The Science and Art of Valuation

Oxford-Princeton Workshop in Math Finance, May 25–26, 2017

Global Derivatives 2017, Barcelona, May 11

University of Oxford, Quantitative Finance Seminar, 26/11/2015

Fields Institute Seminar, Toronto, October 31th 2014

Plenary 30th anniversary AFFI Conference, Lyon, 28-31 May 2013

2014 International Conference on Financial Engineering & Innovation,

Tongji University,Shanghai, March 14-16

Cambridge, Isaac Newton Institute, March 28, 2013

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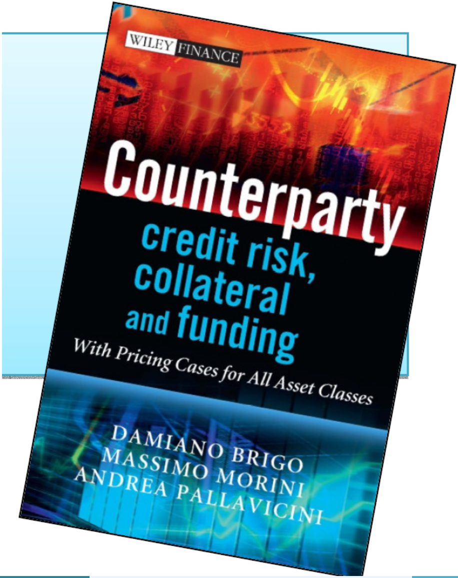
Presentation based on Book (working on 2nd Edition)



A Little History: CVA and DVA can be sizeable. Citigroup:

1Q 2009: “Revenues also included [...] a net 2.5$ billion positive Credit Valuation Adjustment (CVA) on derivative positions, excluding monolines, mainly due to the widening of Citi’s CDS spreads” (DVA)

CVA mark to market losses: BIS

”During the financial crisis, however, roughly two-thirds of losses attributed to counterparty credit risk were due to CVA losses and only about one-third were due to actual defaults.”

Collateral not always effective as a guarantee: B. et al [19]

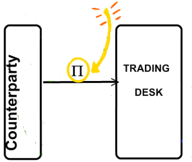
For trades subject to strong contagion at default of the counterparty, like CDS, collateral can leave a sizeable CVA [Initial Margin?]

FVA can be sizeable too. JP Morgan:

Wall St Journal, Jan 14, 2014: ‘[...] So what is a funding valuation adjustment, (FVA) and why did it cost J.P. Morgan Chase $1.5 billion?’

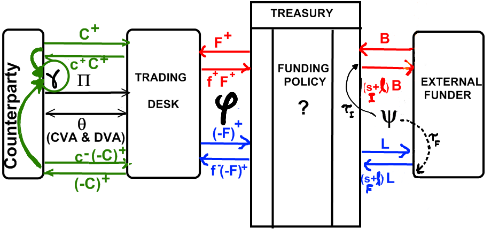
A Quant's trip from the old world to the new one

The old days context:



CLASSIC DISCOUNTED
CASH FLOWS

Now:



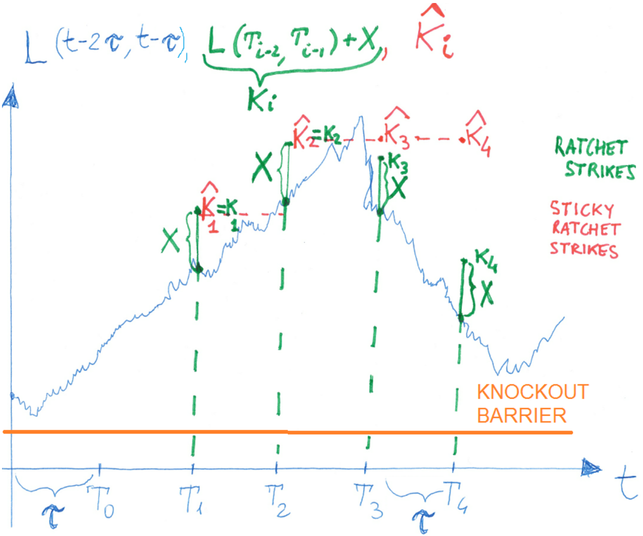
Yesterday $\leftarrow$The Valuation Quant $\rightarrow$ Today

PhD in pure mathematics,
physics... little/no knowledge of:

how the bank works,
regulation, accounting standards,
profit targets, trading costs,
finance & economics.

