Global
Special Report

Charting a Course Through the CDS Big Bang

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Abstract

Following the recent introduction of new forms of Credit Default Swap (CDS) contracts expressed as upfront payments plus a fixed coupon, this note examines the methodology suggested by Barclays Capital, Goldman Sachs, JPMorgan, Markit (BGJM)/ISDA (2009), for conversion of CDS quotes between upfront and running. The proposed flat hazard rate (FHR) conversion method is to be understood as a rule-of-thumb single-contract quoting mechanism rather than as a modelling device. For example, an hypothetical investor who would put the FHR converted running spreads into her old running CDS library would strip wrong hazard rates, inconsistent with those coming directly from the quoted term structure of upfronts.

This new methodology appears mostly as a device to transit the market towards adoption of the new upfront CDS as direct trading products while maintaining a semblance of running quotes for investors who may be suffering the transition. We caution though that

- the conversion done with proper hazard rates consistent across term would produce different results;
- the quantities involved in the conversion should not be used as modelling tools anywhere; and
- for highly distressed names with a high upfront paid by the protection buyer, the conversion to running spreads fails unless, as we propose, a third recovery scenario of 0% is added to the suggested 20% and 40%.

This paper is not meant as a criticism of the proposed standardization of the conversion method but as a warning on the confusion this may generate when the method is not used carefully.

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Table of Contents

1. Introduction

Recently there has been a proposal in Barclays, Goldman Sachs, JPMorgan, and Markit (BGJM)(2009), seconded by ISDA[1] for an imminent change in the convention for quoting CDS. In a traditional running CDS contract a spread is paid throughout the life of the deal, with this spread being set so that the premium and default legs match at inception. In the proposal the running spread will be fixed at S_c, equal to 100bps or 500bps depending of the quality of the credit. Individual CDS will vary in the required upfront payment (an amount to be exchanged immediately upon entering the contract). The recovery is also standardised to two possible values, again depending on the credit quality: 20% or 40%.

In this paper we briefly review a widely accepted CDS model and show how it can be used to convert between running spreads and upfronts. We contrast this with the proposed "flat hazard rate" (FHR) convention method in BGJM/ISDA (2009) and show that there is a material difference which could lead to significant inconsistencies and arbitrage opportunities should the converted spread be taken as the spread of a real running CDS product. The FHR methodology however works and avoids inconsistencies provided that:

- It is applied, as is meant, to a universe where each CDS name has just one quoted maturity. As this does not happen in reality, having more maturities on the same name, this will work only if traded CDS prices will be upfront ones, so that running spreads of CDS will not appear directly in trading in the way they used to appear in the earlier running market.
- It is used only as a quoting mechanism in that FHR is a method to go from traded upfront quotes to a *semblance* of fair-spread quantities and back again without loosing information. The semblance of fair spread is not the actual fair spread that one would have computed in a real running CDS contract that used to appear in the market.
- The FHR converted running spreads for CDS with different maturities are not used to strip hazard rates or to calibrate models across term.

In the paper we highlight the conversion methodology and point out the errors one could face when using the conversion outside of the context for which it is meant.

Finally, the choice of two recovery scenarios limited to 20% and 40% poses some problems and deserves further comment. For highly distressed names with high upfront paid by the protection buyer, the conversion to running spread fails if the upfront plus 20% recovery is larger than one. That is why we suggest that if one is to limit the possible recovery scenarios as in the proposal, adding the 0% recovery case to the proposed 20% and 40% guarantees the existence of the converted running spread.

To put this paper in the context of the more general literature on CDS models, including CDS options and volatility, both single and multi-name, we point out that this paper deals with deterministic hazard rates, ignoring credit spread volatility.

[1] www.cdsmodel.com says on March 2009: "The ISDA CDS Standard Model is a source code for CDS calculations and can be downloaded freely through this website. The source code is copyright of ISDA and available under an Open Source license.[...] As the CDS market evolves to trade single name contracts with a fixed coupon and upfront payment, it is critical for CDS investors to match the upfront payment amounts and to be able to translate upfront quotations to spread quotations and vice versa in a standardized manner.[...]"

