# Multi Currency Credit Default Swaps
# Quanto effects and FX devaluation jumps

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# Default: Reduced-form intensity modelling & Cox process

Let us consider a probability space $(\Omega, \mathcal{G}, \mathbb{Q}, (\mathcal{G}_t))$ satisfying the usual hypothesis and a process $(\lambda_t, t \geqslant 0)$, the intensity, defined on this space. Let $\Lambda(s) = \int_0^s \lambda_u du$.

Default is a jump process $(D_t, t \geqslant 0)$ with the property that $\lambda_t$ is the $\mathcal{G}_t-$intensity of $D$. We only focus on the first jump time of $D$ and we call it $\tau$, the default time.

Let us consider

- the filtration generated by $\lambda$ and "default-free processes" (eg short rate $r_t$), $(\mathcal{F}_t)$;
- the filtration generated by $D$, $\mathcal{H}_t = \sigma((\tau < u), u \leqslant t)$ ("default monitoring")
- separable filtration assumption: Total filtration $\mathcal{G}_t = (\mathcal{F}_t) \vee (\mathcal{H}_t)$;
- a r.v. $\xi \sim \exp(1)$ *"jump to default risk"*, that is independent of $(\mathcal{F}_t)$.

Let us define

$$\tau := \Lambda^{-1}(\xi)$$

assuming $\lambda > 0$. From the default time definition we get that

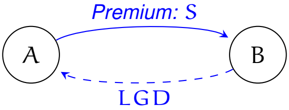
$$\mathbb{Q}(\tau \in [t, t+dt) | \tau > t, \mathcal{F}_t) = \lambda_t \, dt, \quad (\lambda_t \, dt \text{ is a } \textit{"local default probability"});$$

$$\mathbb{Q}(\tau > T | \mathcal{F}_t) = \mathbb{E}\left[\mathbb{Q}\left(\int_0^T \lambda_s \, ds < \xi \middle| \mathcal{F}_T\right) | \mathcal{F}_t\right] = \mathbb{E}\left[e^{-\int_0^T \lambda_s \, ds} | \mathcal{F}_t\right] \quad (\lambda \text{ also } \textit{"credit spread"}).$$

# Credit Defaul Swaps

A CDS is a contract between two parties $A$ and $B$ written with respect to a set of securities issued by the reference entity $C$ where

- before default of the reference entity or until a final maturity, one party [protection buyer] pays the other [protection seller] a protection premium; this can be paid upfront, running or both;
- upon default of the reference entity, if this happens before the final maturity, the protection seller pays the buyer a *loss given default* on the reference entity's securities.




For a running CDS with premium spread $S^c$, the premium and the protection leg cash flows discounted back at time 0 and not yet present valued are given respectively by

$$\Pi^{\text{Premium}} = S^c \sum_{i=0}^{N} \mathbb{1}_{\tau > T_i} \alpha_i D^{ccy}(0, T_i) + \text{accrual-term}, \quad \Pi^{\text{Protection}} = \text{LGD}\, \mathbb{1}_{\tau \leqslant T_N} D^{ccy}(0, \tau)$$

where accrual-term $= S^c(\tau - T_{\text{coupon before } \tau}) D^{ccy}(0, \tau) 1_{\tau < T_N}$ and

- $(T_0, \ldots, T_N)$ quarterly spaced payment times, $\alpha_i$ year fraction between $T_{i-1}$ & $T_i$;
- $D^{ccy}(t, T)$ stochastic discount factor for currency $ccy$ at time $t$ for maturity $T$;

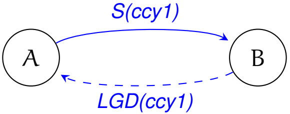
# Multi-currency CDS

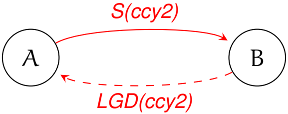
Running CDS is often quoted via the spread $S^c$ that matches the two legs ("par spread").

CDSs on a given entity can be traded in different currencies.

We will consider the following currencies:

- for each CDS we denote the currency in which premium leg and protection leg are settled as the contractual currency;
- for each reference entity we denote the contractual currency corresponding to the most liquid CDS in the market as the liquid currency.




One would prefer to buy protection against the default of Italy in USD rather than EUR.