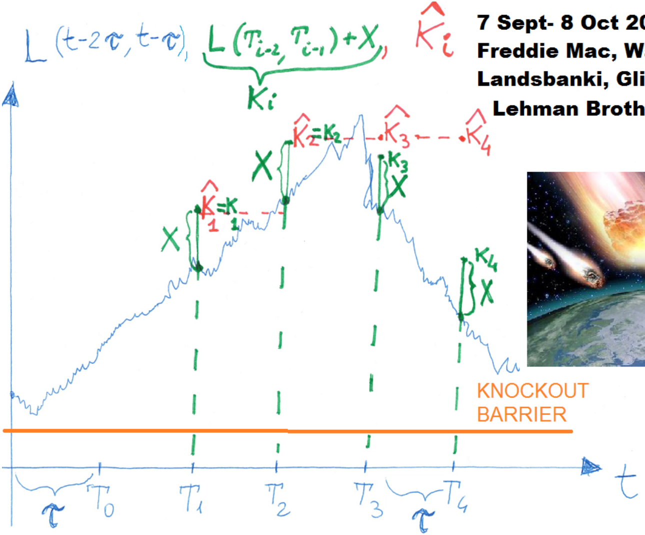
?

Old world specimen: Sticky Ratchet Knockout Cap



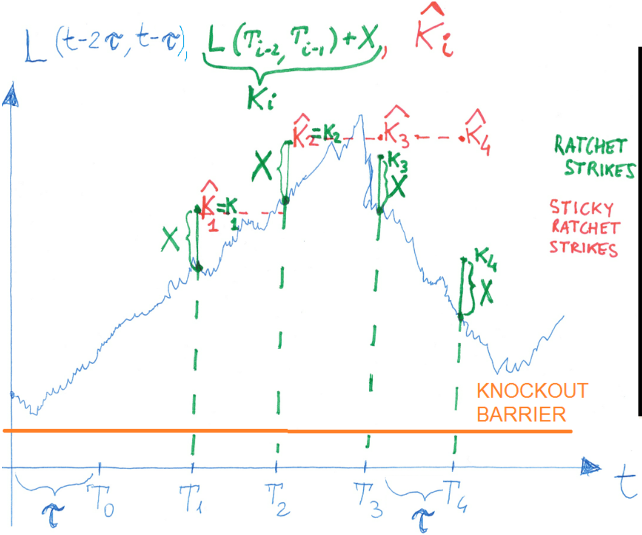
And: **No credit/margin/funding/capital, only basic asset class**

Old world specimen: Sticky Ratchet Knockout Cap

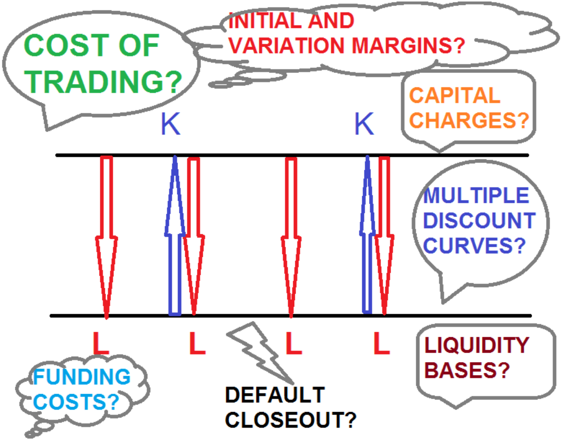


7 Sept- 8 Oct 2008: Fannie Mae, Freddie Mac, Washington Mutual, Landsbanki, Glitnir, Kaupthing, Lehman Brothers (Merril Lynch)

Old world specimen: Sticky Ratchet Knockout Cap



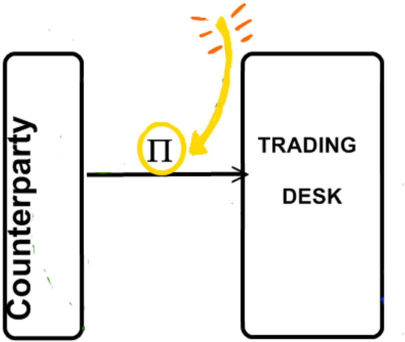
New world specimen: hello again IRS, but with....