Models for credit spread volatility and CDS options have been presented in the literature both in the instantaneous credit spread and on the market credit spread framework. Jamshidian (2004) and Brigo (2005) analyzed the market formula for CDS options in full mathematical rigour, resorting to different approaches to deal with vanishing numeraires. Brigo and Cousot (2006) compare the shifted stochastic intensity square root model for the instantaneous CDS spread with the market valuation formula based on a lognormal forward CDS spread. Brigo and El-Bachir (2009) present a closed form formula for CDS options in a stochastic square root credit spread model with jumps. We should also mention that credit spread volatility is quite relevant when dealing with counterparty risk for CDS. This is analyzed in Brigo and Chourdakis (2008) for the unilateral case, and in Brigo and Capponi (2008) for the bilateral one. Finally, besides single name models, valuation of collateralized debt obligations (CDO's) or Credit index options (CIO) referencing multiple names require dynamic modelling of correlation, or of the loss process in aggregate and of the index spread. One of the few arbitrage free models consistent across the CDO capital structure and maturities is in Brigo, Pallavicini and Torresetti (2007), who also explain how this model was highlighting the possibility of default of sectors even before the crisis started in 2007. Finally, the extension of the CDS option to the multi-name context is addressed in Morini and Brigo (2007) and Brigo and Morini (2009), where the analysis of vanishing numeraires is extended to multi-name CDS options in relationship with the crisis.

2. Running and Upfront Credit Default Swaps

2.1. Spot Running CDS Contract

We recall briefly some basic definitions for running CDS's that dominated the market until 2009. There have been upfront CDS's before 2009, but they used to charge the whole cost of protection upfront and hence have no running spread. This was prevalent for names with very poor or deteriorated credit quality. The new upfront CDS put forward in the so called "big bang" is different though, as we explain later on.

Let t be the current time and $T_1,\dots,T_n$ be the times when the *protection buyer* pays to the *protection seller* a coupon $C(T_i)$ on the notional N for protection against default of a specified entity. The protection buyer pays this coupon until the lesser of the random default time τ and the contract expiry T_n. If the *default time* τ is before the maturity T_n the protection seller pays to the protection buyer the notional amount minus recovery. The leg which pays upon default is called the *protection leg* or *default leg* and the leg which pays the coupons is called the *premium leg*. The lost notional minus recovery is called the *loss given default*, LGD and can be expressed in terms of a *recovery rate*, R, as $LGD = N\cdot(1-R)$. The spread S associated with the coupon payments $C(T_i)$ is called the *contractual* or *fixed spread*. This spread is typically fixed when the trade is executed and held constant to maturity.

$$\begin{array}{|c|ccc|c|} \hline Protection & \rightarrow & protection\;\; LGD \;\; at\ default\ \tau\ if\ \ 0<\tau\le T_n & \rightarrow & Protection \\ Seller\ B & \leftarrow & spread\;\; S\ \ at\ \ T_1,\dots,T_n\ \ or\ until\ default\ \ \tau & \leftarrow & Buyer\ A \\ \hline \end{array}$$

The dates T_i are typically fixed as quarterly dates during the year called IMM-Dates (20th March, 20th June, 20th September, 20th December) where adjustments are made should any date fall on a holiday.

We explicitly point out that we assume the offered protection amount LGD to be independent of the default time τ, thus we can do calculations with a deterministic LGD. The time T_i is computed by the number of days to the i-th coupon payment date and divided by the daycount convention (eg 360).

Using terminology from the market, the deal is struck on the *trade date* for protection starting on a specified *effective date*. Upfronts and accrued amounts are exchanged on the *cash settlement date*. In the mathematical exposition that follows we make the simplifying assumption that all three dates coincide.

This form of the CDS contract, that has been dominant until 2009, is expected to be replaced by the upfront CDS contract described below, at least for North American corporate CDS.

2.2. Premium and Protection Legs, and Spot Running CDS Spreads

Let $D(t,T)$ denote the discount factors at time t for maturity T and assume them to be independent of the default time τ.

As we are going to illustrate, all CDS valuation terms, observed at time 0, can be expressed using survival probabilities, also observed at time 0:

$$Surv(T) := Prob(\tau > T),$$

i.e. the probability that the name survives time T. It follows that the default probability is

$$Prob(\tau \le T) = 1 - Surv(T). \qquad (1)$$

Let the interval between coupon payment $i-1$ and i be denoted $\Delta_i = T_i - T_{i-1}$ with $T_0 = 0$. The value of the premium leg of a CDS at time 0 can be decomposed into the value of the premium leg paying one basis point, the $DV01$[2], scaled by the spread (in basis points):

$$PremLeg_{0,n}(S; D(0,\cdot), Surv(\cdot)) = S \cdot DV01_{0,n}(D(0,\cdot), Surv(\cdot)) \qquad (2)$$

where

$$DV01_{0,n}(D(0,\cdot), Surv(\cdot)) := \sum_{i=1}^{n}\left[D(0,T_i)\Delta_i Surv(T_i) - \int_{T_{i-1}}^{T_i} D(0,s)\Delta_i \frac{s - T_{i-1}}{T_i - T_{i-1}} d\,Surv(s) \right].$$