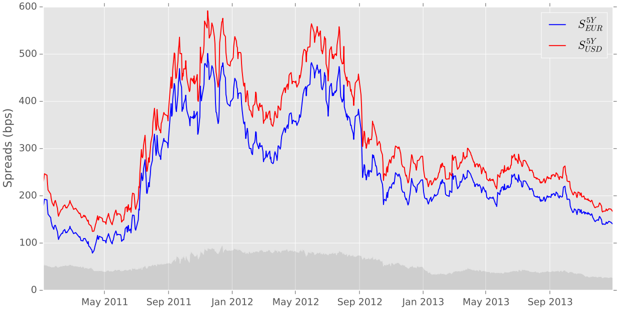
We will always take the liquid currency economy as the reference pricing measure.

When CDSs are traded in contractual currencies different from the liquid one, a joint model for the reference entity's credit worthiness and the FX rate is needed to price the basis between the par-spreads.

# A look at the market

## Italy's case

1. Italy's CDS in EUR:
   - contractual currency: EUR
   - liquid currency: USD
2. Italy's CDS in USD:
   - contractual currency: USD
   - liquid currency: USD



# Mathematical framework for multi-currency pricing

We will consider the case where the contractual and liquid currencies are different. Consider

- a risk neutral measure $\mathbb{Q}$ with numeraire $(B_t, t \geqslant 0)$ with

$$dB_t = r(t)B_t\,dt, \quad B_0 = 1$$

  is associated to the liquid currency:
- a risk-neutral measure $\hat{\mathbb{Q}}$ with numeraire $(\hat{B}_t, t \geqslant 0)$ where

$$d\hat{B}_t = \hat{r}(t)\hat{B}_t\,dt, \quad \hat{B}_0 = 1$$

  is associated to the contractual currency;
- an exchange rate $(Z_t, t \geqslant 0)$ between the currencies of the two economies ($Z$ is the price of one unit of the contractual currency in the liquid currency);
- interest rates are deterministic functions of time, although we will keep notation general in view of generalizations

# Mathematical framework for multi-currency pricing

Recall: $\hat{B}$ contractual currency, $B$ liquid currency.
The link between the two measure is provided by the Radon-Nikodym derivative $L_T := \frac{d\hat{\mathbb{Q}}}{d\mathbb{Q}}|_{\mathcal{G}_T}$. This can be calculated using a change of numeraire argument and a generic function $\phi_T$ representing a payoff denominated in the liquid currency. The price of the liquid currency payoff $\phi$ is then $\mathbb{E}_t\left[\frac{B_t}{B_T}\phi_T\right]$, satisfying the usual

$$\frac{1}{Z_t}\mathbb{E}_t\left[\frac{B_t}{B_T}\phi_T\right] = \hat{\mathbb{E}}_t\left[\frac{\hat{B}_t}{\hat{B}_T}\frac{\phi_T}{Z_T}\right] \Rightarrow \mathbb{E}_t\left[\frac{B_t}{B_T}\phi_T\right] = \hat{\mathbb{E}}_t\left[\frac{\hat{B}_t Z_t}{\hat{B}_T Z_T}\phi_T\right] = \mathbb{E}_t\left[\frac{\hat{B}_t Z_t}{\hat{B}_T Z_T}\phi_T \frac{d\hat{\mathbb{Q}}}{d\mathbb{Q}}|_{\mathcal{G}_T}\right]$$

“First discount liquid, then price liquid, then change to contractual=
=first change to contractual, then discount contractual, then price contractual"
The liquid-ccy expectation can be rewritten also as

$$\mathbb{E}_t\left[\frac{B_t}{B_T}\phi_T\right] = \mathbb{E}_t\left[\frac{B_t \hat{B}_T Z_T}{B_T \hat{B}_t Z_t}\frac{\hat{B}_t Z_t}{\hat{B}_T Z_T}\phi_T\right].$$