Complex payoff, simple system → Simple payoffs, complex system

Let's build this complex risk/cost system one step at the time.

Basic payout

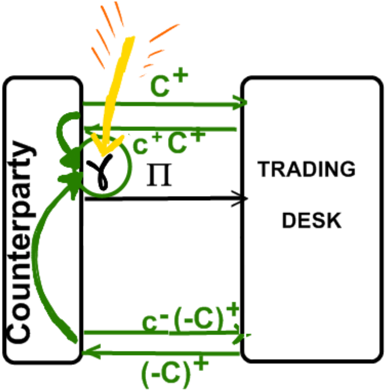


CLASSIC DISCOUNTED CASH FLOWS

Start from derivative's basic cash flows as in old world

$$V_t \quad := \quad \mathbb{E}_t[\,\Pi(t, T \wedge \tau) + \ldots\,] \quad (V = \text{Expected[DiscountedCashFlows]})$$

τ is the first default time between bank and counterparty



Adding pre-default
Collateral Flows

Basic Payout with Collateral Costs & Benefits

- Collateralization procedure cash flows.

$$V_t := \mathbb{E}_t[\,\Pi(t, T \wedge \tau) + \gamma(t, T \wedge \tau; C) + \dots]$$

where

$\longrightarrow$ C_t is the collateral account defined by the CSA,

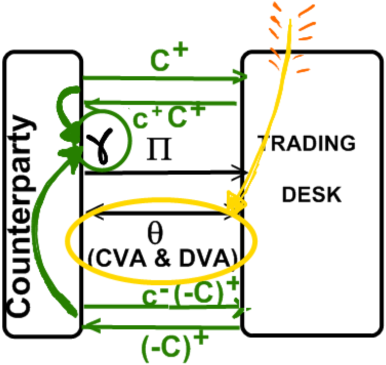
$\longrightarrow$ γ is the collateral margining cost (of carry).

- $$\gamma \approx -\sum_{k=1}^{n-1} 1_{\{t \le t_k < \tau \wedge T\}} D(t, t_k) C_{t_k} (t_{k+1} - t_k)(\tilde{c}_{t_k} - r_{t_k})$$

$$\tilde{c} = c^+ \ \text{ if } \ C > 0, \qquad \tilde{c} = c^- \ \text{ if } \ C < 0.$$

Note that if the collateral rates in $\tilde{c}$ are both equal to the risk free rate, then this term is zero.

- The second expected value: ColVA/VMVA



Adding default closeout Cash Flows (trading CVA and DVA after collateralization)

Close-Out: Trading-CVA/DVA after Collateralization

- Third contribution: cash flow at 1st default

$$V_t := \mathbb{E}_t\big[\,\Pi + \gamma + 1_{\{\tau<T\}} D(t,\tau)\theta_\tau(C,\varepsilon) + \ldots\big]$$

where ε_τ is the close-out amount, or residual value of the deal at default, "exposure at default". We can include liquidation delays.

- Replacement closeout, $\varepsilon_\tau = V_\tau \quad \Rightarrow$ nonlinearity, difficult!

