\} : u \le t)$ is the right-continuous filtration generated by the default event). We set $\mathbb{E}_t(\cdot) := \mathbb{E}(\cdot|\mathcal{G}_t)$, the risk neutral expectation leading to prices.

Let us call T the final maturity of the payoff we need to evaluate. If $\tau > T$ there is no default of the counterparty during the life of the product and the counterparty has no problems in repaying the investors. On the contrary, if $\tau \le T$ the counterparty cannot fulfill its obligations and the following happens. At τ the Net Present Value (NPV) of the residual payoff until maturity is computed: If this NPV is negative (respectively positive) for the investor (defaulted counterparty), it is completely paid (received) by the investor (counterparty) itself. If the NPV is positive (negative) for the investor (counterparty), only a recovery fraction $\textsc{Rec}$ of the NPV is exchanged.

Let us call $\Pi^D(t,T)$ (sometimes abbreviated into $\Pi^D(t)$) the discounted payoff of a generic defaultable claim at t and $\textsc{Cashflows}(u,s)$ the net cash flows of the claim without default between time u and time s, discounted back at u, all payoffs seen from the point of view of the "investor" (i.e. the company facing counterparty risk). Then we have $\mathrm{NPV}(\tau) = \mathbb{E}_\tau\{\textsc{Cashflows}(\tau,T)\}$ and

$$\begin{aligned}\Pi^D(t) &= \mathbf{1}_{\{\tau>T\}}\textsc{Cashflows}(t,T) + \\ &\quad \mathbf{1}_{\{t<\tau\le T\}}\left[\textsc{Cashflows}(t,\tau) + D(t,\tau)\left(\textsc{Rec}\,(\mathrm{NPV}(\tau))^+ - (-\mathrm{NPV}(\tau))^+\right)\right] \end{aligned} \tag{2.1}$$

being $D(u,v)$ the stochastic discount factor at time u for maturity v. This last expression is the general price of the payoff under counterparty risk. Indeed, if there is no early default this expression reduces to risk neutral valuation of the payoff (first term in the right hand side); in case of early default, the payments due before default occurs are received (second term), and then if the residual net present value is positive only a recovery of it is received (third term), whereas if it is negative it is paid in full (fourth term).

Calling $\Pi(t)$ the discounted payoff for an equivalent claim with a default-free counterparty, i.e. $\Pi(t) = \textsc{Cashflows}(t,T)$, it is possible to prove the following

Proposition 2.1. (General counterparty risk pricing formula). *At valuation time t, and on $\{\tau > t\}$, the price of our payoff under counterparty risk is*

$$\mathbb{E}_t\{\Pi^D(t)\} = \mathbb{E}_t\{\Pi(t)\} - \underbrace{\textsc{Lgd}\ \mathbb{E}_t\{\mathbf{1}_{\{t<\tau\le T\}}D(t,\tau)\,(NPV(\tau))^+\}}_{\text{Positive counterparty-risk adjustment}} \tag{2.2}$$

where $\textsc{Lgd} = 1 - \textsc{Rec}$ is the Loss Given Default *and the recovery fraction $\textsc{Rec}$ is assumed to be deterministic. It is clear that the value of a defaultable claim is the value of the corresponding default-free claim minus an option part, in the specific a call option (with zero strike) on the residual NPV giving nonzero contribution only in scenarios where $\tau \le T$. Counterparty risk adds an optionality level to the original payoff.*

For a proof see for example Brigo and Masetti (2006).

Notice finally that the previous formula can be approximated as follows. Take $t = 0$ for simplicity and write, on a discretization time grid $T_0, T_1, \ldots, T_b = T$,

$$\begin{aligned}\mathbb{E}[\Pi^D(0,T_b)] &= \mathbb{E}[\Pi(0,T_b)] - \mathrm{L_{GD}} \textstyle\sum_{j=1}^{b} \mathbb{E}[\mathbf{1}_{\{T_{j-1}<\tau\le T_j\}} D(0,\tau)(\mathbb{E}_\tau \Pi(\tau,T_b))^+] \\ &\approx \mathbb{E}[\Pi(0,T_b)] - \underbrace{\mathrm{L_{GD}} \sum_{j=1}^{b} \mathbb{E}[\mathbf{1}_{\{T_{j-1}<\tau\le T_j\}} D(0,T_j)(\mathbb{E}_{T_j} \Pi(T_j,T_b))^+]}_{\text{approximated (positive) adjustment}}\end{aligned} \quad (2.3)$$

where the approximation consists in postponing the default time to the first T_i following τ. From this last expression, under independence between Π and τ, one can factor the outer expectation inside the summation in products of default probabilities times option prices. This way we would not need a default model but only survival probabilities and an option model for the underling market of Π. This is only possible, in our case, if the default interest-rates correlation is zero. This is what led to earlier results on swaps with counterparty risk in Brigo and Masetti (2006). In this paper we do not assume zero correlation, so that in general we need to compute the counterparty risk without factoring the expectations. To do so we need a default model, to be correlated with the basic interest rate market.