From there, we can deduce the Radon-Nikodym derivative to be

$$L_t = \frac{Z_t \hat{B}_t}{Z_0 B_t}, \quad L_0 = 1,$$

# FX rates no-arbitrage dynamics

The fact that $(L_t, t \geqslant 0)$ is a $\mathcal{G}$-martingale in $\mathbb{Q}$ can be used to deduce no-arbitrage constraints. For example, if $r$ and $\hat{r}$ are deterministic functions of time and if $(Z_t, t \geqslant 0)$ is a GBM

$$dZ_t = \mu^Z Z_t \, dt + \sigma Z_t \, dW_t, \quad Z_0 = z,$$

we can deduce the dynamics of the Radon-Nikodym derivative $L$ as

$$\begin{aligned} dL_t = d\left(\frac{\hat{B}_t}{B_t}\frac{Z_t}{Z_0}\right) &= \frac{\hat{B}_t}{B_t Z_0}(dZ_t + \hat{r}Z_t \, dt - rZ_t \, dt), \\ &= \frac{\hat{B}_t}{B_t Z_0}(\mu^Z Z_t \, dt + \sigma Z_t \, dW_t + \hat{r}Z_t \, dt - rZ_t \, dt), \quad L_0 = 1. \end{aligned}$$

The martingale condition on $L$ is given by $\mathbb{E}_t\left[dL_t\right] = 0$, from which we can deduce the FX drift rate:

$$\mu^Z = r(t) - \hat{r}(t)$$

# Symmetry for FX rates

What if we had started from $\frac{d\hat{\mathbb{Q}}}{d\mathbb{Q}}|_{\mathcal{G}_T}$ and from the reciprocal FX rate, $(X_t, t \geqslant 0)$ where $X = {}^1/z$, instead?

- If $(Z_t, t \geqslant 0)$ (or $X$) are GBM, it doesn't matter which RN derivative we start from. If, for example, we first start from $L$, from which we calculate the no-arbitrage drift for $Z$ as above $(r - \hat{r})$, we subsequently deduce $X$'s dynamics though Ito's formula for $1/Z$, and we finally use Girsanov's theorem to move $X$'s dynamics under the measure $\hat{\mathbb{Q}}$, we obtain
$$\mu^X = \hat{r}(t) - r(t)$$
- This is the same drift we would have obtained postulating a GBM for $X$ and deriving its drift starting from $\frac{d\hat{\mathbb{Q}}}{d\mathbb{Q}}|_{\mathcal{G}_T}$.
- The same cannot be guaranteed to happen for other type of stochastic process for $Z$ or $X$ (e.g. stoch vol in FX, CEV...).

# Quanto survival probabilities

- We start from the price of defaultable zero-coupon bond:

$$\hat{\mathbb{E}}_t\left[\frac{\hat{B}_t}{\hat{B}_T}\,\mathbb{1}_{\tau>T}\right] = \mathbb{E}_t\left[\frac{\hat{B}_t}{\hat{B}_T}\,\mathbb{1}_{\tau>T}\,\frac{d\hat{\mathbb{Q}}}{d\mathbb{Q}}\right] =$$

This can be written as

$$= \frac{B(t,T)}{Z_t}\mathbb{E}_t\left[Z_T\mathbb{1}_{\tau>T}\right] \quad (*),$$

where $B(t,T) = D(t,T) = B_t/B_T$ is the discount factor from time T to time $t \leqslant T$. Remember we are assuming deterministic risk free discount rates.

- We define the quanto-adjusted survival probability as

$$\hat{p}_t(T) := \frac{\hat{\mathbb{E}}_t\left[\frac{\hat{B}_t}{\hat{B}_T}\,\mathbb{1}_{\tau>T}\right]}{\hat{B}(t,T)}$$

- We will often use $(U_t, t \geqslant 0)$

$$U_t := Z_t\hat{\mathbb{E}}_t\left[\frac{\hat{B}_t}{\hat{B}_T}\,\mathbb{1}_{\tau>T}\right] = by(*)above = B(t,T)\mathbb{E}_t\left[Z_T\mathbb{1}_{\tau>T}\right]$$

which is therefore a $\mathbb{Q}$-price and as such has drift $rUdt$.

# Default intensity modelling

Although for credit spreads the square root processes are better in terms of tractability and closed form solutions (B. et al [2, 3]), for lognormal consistency in this work we model the intensity $\lambda$ under the liquid measure $\mathbb{Q}$ as an exponential Ornstein-Uhlenbeck process

$$\begin{aligned} \lambda_t &= e^{Y_t} \\ dY_t &= a(b - Y_t)dt + \sigma^Y dW_t^Y \qquad\qquad Y_0 = y, \end{aligned}$$

which leads to the default time $\tau = \Lambda^{-1}(\xi)$, $\xi \sim \exp(1)$ and recall

- $\lambda$ is instantaneous credit spread, $\lambda_t dt$ local default probability in $[t, t + dt)$;
- $\xi$ is jump to default risk.