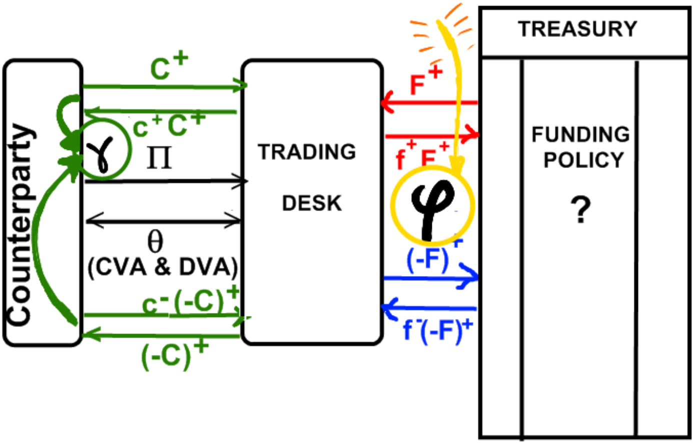
$$V_t := \mathbb{E}_t\big[\,\Pi + \gamma + 1_{\{\tau<T\}} D(t,\tau)\theta_\tau(C, V_\tau) + \ldots\big]$$

- Default cash flow θ_τ calculated by ISDA documentation

$$\theta_\tau(C,\varepsilon) \quad := \quad \varepsilon_\tau - 1_{\{\tau=\tau_C<\tau_I\}}\Pi_{\text{CVAcoll}} + 1_{\{\tau=\tau_I<\tau_C\}}\Pi_{\text{DVAcoll}}$$

- For example, in case of re-hypothecation (same LGD),

$$\Pi_{\text{DVAcoll}} = \mathrm{L{\scriptstyle GD}}_I(-(\varepsilon_\tau - C_{\tau-}))^+, \quad \Pi_{\text{CVAcoll}} = \mathrm{L{\scriptstyle GD}}_C(\varepsilon_\tau - C_{\tau-})^+.$$



Adding costs of funding the trade through the treasury (and offsetting Repo market)

Funding Costs of the Replication Strategy

- As fourth contribution we consider the cost of funding (carry) for the trade accounts and we add the relevant cash flows ([79]).

$$V_t := \mathbb{E}_t\big[\,\Pi + \gamma + 1_{\{\tau<T\}} D\,\theta + \varphi(t, T \wedge \tau; F, H)\,\big]$$

  The last term is related to FVA.

  $\longrightarrow$ F_t is the cash account for the replication of the trade,
  $\longrightarrow$ H_t is the risky-asset account in the replication,
  $\longrightarrow$ φ is cost/benefit of carry for the cash F and hedging H accounts.

- In classical Black Scholes for a call option,

$$V_t^{\text{Call}} = \Delta_t S_t + \eta_t B_t =: H_t + F_t, \ \ (\text{no} \ \ \tau, \ \ C = \gamma = \varphi = 0).$$

Funding Costs of the Replication Strategy

- If H is perfectly collateralized with collateral re-hypothecation ($H = 0$ as we fund the hedge via its collateral)

$$\varphi(t, \tau \wedge T) \approx \sum_{j=1}^{m-1} 1_{t \le t_j < \tau \wedge T} D(t, t_j) \alpha_k \left[\underbrace{-(F_{t_j})^+ \left(f_{t_j}^+ - r_{t_j} \right)}_{\mathbb{E}:\text{Funding Cost FCA}} + \underbrace{(-F_{t_j})^+ \left(f_{t_j}^- - r_{t_j} \right)}_{\mathbb{E}:\text{Benefit FBA}} \right]$$

- If further treasury borrows/lends at risk free $\tilde{f} = r \Rightarrow \varphi = FVA = 0$.

Funding rates and bank policy

f^+ & f^- are policy driven. EG, $f^+ = r + s^I + \ell^+$, $f^- = r + s^I + \ell^-$ (s^I is our bank spread, typically $s^I = \lambda^I \,\mathrm{L_{GD}}_I$), ℓ are liquidity bases driven by treasury policy and market (CDS-Bond basis).