The $DV01$ is the discounted sum of the premium payments weighted by the probability of receiving them. This formula is indeed model independent given the initial zero coupon curve (bonds) at time 0 observed in the market (ie $D(0,\cdot)$) and given the survival probabilities $Surv(\cdot)$ at time 0.

A similar formula holds for the protection leg, again under independence between the default time τ and interest rates:

$$ProtecLeg_{0,n}(LGD; D(0,\cdot), Surv(\cdot)) = -LGD \int_0^{T_n} D(0,s)\, d\ Surv(s). \qquad (3)$$

Here protection starts from today. This formula too is model independent given the initial zero coupon curve (bonds) at time 0 observed in the market and given the survival probabilities at time 0.

The Stieltjes integrals with respect to survival probabilities given in the above formulas can be well approximated numerically by Riemann-Stieltjes sums provided a low enough discretization time step is taken. We usually consider the step-size to be between 10 and 30 days.

[2] sometimes referred to as $PV01$

The running CDS spread at time 0 is obtained as the fair spread $S = S_{0,n}^{mkt\ MID}$ that equates the protection and premium legs[3]:

$$S_{0,n} = \frac{ProtecLeg_{0,n}(LGD; D(0,\cdot), Surv(\cdot))}{DV01_{0,n}(D(0,\cdot), Surv(\cdot))}. \quad (4)$$

Notice that if at time 0 we have a CDS whose spread in the premium leg is $S \neq S_{0,n}$, we can also write the value of the CDS to the protection seller as

$$CDS_{0,n}(S; LGD; D(0,\cdot), Surv(\cdot)) = (S - S_{0,n})\ DV01_{0,n}(D(0,\cdot), Surv(\cdot)).$$

In practice the market quotes CDS spreads at a fixed set of maturities, e.g. $T_n \in \{1y, 3y, 5y, 7y, 10y\}$, and our model $Surv(t)$ must take into account consistently all of these quotes. Using the fact that the market spread is the fair spread and thus the one that equates premium and default legs we can solve

$$ProtecLeg_{0,n}(LGD; D(0,\cdot), Surv(\cdot))) = PremLeg_{0,n}(S_{0,n}^{mktMID}; D(0,\cdot), Surv(\cdot)) \quad (5)$$

in portions of $Surv(\cdot)$ starting from $T_n = 1y$, finding the market implied survival $\{Surv(t), t \leq 1y\}$; plugging this into the $T_n = 2y$ CDS legs formulas, and then solving the same equation with $T_n = 2y$, we find the market implied survival $\{Surv(t), t \in (1y, 2y]\}$, and so on up to $T_n = 10y$. The conditions for a valid survival curve are $Surv(0) = 1$, $Surv(t) \geq 0$ and $Surv(\cdot)$ needs to be decreasing.

This is a way to strip survival probabilities from CDS quotes in a model independent way. In practice this would be hard because the choice of $Surv(t)$ to solve equation (5) is not unique. We therefore need to constrain the form of $Surv(t)$ and the market usually adopts $\exp(-\int_0^t h(s)ds)$ (where h is typically piecewise constant in time) so that the default times τ are exponentially distributed. This is an assumption. To be precise, let $h(t)$ be the piecewise constant intensity or "hazard rate" and $H(t) = \int_0^t h(s)ds$ the cumulated intensity function satisfying

$$Surv(t) = \exp(-H(t)), \quad Prob\{s < \tau \leq t\} = \exp(-H(s)) - \exp(-H(t)).$$

In this case one can derive a formula for CDS prices based on integrals (summations) of h, and on the initial interest-rate curve, resulting from the above expectation. The two legs look like

$$ProtecLeg_{0,n}(LGD; D(0,\cdot), H(\cdot)) \approx LGD \sum_{t_j=0}^{T_n} D\left(0, \frac{t_{j+1} + t_j}{2}\right)(e^{-H(t_j)} - e^{-H(t_{j+1})}), \quad (6)$$