Dependence between the credit component and the FX component is modeled though instantaneous correlation, $\rho$,

$$\begin{aligned} dZ_t &= \mu^Z Z_t dt + \sigma^Z Z_t dW_t^Z, \qquad\qquad Z_0 = z; \\ d\langle W^Y, W^Z\rangle_t &= \rho\, dt \end{aligned}$$

# Impact of correlation

- Example: CDS on Italy, EUR=Contractual ccy, USD= liquid ccy.
- $Z$ is the amount of USD needed to get one unit of EUR.
- If $\rho$ negative $\Rightarrow$ intensity tends to grow when FX rate $Z$ decreases.
- When $\lambda \uparrow$ default of Italy becomes more likely and the amount of USD needed to get one EUR will tend to decrease (EUR devaluation), so that the EUR protection offered by the CDS will be worth less when benchmarked against the liquid USD CDS
- This results in a lower par spread for the EUR CDS.
- Positive correlation will lead to a larger par spread.
- More generally, ceteris paribus, we expect the par spread to increase with the correlation.

# The pricing equation

The dependency of survival probability value of the default event that is given by the additional process $D_t = \mathbb{1}_{\tau<t}$ must be explicitly considered. $(U_t, t \geqslant 0)$ can be represented as some function $f(t, X_t, Y_t, D_t)$ and we have (Itô's lemma with jumps)

$$\begin{aligned} dU_t = {} & rf\,dt + \partial_t f\,dt + \partial_z f\left(\mu^Z z\,dt + \sigma^Z z\,dW_t^Z\right) + \partial_y f\left(a(b - Y_t)\,dt + \sigma^Y\,dW_t^Y\right) \\ & + \frac{1}{2}\left(\sigma^Z z\right)^2 \partial_{zz} f\,dt + \frac{1}{2}\left(\sigma^Y\right)^2 \partial_{yy} f\,dt + \rho\sigma^Z\sigma^Y z\partial_{zy} f\,dt + \Delta f\,dD_t, \end{aligned}$$

where $\Delta f = f(t, x, y, 1) - f(t, x, y, 0)$ is the jump to default of the survival probability. To write the pricing equation, it has to be noted that $D$ is not a martingale, so its compensator must be calculated. The process $(M_t, t \geqslant 0)$ given by

$$M_t = D_t - \int_0^t (1 - D_s)\lambda_s\,ds \tag{1}$$

is a $\mathcal{G}$-martingale in $\mathbb{Q}$.
Imposing that the drift of $dU$ to be $rU_t = rf$ yields

# The pricing equation

$$\partial_t f + \mu^Z z \partial_z f + a(b - Y_t)\partial_y f + \frac{1}{2}\left(\sigma^Z z\right)^2 \partial_{zz} f \\ + \frac{1}{2}\left(\sigma^Y\right)^2 \partial_{yy} f + \rho \sigma^Z \sigma^Y z \partial_{zy} f + e^y (1 - d)\Delta f = 0,$$

The PDE for $f$ can be solved it in two steps:

1. calculating first $u(t, x, y) := f(t, x, y, 1)$;
2. using $u$ to solve for $v(t, x, y) := f(t, x, y, 0)$

They only get coupled through the term $e^y(1 - d)\Delta f$ and that term only enters the $v-$equation. Final conditions for the two functions are respectively given by

$$v(T, z, y) = f(T, z, y, 0) = z;$$
$$u(T, z, y) = f(T, z, y, 1) = 0$$

(see Jeanblanc et al [1] for a more general discussion).

# Two pricing equations

The PDE for $u$ is given by

$$\begin{aligned}\partial_t u &= -\mu^Z z \partial_z u - a(b-y)\partial_y u - \frac{1}{2}\left(\sigma^Z z\right)^2 \partial_{zz} u \\ &\quad - \frac{1}{2}\left(\sigma^Y\right)^2 \partial_{yy} u - \rho \sigma^Z \sigma^Y z \partial_{zy} u \\ u(T, z, y) &= 0.\end{aligned}$$

and leads to $u \equiv 0$. The PDE for $v$ is then given by ($\Delta f = v$):

$$\begin{aligned}\partial_t v &= -\mu^Z z \partial_z v - a(b-y)\partial_y v - \frac{1}{2}\left(\sigma^Z z\right)^2 \partial_{zz} v \\ &\quad - \frac{1}{2}\left(\sigma^Y\right)^2 \partial_{yy} v - \rho \sigma^Z \sigma^Y z \partial_{zy} v + e^y\, v \\ v(T, z, y) &= z.\end{aligned}$$