Putting everything together

$$(*) \quad V_t = \mathbb{E}_t\left[\underbrace{\Pi(t, T\wedge\tau)}_{Basic\ cash\ flows} + \underbrace{\gamma(t)}_{collateral\ carry} + \underbrace{1_{\{\tau<T\}} D(t,\tau)\theta_\tau}_{closeout\ cashflows} + \underbrace{\varphi(t)}_{funding\ carry} \right]$$

Can we interpret:

$$\mathbb{E}_t\left[\Pi + 1_{\{\tau<T\}} D(t,\tau)\theta\right] : \quad \text{RiskFree Price + DVA - CVA?}$$

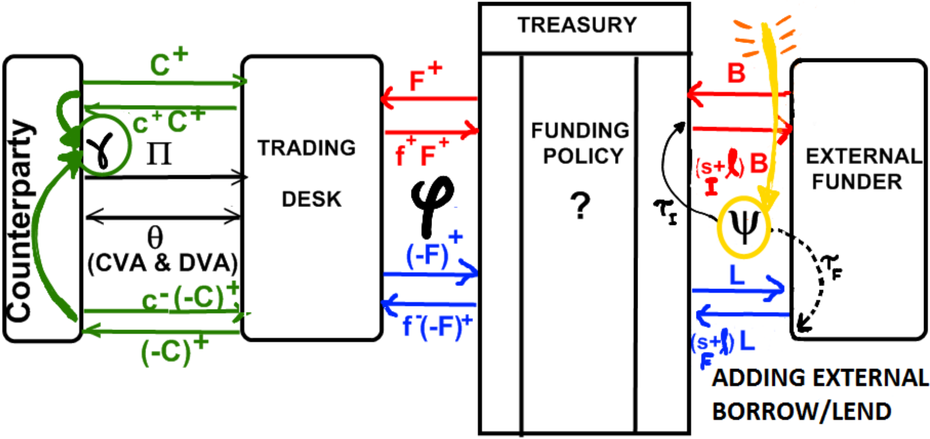
$$\mathbb{E}_t[\gamma + \varphi] : \quad \text{Funding adjustment ColVA+FVA?}$$

Not really. $V_t = F_t + H_t + C_t$ (re–hypo) $\Rightarrow \varphi$ term depends on future $F = V - H - C$ to distinguish f^+ & f^-, & the closeout depends on future V too. All terms depend on V (all risks), no neat separation.

Nonlinear Equation! (FBSDE, SL-PDE)

$$V_t = \mathbb{E}_t\left[\Pi + \gamma(t,V) + 1_{\{\tau<T\}} D(t,\tau)\theta_\tau(V) + \varphi(t,V)\right]$$

Treasury CVA & DVA



FCA and FBA eplained by the costs and benefits the bank faces in borrowing/lending externally to service our (and others) trade ($\mathbb{E}\psi$)

Kapital Valuation Adjustment?

Bank may charge a client a margin to keep the bank profit target (RAROC) in a desired range.   RAROC = Expected P&L / VaR

Start a new trade with client. This generates new risk and triggers corresponding RWA/VaR calculation.

This reduces RAROC by increasing its VaR denominator. The improved P&L with the new trade may not be enough to compensate.

To keep RAROC as before bank needs to increase numerator. Charge a margin - KVA?- that compensates RAROC worsening.

Bank treasury may charge KVA to the trading desk and the trading desk may charge it to the client, or use it only for profitability analysis.

CVA is itself subject to capital requirements, so we could have a KVA on CVA. Additive adjustments, really?

Nonlinearities

Should we embrace nonlinearities or keep them at arm's length?

Aggregation–dependent and asymmetric valuation

Valuation is aggregation dependent.

$$V(P_1 + P_2) \neq V(P_1) + V(P_2).$$

More: Without costs/risks, price to one entity is minus price to the other one. No more with credit/capital/funding.

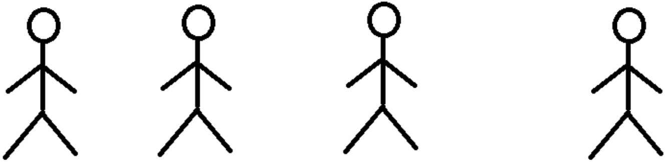
Once aggregation is set, valuation is non–separable. Holistic consistent modeling across trading desks & asset classes needed

Quants have a hard time with nonlinearities, **how about managers**?