[3] actually bid and ask quotes are available for this fair S

$$PremLeg_{0,n}(S; D(0,\cdot), H(\cdot)) = S \quad DV01_{0,b}(D(0,\cdot), H(\cdot)), \tag{7}$$

$$DV01_{0,n}(D(0,\cdot), H(\cdot)) \approx \sum_{i=1}^{n} D(0,T_i)\Delta_i e^{-H(T_i)} - \sum_{i=1}^{n} D\left(0, \frac{T_i + T_{i-1}}{2}\right)\frac{\Delta_i}{2}(e^{-H(T_i)} - e^{-H(T_{i-1})}), \tag{8}$$

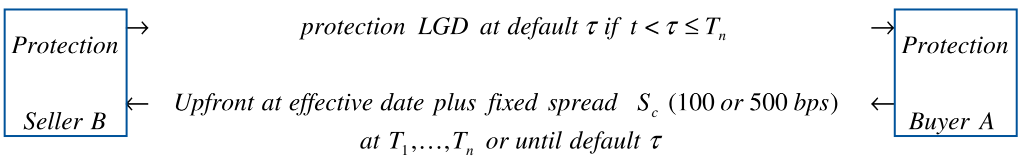
where we approximated the integrals with Riemann-Stieltjes sums on the fine grid $\{t_0, \ldots\}$.

Under this setup the calibration procedure described for $Surv(t)$ reduces to finding the constant value of $h(t)$ in $[0,1Y]$ that reproduces the one year quote and then the value in $[1Y,3Y]$ that reproduces the three year quote and so on.

It is important to point out that in more demanding modeling tasks the actual model one assumes for τ is more complex and may involve stochastic intensity either directly or through stochastic modeling of the S dynamics itself. Even so, the h are retained as a mere quoting mechanism for CDS rate market quotes, and may be taken as inputs in the calibration of more complex models. See for example the discussion on the role of credit spread volatility in counterparty risk for credit default swaps in Brigo and Chourdakis (2008).

2.3. Upfront CDS with Fixed Running Spread

Traditionally most CDS are traded as a fixed running spread paid throughout the life of the contract. Recently the market has turned towards upfront CDS, where in addition to a (different) fixed running spread there is an immediate (upfront) payment when the deal is entered.



In this new formulation, instead of choosing the spread to equate the value of the contract legs to the protection buyer and seller, the spread is fixed at the same level for all contracts and the upfront is chosen as an add-on at the initial time to match again the legs.

The recent suggestions in BGJM/ISDA (2009) use just one of two running spreads, 100 bps for investment grade CDS and 500 bps for high yield CDS. The recovery is also restricted similarly to be either 40% or 20%. The upfront payment can be negative or positive, based on where the corresponding fair spread would be with respect to the fixed spread and on possible recovery differences.

3. Conversion Between Running and Upfront Spreads

For converting upfront CDS spread quotes into running or viceversa there are essentially two possibilities. The first one is consistent with the whole term structure of hazard rates and allows also for hedging portfolios with several positions. The second one is merely a quoting mechanism that has to be used very carefully and only at a single deal basis, in order to avoid possible dangers.

3.1. Running to Upfront with a Consistent Term Structure of Hazard Rates

The upfront is just the amount that makes the contract fair, in that the upfront added to the value of the premium leg with the contractual spread S_c (100 or 500 bps) matches the protection leg value. It is therefore straightforward to convert the running spread into an upfront and a new given fixed contractual spread. The upfront is simply the present value of the payer CDS contract having the new contractual (100 or 500bps) spread in the premium leg. Given the hazard rate curve $h(\cdot)$ calibrated to the running CDS spreads for several maturities, the market running spread $S_{0,n}$, and the contractual spread S_c, the upfront for a maturity T_n is simply

$$Upfr_{0,n} = ProtecLeg_{0,n}(LGD; D(0,\cdot), H(\cdot)) - S_c DV01_{0,n}(D(0,\cdot), H(\cdot)).$$

or, equivalently,

$$Upfr_{0,n} = (S_{0,n} - S_c)\ DV01_{0,n}(D(0,\cdot), H(\cdot))$$

3.2. Upfront to Running with a Consistent Term Structure of Hazard Rates

If we have the upfront for several maturities and we wish to move to running spread taking into account the term structure of CDS consistently, we need first to strip hazard rates from the spanning upfront quotes, and then use the hazard rates to obtain the runnings. This is done as follows: Solve in $h(\cdot)$ with $H(t) = \int_0^t h(s)\,ds$ the equations

$$Upfr_{0,n} + S_c\ DV01_{0,n}(D(0,\cdot), H(\cdot)) = ProtecLeg_{0,n}(LGD; D(0,\cdot), H(\cdot))$$

If we are given $Upfr_{0,n}^{mkt\ MID}$ for different maturities T_n, we can assume as before a *piecewise constant* h, and invert prices in an iterative way as T_n increases, deriving at each time the new part of h that is consistent with the Upfr for the new increased maturity.