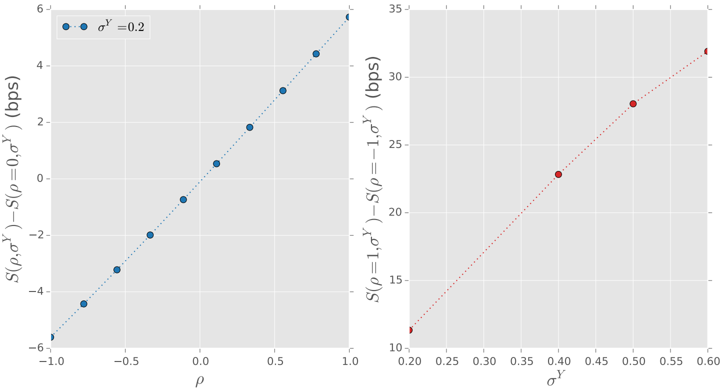
# The pricing equation

$u$ and $v$ can be interpreted as pre-default and post-default values of a derivative with payoff function $\phi$:

$$\begin{aligned} f(t,x,y,d) &= \mathbb{E}_t\left[\phi(X_T,Y_T,D_T)|X_t=x,Y_t=y,D_t=d\right] \\ &= \mathbb{1}_{\tau>t}\mathbb{E}_t\left[\phi(X_T,Y_T,D_T)|X_t=x,Y_t=y,D_t=0\right] \\ &+ \mathbb{1}_{\tau\leqslant t}\mathbb{E}_t\left[\phi(X_T,Y_T,D_T)|X_t=x,Y_t=y,D_t=1\right] = \mathbb{1}_{\tau>t}v(t,x,y)+\mathbb{1}_{\tau\leqslant t}u(t,x,y) \end{aligned}$$

We will solve numerically the PDEs by using an *Alternating Directions Implicit* scheme variant, accurate fourth order in $x$ and second order in $t$ (see [6]).

# Limits of instantaneous correlation on Quanto CDS par spreads $S$




| $z$ | $\mu$ | $\sigma^Z$ | $a$ | $b$ | $y$ | $T$ |
|---|---|---|---|---|---|---|
| 0.8 | 0.0 | 0.1 | 0.0001 | 204.0 | -4.089 | 5.0 |

Italy CDS. Contractual currency: EUR. Liquid Currency: USD. We see that even the maximum excursion of correlation cannot explain the discrepancy between CDS spreads in two currencies, since differences in the markets can reach much larger ranges than what we see here (case of Italy).

# Adding Jump to default: Direct Default-FX contagion

We can account for the devaluation factor directly in the dynamics of $(Z_t, t \geqslant 0)$ by considering

$$dZ_t = \bar{\mu}^Z Z_t \, dt + \sigma^Z Z_t \, dW_t^Z + \boxed{\gamma Z_{t-} \, dD_t}, \quad Z_0 = z,$$

- Take again the example of Italy.
- Now, if Italy defaults, a negative $\gamma$ would push Z down with a jump.
- The amount of USD needed to buy 1 EUR jumps down, we have a intantaneous devaluation jump for the EUR, which makes sense in a scenario of default of Italy.
- As a consequence the CDS offering protection in EUR will be worth much less when benchmarked with that in USD, so that we expect the par spread to go down with a negative $\gamma$.
- A less negative or positive $\gamma$ would instead give us a larger par spread.
- We expect the par spread to increase with $\gamma$.