3 Modeling assumptions

In this section we consider a model that is stochastic both in the interest rates (underlying market) and in the default intensity (counterparty). Joint stochasticity is needed to introduce correlation. The interest-rate sector is modelled according to a short-rate Gaussian shifted two-factor process (hereafter G2++), while the default-intensity sector is modelled according to a square-root process (hereafter CIR++). Details for both model can be found, for example, on Brigo and Mercurio (2001, 2006). The two models are coupled by correlating their Brownian shocks.

3.1 G2++ interest rate model

We assume that the dynamics of the instantaneous-short-rate process under the risk-neutral measure is given by

$$r(t) = x(t) + z(t) + \varphi(t;\alpha), \quad r(0) = r_0, \quad (3.1)$$

where α is a set of parameters and the processes x and z are $\mathcal{F}_t$ adapted and satisfy

$$\begin{aligned} dx(t) &= -a x(t) dt + \sigma dZ_1(t), \quad x(0) = 0, \\ dz(t) &= -b z(t) dt + \eta dZ_2(t), \quad z(0) = 0, \end{aligned} \quad (3.2)$$

where (Z_1, Z_2) is a two-dimensional Brownian motion with instantaneous correlation $\rho_{1,2}$ as from

$$dZ_1(t) dZ_2(t) = \rho_{1,2} dt,$$

where r_0, a, b, σ, η are positive constants, and where $-1 \le \rho_{1,2} \le 1$. These are the parameters entering φ, in that $\alpha = [r_0, a, b, \sigma, \eta, \rho_{1,2}]$. The function $\varphi(\cdot;\alpha)$ is deterministic and well defined in the time interval $[0, T^*]$, with T^* a given time horizon, typically 10, 30 or 50 (years). In particular, $\varphi(0;\alpha) = r_0$. This function can be set to a value automatically calibrating the initial zero coupon curve observed in the market. In our numerical tests we use the market inputs listed in Tables 1 and 2 corresponding to parameters α given by

$$a = 0.0558,\ b = 0.5493,\ \sigma = 0.0093,\ \eta = 0.0138,\ \rho_{1,2} = -0.7$$

3.2 CIR++ stochastic intensity model

For the stochastic intensity model we set

$$\lambda_t = y_t + \psi(t;\beta)\ ,\quad t \geq 0, \tag{3.3}$$

where ψ is a deterministic function, depending on the parameter vector β (which includes y_0), that is integrable on closed intervals. The initial condition y_0 is one more parameter at our disposal: We are free to select its value as long as

$$\psi(0;\beta) = \lambda_0 - y_0\ .$$

We take y to be a Cox Ingersoll Ross process (see for example Brigo and Mercurio (2001) or (2006)):

$$dy_t = \kappa(\mu - y_t)dt + \nu\sqrt{y_t}dZ_3(t),$$

where the parameter vector is $\beta = (\kappa, \mu, \nu, y_0)$, with κ, μ, ν, y_0 positive deterministic constants. As usual, Z is a standard Brownian motion process under the risk neutral measure, representing the stochastic shock in our dynamics. We assume the origin to be inaccessible, i.e.

$$2\kappa\mu > \nu^2.$$

We will often use the integrated quantities

$$\Lambda(t) = \int_0^t \lambda_s ds,\quad Y(t) = \int_0^t y_s ds,\quad \text{and}\quad \Psi(t,\beta) = \int_0^t \psi(s,\beta)ds.$$

3.3 CIR++ model: CDS calibration

Assume that the intensity λ, and the cumulated intensity Λ, are independent of the short rate r, and of interest rates in general. Since in our Cox process setting $\tau = \Lambda^{-1}(\xi)$ with ξ exponential and independent of interest rates, in this zero correlation case the default time τ and interest rate quantities $r, D(s,t), \ldots$ are independent. It follows that (approximated no-accrual receiver) CDS valuation becomes model independent and is given by the formula

$$\text{CDS}_{a,b}(0,R) = R\sum_{i=a+1}^{b} P(0,T_i)\alpha_i\mathbb{Q}(\tau \geq T_i) - \text{L}_{\text{GD}}\sum_{i=a+1}^{b} P(0,T_i)\mathbb{Q}(\tau \in [T_{i-1},T_i]) \tag{3.4}$$

(see for example the Credit chapters in Brigo and Mercurio (2006) for the details). Here R is the periodic premium rate (or "spread") received by the protection seller from the premium leg, until final maturity or until the first T_i following default, whereas $\text{L}_{\text{GD}} = 1 - \text{R}_{\text{EC}}$ is the loss given default protection payment to be paid to the protection buyer in the default (or protection) leg in case of early default, at the first T_i following default.