Once this is done, the running spread can be readily obtained as

$$S_{0,n} = \frac{ProtecLeg_{0,n}(LGD; D(0,\cdot), H(\cdot))}{DV01_{0,n}(D(0,\cdot), H(\cdot))} = \frac{Upfr_{0,n}}{DV01_{0,n}(D(0,\cdot), H(\cdot))} + S_c.$$

Besides being useful for a consistent conversion across term, this tool allows to derive a model consistent with several upfront quantities on different maturities at the same time, so that we can be able to properly handle a portfolio of CDS's across several maturities.

3.3. Conversion Using a Flat Hazard Rate (FHR)

In order to standardize the conversion from running fair spreads (par spreads) to upfronts, BGJM/ISDA (2009) suggested a conversion method, which is reasonably robust. C++ code is provided so that market participants are able to adopt this method.

However, this method suffers from two important drawbacks: first, it is inconsistent across maturities, and second, it leads to different results even when converting single CDS deals between running and upfront when compared to the consistent method.

As a result of this, the method works as a rule-of-thumb metric to uniquely if inconsistently convert traded upfront prices into a semblance of running spreads that are not meant to be spreads of actually traded running CDS. This method works for translating upfront to running spread for a single maturity. The "model" is not intended to price CDS for any other maturity or a portfolio of CDS and thus we are not dealing with a CDS model as mentioned before. In the proposal, the "model" is calibrated to a single upfront CDS quote for a specific maturity (eg $\text{Upfr}_{0,5Y}$). It can only be seen as consistent in the absence of any other quotes for earlier maturities. If other such quotes exist (eg $\text{Upfr}_{0,3Y}$), then they give information about the default probability of the reference entity for an overlapping period of time and this information should be accounted for consistently, which is not possible under the flat hazard rate paradigm.

Example 1 (Inconsistency of the flat hazard rate framework when used for more than one maturity)

The Upfront $\text{Upfr}_{0,5Y}$ *gives information about default over zero to five years and* $\text{Upfr}_{0,3Y}$ *gives information over zero to three years. Suppose there is not a* $2y$ *upfront CDS quoted but that we need to price a 2y running CDS. The proposed methodology is powerless and should not be used since it works only on a single deal level and there is no 2y upfront deal. However, if one would try to force the methodology, one could first use the 3y upfront to get a flat hazard rate translating into a 3y running, and then use that hazard rate to compute the 2y running. Or one could do exactly the same thing with the 5y upfront, getting a flat hazard rate for the 5y running calculation and then use this to compute the 2y running. This would lead to two different running spreads for the 2y, one based on the 3y flat hazard rate and one based on the 5y one. Other examples are possible: suppose the market quotes the 3y and 5y upfront but we need the 4y running. What to do?*

The only possible reasonable method is to strip a term structure of hazard rates consistent with the 3y and 5y at the same time and then pricing the 2y (or 4y) running CDS. This would be consistent. However, if some not too careful investor applies the above FHR methodology, this can lead into ambiguous results. Furthermore, for hedging a portfolio of upfront CDS, the methodology is also quite powerless.

To phrase the flat hazard rate method in terms of our setup, let $\bar{T}$ be the maturity for which we transform the upfront U with a fixed spread S_c into a fair spread S.