Nonlinearity and responsibility

LINEAR VALUATION

V = Vbasic - CVA - FCA + FBA - KVA



Resp. for standard trades

Resp. for credit valuation

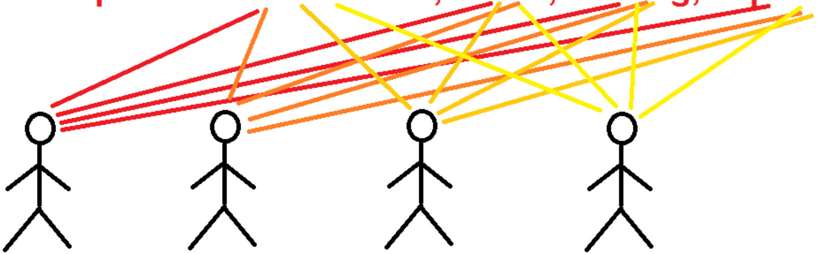
Resp. for funding valuation

Resp. for Capital Costs

Nonlinearity and responsibility

NONLINEAR VALUATION

V non separable: Basic flows, credit, funding, capital



Every trader responsible for every risk.
Ok with enlightened people or with Bees.. ZZZZZZZ...
but with us normal humans? Management problem

Nonlinearity: linearize?

Looks like managers may wish to linearize!
EG, symmetrize borrowing & lending rates, risk free closeout at default.

Nonlinearity Valuation Adjustment (NVA) (B. et al (2014)[25])

NVA analyzes the double counting involved in forcing linearization. Our numerical examples for simple call options show that NVA can easily reach 2 or 3% of the deal value even in relatively standard settings without WWR.

Multiple interest rate curves

Full theory may account also for multiple discount curves (see [78]).

Initial Margins (CCPs, SCSA)

Add the initial margin account flows & customize to relevant initial margin rule, depending on the CCP or SCSA if OTC. IMVA [36].

Price vs Value

Cecil Graham: “What is a cynic?”
Lord Darlington: “A man who knows the **price** of everything, and the **value** of nothing.”
CG: “And a sentimentalist [...] is a man who sees an absurd value in everything, and doesn't know the market price of any single thing”. (Who wrote this?)

Oscar Wilde,
Lady Windermere's Fan

Price of Value? Charging clients?

Adjusted ”price”? Not a price in conventional sense. Use it for cost/profitability analysis or internal fund transfers, but can we charge it to a client straight away? How can client check our price is fair if she has no access to our funding policy, target profit policy & parameters?

We didn't talk much about the art!

What if you don't have data to calibrate market implied credit spreads for CVA? Or implied market-credit correlations for WWR? Any guess on the recovery? (Lehman: [8%, 50%]). Can we really hedge things like jump to default risk? ....

Will new valuation adjustments continue to appear, confusing the picture further and increasing the risk of double counting?

Coming soon: EVA

(Electricity-bill Valuation Adjustment)

Thank you for your attention! Questions?

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Bonus material

The following material did not fit the talk due to time limits.

I include it here for potential questions and follow up.

A Trader's explanation of the funding cash flows φ

1. **Time** t: I wish to buy a call option with maturity T whose current price is $V_t = V(t, S_t)$. I need V_t cash to do that. So I borrow V_t cash from my bank treasury and buy the call.
2. I receive the collateral C_t for the call, that I give to the treasury.
3. Now I wish to hedge the call option I bought. To do this, I plan to repo-borrow $\Delta_t = \partial_S V_t$ stock on the repo-market.
4. To do this, I borrow $H_t = \Delta_t S_t$ cash at time t from the treasury.
5. I repo-borrow an amount Δ_t of stock, posting cash H_t guarantee.
6. I sell the stock I just obtained from the repo to the market, getting back the price H_t in cash.
7. I give H_t back to treasury.
8. Outstanding: I hold the Call; My debt to the treasury is $V_t - C_t$; I am Repo borrowing Δ_t stock.