This formula implies that if we strip survival probabilities from CDS in a model independent way, to calibrate the market CDS quotes we just need to make sure that the survival probabilities we strip from CDS are correctly reproduced by the CIR++ model. Since the survival probabilities in the CIR++ model are given by

$$\mathbb{Q}(\tau > t)_{model} = \mathbb{E}(e^{-\Lambda(t)}) = \mathbb{E}\exp\left(-\Psi(t,\beta) - Y(t)\right) \tag{3.5}$$

we just need to make sure

$$\mathbb{E}\exp\left(-\Psi(t,\beta) - Y(t)\right) = \mathbb{Q}(\tau > t)_{market}$$

from which

$$\Psi(t,\beta) = \ln\left(\frac{\mathbb{E}(e^{-Y(t)})}{\mathbb{Q}(\tau > t)_{market}}\right) = \ln\left(\frac{P^{\text{CIR}}(0,t,y_0;\beta)}{\mathbb{Q}(\tau > t)_{market}}\right) \tag{3.6}$$

where we choose the parameters β in order to have a positive function ψ (i.e. an increasing Ψ) and P^{CIR} is the closed form expression for bond prices in the time homogeneous CIR model with initial condition y_0 and parameters β (see for example Brigo and Mercurio (2001, 2006)). Thus, if ψ is selected according to this last formula, as we will assume from now on, the model is easily and automatically calibrated to the market survival probabilities (possibly stripped from CDS data).

This CDS calibration procedure assumes zero correlation between default and interest rates, so in principle when taking nonzero correlation we cannot adopt it. However, we have seen in Brigo and Alfonsi (2005) and further in Brigo and Mercurio (2006) that the impact of interest-rate / default correlation is typically negligible on CDSs, so that we may retain this calibration procedure even under nonzero correlation, and we will do so in the paper.

Once we have done this and calibrated CDS data through $\psi(\cdot,\beta)$, we are left with the parameters β, which can be used to calibrate further products. However, this will be interesting when single name option data on the credit derivatives market will become more liquid. Currently the bid-ask spreads for single name CDS options are large and suggest to consider these quotes with caution. At the moment we content ourselves of calibrating only CDS's for the credit part. To help specifying β without further data we set some values of the parameters implying possibly reasonable values for the implied volatility of hypothetical CDS options on the counterparty.

In our tests we take stylized flat CDS curves for the counterparty, assuming they imply initial survival probabilities at time 0 consistent with the following hazard function formulation,

$$\mathbb{Q}(\tau > t)_{\text{market}} = \exp(-\gamma t),$$

for a constant deterministic value of γ. This is to be interpreted as a quoting mechanism for survival probabilities and not as a model. Assuming our counterparty CDS's at time 0 for different maturities to imply a given value of γ, we will value counterparty risk under different values of γ. This assumption on CDS spreads is stylized but our aim is checking impacts rather than having an extremely precise valuation.

In our numerical examples we take as values of the intensity volatility parameters y_0, κ, μ, ν the following values:

$$y_0 = 0.0165,\ \kappa = 0.4,\ \mu = 0.026,\ \nu = 0.14$$

Paired with stylized CDS data consistent with survivals $\mathbb{Q}(\tau > t)_{\text{market}} = \exp(-\gamma t)$ for several possible values of γ, these parameters imply the CDS volatilities[1] listed in Table 3.

3.4 Interest-rate / credit-spread correlation

We take the short interest-rate factors x and z and the intensity process y to be correlated, by assuming the driving Brownian motions Z_1, Z_2 and Z_3 to be instantaneously correlated according to

$$dZ_i \, dZ_3 = \rho_{i,3} dt, \quad i \in \{1, 2\}.$$

Notice that the instantaneous correlation between the resulting short rate and the intensity, i.e. the instantaneous interest-rate / credit-spread correlation is

$$\bar{\rho} = \text{Corr}(dr_t, d\lambda_t) = \frac{\sigma\rho_{1,3} + \eta\rho_{2,3}}{\sqrt{\sigma^2 + \eta^2 + 2\sigma\eta\rho_{1,2}}}.$$

We find the limit values of -1, 0 and 1 according to Table 5.

3.5 Adding jumps to the intensity model

CDS volatilities quoted on the market are not liquid, but they are usually higher than the CDS implied volatilities obtained with the CIR++ model. Adding jumps to the intensity model is one means to enhance the implied volatility (see for example Brigo (2005)), and it agrees to hystorical series of the credit spread too. Thus, by following Brigo and El-Bachir (2006, 2007) and Brigo and Pallavicini (2008), we consider also a square-root process with exponential jumps (hereafter JCIR++) for the default-intensity sector of our model.

$$dy_t = \kappa(\mu - y_t)dt + \nu\sqrt{y_t}dZ_3(t) + dJ_t(\zeta_1, \zeta_2),$$

where the parameter vector β is now augmented to include the jump parameters, and each parameter is a positive deterministic constant. As before, Z_3 is a standard Brownian motion process under the risk neutral measure, while the jump part $J_t(\zeta_1, \zeta_2)$ is defined as

$$J_t(\zeta_1, \zeta_2) := \sum_{i=i}^{M_t(\zeta_1)} X_i(\zeta_2)$$

where M is a time-homogeneous Poisson process with intensity ζ_1 (independent of Z), the Xs being exponentially distributed with positive finite mean ζ_2 independent of M (and Z).