# Adding Jump to default: Direct Default-FX contagion

Re-doing calculations as above but accounting for the new jump term in $Z$:

$$
\begin{aligned}
dU_t = {} & rf\,dt + \partial_t f\,dt + \partial_z f\left(\bar{\mu}_Z z\,dt + \sigma^Z z\,dW_t^{(2)} + \gamma^Z z\,dD_t\right) + \partial_y f\left(a(b - Y_t)\,dt + \sigma^Y dW_t^{(1)}\right) \\
& + \frac{1}{2}\left(\sigma^Z z\right)^2 \partial_{zz} f\,dt + \frac{1}{2}\left(\sigma^Y\right)^2 \partial_{yy} f\,dt + \rho\sigma^Z\sigma^Y z\partial_{zy} f\,dt + \Delta f\,dD_t - \partial_z f \Delta Z_t.
\end{aligned}
$$

The pricing equation associated to that is

$$
\begin{aligned}
\partial_t v &= -(r - \hat{r})z\partial_z v - a(b - y)\partial_y v - \frac{1}{2}\left(\sigma^Z z\right)^2 \partial_{zz} v \\
&\quad - \frac{1}{2}\left(\sigma^Y\right)^2 \partial_{yy} v - \rho\sigma^Z\sigma^Y z\partial_{zy} v + e^y\left(v - \gamma^Z z\partial_z v\right) \\
v(T, z, y) &= z,
\end{aligned}
$$

# Hazard rate's dyamics in $\hat{\mathbb{Q}}$

So far we always worked in the benchmark liquid currency measure $\mathbb{Q}$.
However there is an important approximation where we can benefit from deriving the contractual currency measure $\hat{\mathbb{Q}}$ dynamics for the intensity. By Girsanov's theorem:

$$
\begin{aligned}
d\hat{M}_t &= dM_t - \frac{d\langle M, L\rangle_t}{L_t} = dM_t - d\langle D, \gamma^Z D\rangle_t \\
&= dM_t - (1 - D_t)\gamma^Z \lambda_t \, dt \\
&= dD_t - (1 - D_t)(1 + \gamma^Z)\lambda_t \, dt
\end{aligned}
$$

from which the intensity of the default process in $\hat{\mathbb{Q}}$ is given by

$$\hat{\lambda}_t = (1 + \gamma^Z)\lambda_t$$

This is important because there are cases when a CDS par-spread can be suitably approximated by $S \approx (1 - R)\lambda$ (e.g. deterministic constant hazard rate models). In such cases, dividing both sides by LGD=1-R, which is fixed at the same level in both currencies by the auction (Doctor et al. (2010) [5]), an approximated relation can be written for the par-spreads of the contractual and liquid CDSs as

$$\boxed{\hat{S} = (1 + \gamma^Z)S}$$

# FX Symmetry

## Jump diffusion

- Requiring that $(L_t, t \geqslant 0)$ is a $\mathcal{G}$-martingale in $\mathbb{Q}$ leads to

$$\bar{\mu}^Z = r(t) - \hat{r}(t) - \lambda_t \gamma^Z \mathbb{1}_{\tau > t}$$

- The dynamics in $\hat{\mathbb{Q}}$ of $(X_t, t \geqslant 0)$ can be calculated starting from the dynamics of $(Z_t, t \geqslant 0)$

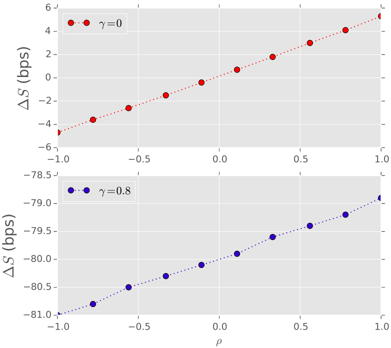
$$dX_t = (\hat{r} - r) X_t \, dt - \sigma^Z X_t \, d\hat{W}_t^Z + X_{t-} \gamma^X \, d\hat{M}_t, \quad X_0 = \frac{1}{z},$$

where

$$\gamma^X = -\frac{\gamma^Z}{1 + \gamma^Z}$$

- That is a dynamics such that $\frac{d\hat{\mathbb{Q}}}{d\mathbb{Q}}|_{\mathcal{G}_t}$ is a $\mathcal{G}$-martingale in $\hat{\mathbb{Q}}$.