The major assumption is that we are using a flat hazard rate h for the time interval $[0,\overline{T}]$. So we are looking for $h>0$ which satisfies $H(T)=hT$ for all T and such that

$$U+S_c\sum_{i=1}^{n}\left[D(0,T_i)\Delta_i e^{-H(T_i)}-D\left(0,\frac{T_i+T_{i-1}}{2}\right)\frac{\Delta_i}{2}(e^{-H(T_i)}-e^{-H(T_{i-1})})\right]$$

$$=(1-R)\sum_{t_j=0}^{\overline{T}}D\left(0,\frac{t_{j+1}+t_j}{2}\right)(e^{-H(t_j)}-e^{-H(t_{j+1})}).$$

This can be calculated by an ordinary root searcher. After obtaining h we get the running S by

$$S=\frac{(1-R)\sum_{t_j=0}^{\overline{T}}D\left(0,\frac{t_{j+1}+t_j}{2}\right)(e^{-H(t_j)}-e^{-H(t_{j+1})})}{\sum_{i=1}^{n}\left[D(0,T_i)\Delta_i e^{-H(T_i)}-D\left(0,\frac{T_i+T_{i-1}}{2}\right)\frac{\Delta_i}{2}(e^{-H(T_i)}-e^{-H(T_{i-1})})\right]}. \quad (9)$$

This however leads to differences with the consistent framework above. In other terms, **even at single deal level the fair running CDS spread calculated with the rough and CDS-term-inconsistent flat hazard rate is different from the running CDS spread calculated with the CDS-term-consistent hazard rate curve**. The difference can be considerable in presence of a strong patterns of the term structure of upfront CDS quotes by maturity. It is therefore clear that this does not involve a problem as long as running CDS are not traded, but if they were, the potential confusion this can create in the market would certainly be a concern. An example of confusion is the following.

Example 2 (Investor with existing pre-upfront CDS libraries based on running spreads).

We consider a hypothetical investor who has developed libraries to strip a term structure of hazard rates from running spreads across maturities. These hazard rates are used as basic modeling tools in pricing other credit derivatives, counterparty risk and other products involving credit features.

With the market switching to upfront CDS, and with input becoming upfront quotes, this investor would now have two choices.

- *The first choice would be to strip directly hazard rates from the upfront CDS of a name from several maturities.*
- *The second choice would be to use the proposed FHR methodology to convert upfront CDS's into running, and then put the FHR converted running spread into the old libraries based on running CDS inputs*

The two procedures would not produce the same result. The only consistent procedure would be the first one, whereas the second procedure would produce a term structure of hazard rates that are inconsistent with the originally traded upfront CDS. This would be a problem when pricing other products.

3.4. The Role of Recovery and Problems with the 20% and 40% Choices

In the proposed conversion method one is not free to choose the recovery. Instead, it is fixed at 40% for senior and 20% for subordinated. In case the contract to be converted featured a different recovery, part of the difference would be absorbed by the flat hazard rate. That is, if the market consensus recovery was 50%, but 40% was used in the conversion, then the hazard rate (and hence the default probability) must move down to balance the larger loss incurred on default. This change in modeling quantities will, of course, affect the conversion.

A potential problem with the conversion method is when the upfront is very high. In some such cases the conversion method would fail to produce a corresponding positive running spread. For example, if the upfront to be paid by the protection buyer is 81%, then converting with the proposed fixed recovery rate of 20% will not work. The only possibility to get a positive flat hazard rate to do the conversion is to lower the recovery rate. Highly distressed names were recently observed for American automobile producers (March 2009).

For cases when the conversion method fails, we suggest to use a third possible value of 0% for the recovery Rate R, if we have to stick to a limited set of recovery scenarios. Since in the conversion we are calculating the hazard rate to one maturity only, a realistic case of a failing conversion method is when $Upr + R > 1$, which can be fixed for $Upr \in [0,1)$ by setting $R = 0$. In fact, a sufficient condition for the upfront paid by the protection buyer not to be convertible into a running spread with the proposed methodology is $Upr + R > 1$. This stems from the default or protection leg of a CDS being bounded from above by $1 - R$. This is why the adoption of a third recovery value, 0, would ease matters in this respect. Indeed, one can show that $Upr + R < 1$ is a sufficient condition to guarantee existence of a flat hazard rate for the upfront paid by the protection buyer.

4. Numerical Examples

We produce examples of conversion from upfront to running and vice versa with the fully consistent model and with the FHR "model". We take market spreads to highlight the differences: we take 10 years final maturity, and we assume flat and 0 interest rates (discounts $D(t,T)$ all equal to 1).

4.1. From Upfront to Running

We start with a term structure of upfront quotes (see Table 1). We put ourselves in the context of Example 2 above, looking at an investor having an old library and stripping hazard rates[4] from a term structure of running CDS spreads and receiving now upfront quotes from the market. We assume recovery in all CDS to be 40%. We aim at comparing the two procedures:

a) Strip hazard rates directly from the upfront, then using those to compute the running spreads.

b) Convert the upfront to running using the FHR methodology.