Notice that the instantaneous correlation between the resulting short rate and the intensity is now reduced due to the jumps, as shown in Table 5, and it is given by

$$\bar{\rho} = \text{Corr}(dr_t, d\lambda_t) = \frac{\sigma\rho_{1,3} + \eta\rho_{2,3}}{\sqrt{\sigma^2 + \eta^2 + 2\sigma\eta\rho_{1,2}}\sqrt{1 + \frac{2\zeta_1\zeta_2^2}{\nu^2 y_t}}}.$$

As in the CIR++ case we assume the independence of the default intensity and interest rates while calibrating, so that, given market implied default probabilities, extracted from

[1]See Brigo (2005, 2006) for a precise notion of CDS implied volatility.

CDS quotes, it is always possible to get a close form formula for $\psi(\cdot,\beta)$ such that the JCIR++ model fits exactly the market default probabilities, see for example Brigo and El-Bachir (2006), reported also in Brigo and Mercurio (2006).

We set the diffusion part intensity parameters for the JCIR++ model to

$$y_0 = 0.035,\ \kappa = 0.35,\ \mu = 0.045,\ \nu = 0.15$$

Then, we consider different possibilities for the values of the jump parameters ζ_1 and ζ_2 for three different choices of the CDS curves to reproduce different realistic market situations, as shown in Table 4.

4 Numerical methods

A Montecarlo simulation is used to value all the payoffs considered in the present paper. We adopt the following prescriptions to implement effectively the algorithm. The standard error of each Montecarlo run is at most on the last digit of numbers reported in tables.

4.1 Discretization scheme

Payoff present values can be calculated with the joint interest-rate and credit model by means of a Montecarlo simulation of the three underlying variables x, z and y, whose joint transition density is needed. The transition density for the G2++ model is known in closed form, while the CIR++ model requires a discretization scheme, leading to a three dimensional Gaussian local discretization. For CIR++ we adopt a discretization with a weekly step and we find similar convergence results both with the *full truncation scheme* introduced by Lord, Koekkoek and Van Dijk (2006) and with the *implied scheme* by Brigo and Alfonsi (2005). In the following we adopt the former scheme.

4.2 Simulating intensity jumps

In order to add the jumps on the intensity process, we first simulate the diffusive part of the process at a fixed set of dates $0 = t_0 < t_1 < \cdots < t_n$, according to the discretization scheme (we adopt the same discretization scheme of the CIR++ model). Then, we compute on each path the number of jumps occuring per time interval and their amplitudes. Finally, the jumps are added by considering all the contribution as occuring at the end of each discretization period.

4.3 Forward expectations

The simulation algorithm allows the counterparty to default on the contract payment dates, unless the time-interval between two payment dates is longer than two months. In such case additional checks on counterparty default are added in order to ensure that the gap between allowed default-times is at most of two months. The calculation of the forward expectation, required by counterparty risk evaluation, as given in equation (2.3) (inner expectation $\mathbb{E}_{T_j}$) is taken by approximating the expectation at the effective default time T_j with a polynomial series in the interest-rate model underlyings, x and z, valued at the first allowed default-time after τ, i.e. at T_j. The coefficients of the series expansion are calculated by means of a least-square regression, as usually done to price Bermudan options, by means of the algorithm by Longstaff and Schwarz (2001).

4.4 Callable payoffs

Counterparty risk for callable payoffs is calculated in two steps. First, given a riskless version of the payoff, the payoff exercise boundary is calculated by a Montecarlo simulation with the Longstaff and Schwarz algorithm. Since the default time is unpredictable from the point of view of the interest-rate sector of the model, the same exercise boundary, as a function of the underlying at exercise date, is assumed to hold also for the default-risky payoff. Then the risky payoff along with the exercise boundary is treated as a standard European default-risky option, given that the continuation value at any relevant time is now a function of the underlying processes.

5 Results and Discussion

We consider the pricing of different payoffs in presence of counterparty risk for three different default probability scenarios (as expressed by hazard rates $\gamma = 3\%$, 5% and 7%) and for three different correlation scenarios ($\bar{\rho} = -1$, 0 and 1). For a detailed description of the payoffs the reader is referred to Brigo and Mercurio (2006).