# Jump to default effect on quanto spreads




$$\Delta S := S(\rho, \gamma) - S(0, 0)$$

| $z$ | $\mu$ | $\sigma^Z$ | $a$ | $b$ | $y$ | $\sigma^Y$ | $T$ |
|---|---|---|---|---|---|---|---|
| 0.8 | 0.0 | 0.1 | 0.0001 | -210.0 | -4.089 | 0.2 | 5.0 |

# Jump to default effect on default probabilities

We can link the ratio of quanto-adjusted and single-currency default probabilities and the factor $\gamma$. For

- small T

the two are linked by

$$1 + \gamma = \frac{1 - \hat{p}_t(T)}{1 - p_t(T)}$$

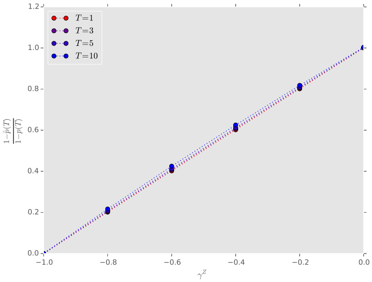



Figure: Low spread (≈ 100bps)

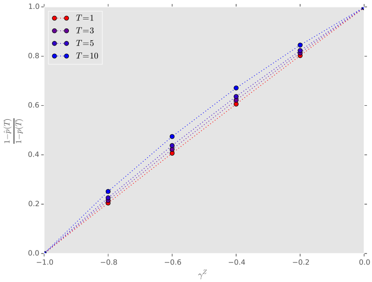
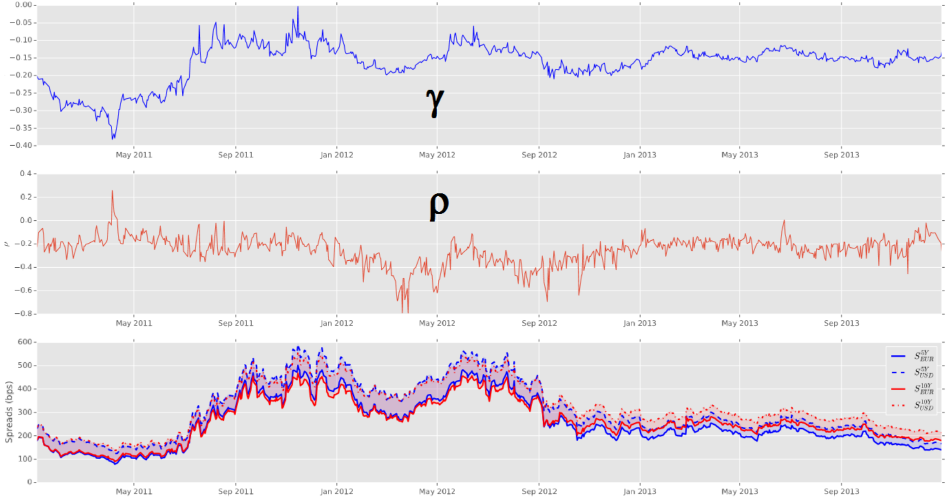



Figure: High spreads, (≈ 700bps).

# CDS spreads on Italy in 2011-2013



# CDS spreads on Italy in 2011-2013

## Linking parameters to market data

Let us recall the change in intensity of default induced by the JTD in the Radon-Nikodym derivative:

$$\hat{\lambda}_t = (1 + \gamma^Z)\lambda_t$$

This relation can be inverted and used as a way to estimate $\gamma$ from market data.

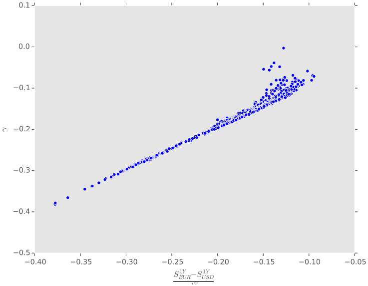



Figure: Relative basis spread for 1Y maturity CDSs.

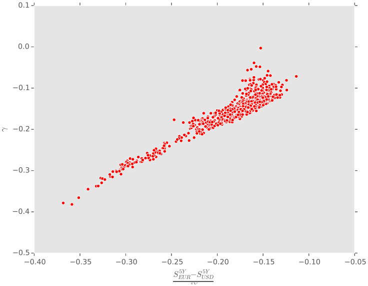



Figure: Relative basis spread for 5Y maturity CDSs.

# Conclusions

Throughout this presentation we showed

- a model that can consistently accounts both for instantaneous correlation between FX and hazard rate and for a devaluation effect;
- that jump-to-default effects in FX rates are needed to account for observed quanto basis in the market;
- how the introduction of jump to default in the RN derivative affects the intensity of the default event in different measures;
- that introducing jump to default effects in the FX rate dynamics does not break the "symmetry";
- Numerical example and a practical recipe to estimate devaluation effect on FX from CDS data.

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