[4] See for example Brigo and Mercurio (2006)

An unaware investor who would then feed the running spreads obtained in (b) to a hazard rate stripper would find hazard rates quite different from the correct ones obtained in (a). The differences in the running spreads from the consistent and the FHR conversion are shown in Table 2. It is clear that the differences are relevant, ranging from about 4 to 75 basis points.

Table 1: Term structure of upfronts for the four reference entities used in the examples

	Upfronts					Recovery
(%)	20 Jun 10	20 Jun 12	20 Jun 14	20 Jun 16	20 Jun 19	Rate
ArcelorMittal Finance SCA	-8.66	-14.79	-17.38	-18.38	-18.06	40
Continental AG	-16.85	-23.15	-25.51	-26.03	-25.94	40
American International Group Inc.	-25.29	-32.58	-34.92	-35.56	-36.44	35
Hitachi, Ltd.	-0.72	-3.00	-5.75	-8.10	-11.80	35

Table 2: Fair and conventional spreads for maturity 20 Jun 2019

	Proper mechanism				
	Rec (%)	Fair Spread	Rec (%)	Conventional	Difference
ArcelorMittal Finance SCA	40	852.57	40	827.17	25.40
Continental AG	40	1,112.90	40	1,037.78	75.12
American International Group Inc.	35	1,523.00	40	1,467.23	55.77
Hitachi, Ltd.	35	234.80	40	238.72	-3.92

4.2. From Running to Upfront

We now consider the case of an investor who has running market quotes for a CDS at multiple maturities and some libraries in place to strip a term structure of hazard rates consistent with all quotes. We aim to compare the differences arising in pricing the corresponding upfront CDS when using either FHR or the proper consistent term structure of hazard rates.

Table 3 shows four corporates as quoted on the 25th March 2009. The PVs for the last maturity obtained using the proper mechanism and the proposed conversion mechanism are given in table 4. The third table in this figure shows the differences between the two, which are as much as 4.17% in our examples. It is clear that the two calculations result in material differences in the PV and thus using the suggested conversion method with any other running CDS model might cause some severe inconsistencies.

Table 3: Term structure of spreads for the four reference entities used in the examples

	Spreads					Recovery
	20 Jun 10	20 Jun 12	20 Jun 14	20 Jun 16	20 Jun 19	Rate (%)
ArcelorMittal Finance SCA	1,287.00	1,109.86	1,009.57	938.57	852.57	40
Continental AG	2,168.64	1,607.98	1,388.67	1,245.89	1,112.90	40
American International Group Inc.	3,197.28	2,274.91	1,913.48	1695.9	1,523.00	35
Hitachi, Ltd.	157.92	195.57	217.21	223.87	234.8	35

Table 4: Present values for a maturity of 20 Jun 2019 using the proper mechanism and the proposed conversion mechanism for a notional of 10,000,000 and a zero upfront payment.

	Proper mechanism				
Name	Fixed Spread	Rec (%)	PV	Premium PV	Protection PV
ArcelorMittal Finance SCA	500	40	1,806,384.70	2,561,739.08	4,368,123.78
Continental AG	500	40	2,593,569.97	2,115,818.22	4,709,388.18
American International Group Inc.	500	35	3,643,573.65	1,780,827.79	5,424,401.43
Hitachi, Ltd.	100	35	1,180,156.83	875,487.26	2,055,644.09
	Conversion mechanism				
Name	Fixed Spread	Rec (%)	PV	Premium PV	Protection PV
ArcelorMittal Finance SCA	500	40	1,914,127.72	2,714,535.72	4,628,663.44
Continental AG	500	40	2,823,185.85	2,303,137.42	5,126,323.27
American International Group Inc.	500	40	3,741,810.30	1,828,841.79	5,570,652.09
Hitachi, Ltd.	100	40	1,150,463.77	853,459.77	2,003,923.55
	Proper - Conversion (as %)				
Name	Fixed Spread	Rec	PV	Premium PV	Protection PV
ArcelorMittal Finance SCA	500	40	-1.08	-1.53	-2.6
Continental AG	500	40	-2.30	-1.87	-4.1
American International Group Inc.	500	40	-0.98	-0.48	-1.4
Hitachi, Ltd.	100	40	0.30	0.22	0.5

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