5.1 Single Interest Rate Swaps (IRS)

In the following we consider payoffs depending on at-the-money fix-receiver forward interest-rate-swap (IRS) paying on the EUR market. These contracts reset a given number of years from trade date and start accruing two business days later. The IRS's fixed legs pay annually a 30E/360 strike rate, while the floating legs pay LIBOR twice per year. The first products we analyze are simple IRS of this kind. We list in Table 6 the counterparty risk adjustment for the 10y IRS and the impact of correlation, for different levels of default probabilities.

We price the counterparty risk for the single IRS also in the case that the default intensity can jump. We list in Table 7 the results. Notice that, in presence of jumps on the default intensity, the correlation impact may be enhanced.

5.2 Netted portfolios of IRS

After single IRS, we consider portfolios of at-the-money IRS either with different starting dates or with different maturities. In particular we focus on the following two portfolios:

1. ($\Pi 1$) given a set of annually spaced dates $\{T_i : i = 0 \dots N\}$, with T_0 at two business days from trade date, consider the portfolio of swaps maturing at each T_i, with $i > 0$, and all starting at T_0. The netting of the portfolio is equal to an amortizing swap with decreasing outstanding.

2. ($\Pi 2$) given the same set of annually spaced dates, consider the portfolio of swaps starting at each T_i, with $i < N$, and all maturing at T_N. The netting of the portfolio is equal to an amortizing swap with increasing outstanding.

We list in Table 6 the counterparty risk adjustment for both portfolios.

5.3 European swaptions

We consider contracts giving the opportunity to enter a receiver IRS at a IRS's reset date. The strike rate in the swap to be entered is fixed at the at-the-money forward swap level observed at option inception, i.e. at trade date. We list in Table 8 the price of both the riskless and the risky contract. In Table 10 the same data are cast in term of Black implied swaption volatility, i.e. we compute the black swaption volatility that would match the counterparty risk adjusted swaption price when put in a default-free Black formula for swaptions. In Table 9 we show an example with payer swaptions instead.

5.4 Bermudan swaptions

We consider contracts giving the opportunity to enter a portfolio of IRS, as defined in Section 5.2, every two business days before the starting of each accruing period of the swap's fix leg. We list in Table 11 the price of entering each portfolio, risky and riskless, along with the price of entering, at the same exercise dates, the contained IRS of longest tenor.

5.5 CMS spread options

We consider a contract[2] on the EUR market starting within two business days which pays, quarterly on an ACT/360 basis and up to maturity t_M, the following exotic index:

$$(L(S_a(t_i) - S_b(t_i)) - K)^+$$

where L and K are positive constants and $S_k(t_i)$, with $k \in \{a, b\}$ and $i = 0 \dots M$, is the constant maturity swap rate (hereafter CMS) fixing two business days before each accruing period starting date t_i, i.e. the at-the-money rate for a IRS with tenor of k years fixing at t_i. We list in Table 12 the option prices, default-risky and riskless.

5.6 Contingent CDS

A Contingent Credit Default Swap (CCDS) is a CDS that, upon the default of the reference credit, pays the loss given default on the residual net present value of a given portfolio if this is positive. The standard CDS instead pays the loss given default on a pre-specified notional amount, which we assumed to be 1 in our earlier CDS formulation (3.4).

It is immediate then that the default leg CCDS valuation, when the CCDS underlying portfolio constituting the protection notional is Π, is simply the counterparty risk adjustment in Formula (2.2). Our adjustments calculations above can then be interpreted also as examples of contingent CDS pricing[3].

The mathematical shape of the CCDS payoff shows that in principle the CCDS would be a ideal instrument to hedge conuterparty risk. However, the mathematical equivalence of the payoffs does not necessarily imply that CCDS be always a convenient solution for hedging counterparty risk, see for example the discussion in Patel (2007).

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[2]See also Mercurio and Pallavicini (2005) for a detailed discussion of CMS spread option pricing.

[3]We are grateful to Gloria Ikosi of the Federal Deposit Insurance Corporation in Washington DC for helpful correspondence on this subject

6 Results Interpretation and Conclusions

In the paper we have found that counterparty risk has a relevant impact on interest-rate payoffs prices and that, in turn, correlation between interest-rates and default (intensity) has a relevant impact on the adjustment due to counterparty risk. The same applies to Contingent Credit Default Swap pricing, given the strong analogies with counterparty risk valuation. We have analyzed the pattern of such impacts as products characteristics and tenor structures change through some fundamental numerical examples and we have found stable and reasonable patterns. In particular, the (positive) counterparty risk adjustment to be subtracted from the default free price decreases with correlation for receiver payoffs (IRS, IRS portfolios, European and Bermudan Swaptions). This is to be expected. If default intensities increase, with high positive correlation their correlated interest rates will increase more than with low correlation; since when interest rates increase a receiver swaption value decreases, we see that ceteris paribus a higher correlation implies a lower value for the swaptions impacting the adjustment, so that with higher correlation the adjustment absolute value decreases. The analogous adjustment for payer payoffs increases with correlation instead, as is to be expected.

In general our results, including the CMS spread options, confirm the counterparty risk adjustment to be relevant and the impact of correlation on counterparty risk to be relevant in turn, expecially in presence of jumps on default intensity, as can be required in order to achieve higher implied volatilites for CDS options. We have found the following further stylized facts, holding throughout all payoffs. As the default probability implied by the counterparty CDS increases, the size of the adjustment due to counterparty risk increases as well, but the impact of correlation on it decreases. This is financially reasonable: Given large default probabilities for the counterparty, fine details on the dynamics such as the correlation with interest rates become less relevant, everything being wiped out by massive defaults anyway. On the contrary, with smaller default probabilities, the fine structure of the dynamics and correlation in particular is more important.

The conclusion is that we should take into account interest-rate/ default correlation in valuing counteparty risky interest-rate payoffs, especially when the default probabilities are not extremely high.

References

[1] Brigo, D. (2005). Market Models for CDS Options and Callable Floaters, *Risk*, January issue. Also in: *Derivatives Trading and Option Pricing*, Dunbar N. (Editor), Risk Books, 2005.

[2] Brigo, D. (2006). Constant Maturity Credit Default Swap Valuation with Market Models, *Risk*, June issue.

[3] Brigo, D., and Alfonsi, A. (2005) Credit Default Swaps Calibration and Derivatives Pricing with the SSRD Stochastic Intensity Model, Finance and Stochastic, Vol. 9, N. 1.

[4] Brigo, D., and El-Bachir, N. (2006) Credit Derivatives Pricing with a Smile-Extended Jump Stochastic Intensity Model. ICMA Research Report, available at `http://papers.ssrn.com/sol3/papers.cfm?abstract_id=950208`

[5] Brigo, D., and El-Bachir, N. (2007) An Exact Formula for Default Swaptions Pricing in the SSRJD Stochastic Intensity Model. ICMA Research Report, available at `http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1029903`

[6] Brigo, D., and Masetti, M. (2006) Risk Neutral Pricing of Counterparty Risk. In *Counterparty Credit Risk Modeling: Risk Management, Pricing and Regulation*, ed. Pykhtin, M., Risk Books, London.

[7] Brigo, D., Mercurio, F. (2001) *Interest Rate Models: Theory and Practice*, Springer Verlag.

[8] Brigo, D., Mercurio, F. (2006) *Interest Rate Models: Theory and Practice - with Smile, Inflation and Credit*, Second Edition, Springer Verlag.

[9] Brigo, D. and Pallavicini, A. (2008) Counterparty risk and Contingent CDS with stochastic intensity hybrid models, *Risk*, February issue.

[10] Brigo, D., Morini, M. (2006) Structural credit calibration, *Risk*, April issue.

[11] Cherubini, U. (2005) Counterparty Risk in Derivatives and Collateral Policies: The Replicating Portfolio Approach. In: *Proceedings of the Counterparty Credit Risk 2005 C.R.E.D.I.T. conference*, Venice, Sept 22-23, Vol 1.

[12] Longstaff, F. A., Schwarz, E. S. (2001) Valuing American options by simulation: a simple least-squares approach. *Review of Financial Studies*, 14, 113-147.

[13] Lord, R., Koekkoek, R., and Van Dijk, D.J.C. (2006). A Comparison of Biased Simulation Schemes for Stochastic Volatility Models. Working paper.

[14] Mercurio, F., Pallavicini, A. (2005) Mixing Gaussian Models to Price CMS Derivatives. Working paper available at: `http://ssrn.com/abstract=872708`

[15] Patel, N. (2007). In the Shadows, Risk Magazine, June issue

[16] Sorensen, E.H., and Bollier, T. F. (1994) Pricing Swap Default Risk. *Financial Analysts Journal*, 50, 23-33.

A Appendix: Detailed outputs

Date	Rate	Date	Rate	Date	Rate	Date	Rate
26-Jun-06	2.83%	20-Sep-07	3.46%	27-Jun-16	4.19%	27-Jun-28	4.51%
27-Jun-06	2.83%	19-Dec-07	3.52%	27-Jun-17	4.23%	27-Jun-29	4.51%
28-Jun-06	2.83%	19-Mar-08	3.57%	27-Jun-18	4.27%	27-Jun-30	4.52%
04-Jul-06	2.87%	19-Jun-08	3.61%	27-Jun-19	4.31%	27-Jun-31	4.52%
11-Jul-06	2.87%	18-Sep-08	3.65%	29-Jun-20	4.35%	28-Jun-32	4.52%
18-Jul-06	2.87%	29-Jun-09	3.75%	28-Jun-21	4.38%	27-Jun-33	4.52%
27-Jul-06	2.88%	28-Jun-10	3.84%	27-Jun-22	4.41%	27-Jun-34	4.52%
28-Aug-06	2.92%	27-Jun-11	3.91%	27-Jun-23	4.43%	27-Jun-35	4.52%
20-Sep-06	2.96%	27-Jun-12	3.98%	27-Jun-24	4.45%	27-Jun-36	4.52%
20-Dec-06	3.14%	27-Jun-13	4.03%	27-Jun-25	4.47%	27-Jun-46	4.49%
20-Mar-07	3.27%	27-Jun-14	4.09%	29-Jun-26	4.48%	27-Jun-56	4.46%
21-Jun-07	3.38%	29-Jun-15	4.14%	28-Jun-27	4.50%		

Table 1: EUR zero-coupon continuously-compounded spot rates (ACT/360) observed on June,23 2006.

	Tenor						
Expiry	1y	2y	5y	7y	10y	15y	20y
1y	17.51%	15.86%	14.63%	14.20%	13.41%	12.14%	11.16%
2y	16.05%	15.26%	14.55%	14.09%	13.29%	12.03%	11.09%
3y	15.58%	15.06%	14.43%	13.92%	13.10%	11.87%	10.96%
4y	15.29%	14.90%	14.20%	13.67%	12.85%	11.66%	10.79%
5y	15.05%	14.67%	13.90%	13.36%	12.55%	11.42%	10.60%
7y	14.39%	14.00%	13.22%	12.70%	11.96%	10.95%	10.20%
10y	13.25%	12.94%	12.23%	11.79%	11.17%	10.31%	9.65%
15y	11.87%	11.64%	11.11%	10.76%	10.26%	9.52%	8.89%
20y	11.09%	10.92%	10.45%	10.14%	9.67%	8.91%	8.27%

Table 2: Market at-the-money swaption volatilities observed on June,23 2006.

γ	$\boldsymbol{\sigma}_{\text{impl}}$			
	1x1	1x4	4x1	1x9
3%	42%	25%	26%	15%
5%	25%	15%	15%	9%
7%	18%	11%	11%	7%

Table 3: Black volatilities for CDS options implied by CIR++ model (with parameters $y_0 = 0.0165$, $\kappa = 0.4$, $\mu = 0.026$, $\nu = 0.14$) for different choices of the default-probability parameter γ. Interest rates are modelled according to section 3.1 and $\bar{\rho} = 0$.

$\boldsymbol{\zeta_1}$	$\boldsymbol{\zeta_2}$	$\boldsymbol{\sigma}_{\text{impl}}$	$\boldsymbol{R}$			
		1x5	1	3	5	10
0	0	28%	2.59%	2.71%	2.77%	2.84%
0.1	0.1	40%	2.89%	3.37%	3.64%	3.93%
0.15	0.15	57%	3.25%	4.12%	4.58%	5.07%

Table 4: Black volatilities for CDS options implied by JCIR++ model (with parameters $y_0 = 0.035$, $\kappa = 0.35$, $\mu = 0.045$, $\nu = 0.15$) for different choices of the jump parameters. Interest rates are modelled according to section 3.1 and $\bar{\rho} = 0$.

$\boldsymbol{\rho_{1,3}}$		4.05%	0.00%	-4.05%
$\boldsymbol{\rho_{2,3}}$		-74.19%	0.00%	74.19%
$\boldsymbol{\zeta_1}$	$\boldsymbol{\zeta_2}$	$\boldsymbol{\bar{\rho}}$		
0	0	-100.00%	0.00%	100.00%
0.1	0.1	-67.29%	0.00%	67.29%
0.15	0.15	-51.67%	0.00%	51.67%

Table 5: Values of model instantaneous correlations $\rho_{1,3}$ and $\rho_{2,3}$ ensuring special interest-rate / credit-spread instantaneous correlations $\bar{\rho}$ for the chosen interest-rate and intensity dynamics parameters. Notice that the instantaneous correlations are state dependent in presence of jumps, i.e. when $\zeta_1 > 0$ and $\zeta_2 > 0$, so that the last two rows of the table are only indicative values obtained in the limit $y_t \longrightarrow \mu + \zeta_1\zeta_2/\kappa$.

γ	$\bar{\rho}$	**Π1**	**Π2**	**IRS**
3%	-1	-140	-294	-36
	0	-84	-190	-22
	1	-47	-115	-13
5%	-1	-181	-377	-46
	0	-132	-290	-34
	1	-99	-227	-26
7%	-1	-218	-447	-54
	0	-173	-369	-44
	1	-143	-316	-37

Table 6: Counterparty risk price for receiver IRS portfolio defined in section 5.2 for a maturity of ten years, along with the counterparty risk price for a ten year swap. Every IRS, constituting the portfolios, has unitary notional. Prices are in basis points.

ζ_1	ζ_2	$\bar{\rho}$	**10y**
0	0	-100%	-56(0)
		0	-45(0)
		100%	-37(0)
0.1	0.1	-67%	-69(1)
		0	-58(0)
		67%	-50(1)
0.15	0.15	-52%	-93(4)
		0	-71(3)
		52%	-57(3)

Table 7: Counterparty risk price for ten year receiver IRS defined in section 5.1 for three different calibrations of the JCIR++ model with jumps as given in Table 4. Prices are in basis points and are followed within brackets by the statistical error of the Monte Carlo.

γ	$\bar{\rho}$	**1x5**	**5x5**	**10x5**	**20x5**
3%	-1	-14	-37	-53	-56
	0	-9	-27	-42	-48
	1	-6	-19	-34	-41
5%	-1	-19	-50	-71	-70
	0	-14	-41	-61	-65
	1	-11	-35	-55	-61
7%	-1	-23	-61	-84	-79
	0	-19	-53	-77	-75
	1	-16	-47	-72	-73
riskless		106	205	215	157

γ	$\bar{\rho}$	**1x10**	**5x10**	**10x10**	**20x10**
3%	-1	-38	-78	-98	-98
	0	-25	-56	-78	-83
	1	-16	-43	-64	-72
riskless		184	342	353	256

γ	$\bar{\rho}$	**1x20**	**5x20**	**10x20**	**20x20**
3%	-1	-87	-140	-160	-150
	0	-61	-107	-129	-131
	1	-45	-83	-107	-114
riskless		261	474	486	354

Table 8: Counterparty risk price for European receiver swaptions defined in section 5.3 for different expiries and tenors. Riskless prices are listed too. Contracts have has unitary notional. Prices are in basis points.

γ	$\bar{\rho}$	**1x5**	**5x5**	**10x5**	**20x5**
3%	-1	-6	-20	-33	-40
	0	-10	-28	-44	-50
	1	-16	-39	-56	-58
riskless		106	205	215	157

Table 9: Counterparty risk price for European payer swaptions defined in section 5.3 for different expiries and tenors. Riskless prices are listed too. Contracts have has unitary notional. Prices are in basis points.

γ	$\bar{\rho}$	**1x5**	**5x5**	**10x5**	**20x5**
3%	-1	-1.96%	-2.52%	-3.06%	-3.74%
	0	-1.26%	-1.82%	-2.38%	-3.20%
	1	-0.77%	-1.32%	-1.93%	-2.78%
5%	-1	-2.60%	-3.40%	-4.06%	-4.71%
	0	-1.96%	-2.78%	-3.51%	-4.37%
	1	-1.54%	-2.35%	-3.16%	-4.09%
7%	-1	-3.19%	-4.14%	-4.81%	-5.32%
	0	-2.62%	-3.60%	-4.39%	-5.06%
	1	-2.22%	-3.23%	-4.11%	-4.89%
riskless		14.63%	13.90%	12.23%	10.45%

γ	$\bar{\rho}$	**1x10**	**5x10**	**10x10**	**20x10**
3%	-1	-2.74%	-2.86%	-3.14%	-3.72%
	0	-1.84%	-2.08%	-2.50%	-3.17%
	1	-1.19%	-1.59%	-2.03%	-2.75%
riskless		13.41%	12.55%	11.17%	9.67%

γ	$\bar{\rho}$	**1x20**	**5x20**	**10x20**	**20x20**
3%	-1	-3.71%	-3.14%	-3.19%	-3.53%
	0	-2.63%	-2.40%	-2.57%	-3.09%
	1	-1.95%	-1.87%	-2.14%	-2.68%
riskless		11.16%	10.60%	9.65%	8.27%

Table 10: Counterparty risk implied volatilities for European receiver swaptions defined in section 5.3 for different expiries and tenors. Riskless implied volatilities are listed too. Contracts have a unit notional.

γ	$\bar{\rho}$	**Π1**	**Π2**	**IRS**
3%	-1	-197	-387	-47
	0	-140	-289	-34
	1	-101	-219	-25
5%	-1	-272	-528	-65
	0	-223	-446	-54
	1	-188	-387	-46
7%	-1	-340	-652	-80
	0	-295	-578	-70
	1	-266	-529	-63
riskless		1083	1917	240

Table 11: Counterparty risk price for callable receiver IRS portfolio defined in section 5.4 for a maturity of ten years, along with the counterparty risk price for a spot-starting ten year bermuda swaption. Riskless prices are listed too. Every IRS, constituting the portfolios, has unitary notional. Prices are in basis points.

γ	$\bar{\rho}$	**5y**	**10y**	**20y**
3%	-1	-5	-16	-34
	0	-4	-11	-24
	1	-2	-8	-18
5%	-1	-7	-22	-44
	0	-6	-17	-37
	1	-5	-15	-31
7%	-1	-9	-26	-52
	0	-7	-23	-46
	1	-6	-20	-42
riskless		58	122	182

Table 12: Counterparty risk price for CMS spread options defined in section 5.5 with $L = 15$, $K = 15\%$, $a = 10y$, $b = 2y$ and three different maturities $t_M \in \{5y, 10y, 15y\}$. Riskless prices are listed too. Prices are in basis points.