A Trader's explanation of the funding cash flows φ

9. **Time $t + dt$:** I need to close the repo. To do that I need to give back Δ_t stock. I need to buy this stock from the market. To do that I need $\Delta_t S_{t+dt}$ cash.
10. I thus borrow $\Delta_t S_{t+dt}$ cash from the bank treasury.
11. I buy Δ_t stock and I give it back to close the repo and I get back the cash H_t deposited at time t plus interest $h_t H_t$.
12. I give back to the treasury the cash H_t I just obtained, so that the net value of the repo operation has been

$$H_t(1 + h_t\, dt) - \Delta_t S_{t+dt} = -\Delta_t\, dS_t + h_t H_t\, dt$$

Notice that this $-\Delta_t dS_t$ is the right amount I needed to hedge V in a classic delta hedging setting.

13. I close the derivative position, the call option, and get V_{t+dt} cash.

A Trader's explanation of the funding cash flows φ

14. I have to pay back the collateral plus interest, so I ask the treasury the amount $C_t(1 + c_t\,dt)$ that I give back to the counterparty.
15. My outstanding debt plus interest (at rate f) to the treasury is $V_t - C_t + C_t(1 + c_t\,dt) + (V_t - C_t)f_t\,dt = V_t(1 + f_t\,dt) + C_t(c_t - f_t\,dt)$. I then give to the treasury the cash V_{t+dt} I just obtained, the net effect being

$$V_{t+dt} - V_t(1 + f_t\,dt) - C_t(c_t - f_t)\,dt = dV_t - f_t V_t\,dt - C_t(c_t - f_t)\,dt$$

16. I now have that the total amount of flows is :

$$-\Delta_t\,dS_t + h_t H_t\,dt + dV_t - f_t V_t\,dt - C_t(c_t - f_t)\,dt$$

A Trader's explanation of the funding cash flows φ

17 Now I present–value the above flows in t in a risk neutral setting.

$$\mathbb{E}_t[-\Delta_t\, dS_t + h_t H_t\, dt + dV_t - f_t V_t\, dt - C_t(c_t - f_t)\, dt] =$$

$$= -\Delta_t(r_t - h_t)S_t\, dt + (r_t - f_t)V_t\, dt - C_t(c_t - f_t)\, dt - d\varphi(t)$$

$$= -H_t(r_t - h_t)\, dt + (r_t - f_t)(H_t + F_t + C_t)\, dt - C_t(c_t - f_t)\, dt - d\varphi(t)$$

$$= (h_t - f_t)H_t\, dt + (r_t - f_t)F_t\, dt + (r_t - c_t)C_t\, dt - d\varphi(t)$$

This derivation holds assuming that $\mathbb{E}_t[dS_t] = r_t S_t\, dt$ and $\mathbb{E}_t[dV_t] = r_t V_t\, dt - d\varphi(t)$, where $d\varphi$ is a dividend of V in $[t, t + dt)$ expressing the funding costs. Setting the above expression to zero we obtain

$$d\varphi(t) = (h_t - f_t)H_t\, dt + (r_t - f_t)F_t\, dt + (r_t - c_t)C_t\, dt$$

which coincides with the definition given earlier.

Treasury CVA & DVA

Include default risk of funder and funded ψ, leading to CVA_F & DVA_F.

$$V_t = \mathbb{E}_t\big[\,\Pi(t, T \wedge \tau) + \gamma(t) + 1_{\{\tau<T\}} D(t,\tau)\theta_\tau + \varphi(t) + \psi(t, \tau_F, \tau)\,\big]$$

FVA $= -$FCA$+$FBA from f^+ & f^- largely offset by $\mathbb{E}\psi$ after immersion, approx & linearization.

Assume $H = 0$ (perfectly collateralized hedge with re–hypothecation), once f^+ & f^- are decided by policy, under *immersion*

- Underlying $\Pi(t, T)$ is not credit sensitive, technically $\mathcal{F}_t$-measurable; $\mathcal{F}$ pre-default filtration, $\mathcal{G}$ full filtration.
- τ_I and τ_C and τ_F are $\mathcal{F}$ conditionally independent (credit spreads can be correlated, jumps to default are independent);

we obtain a practical decomposition of price into

V = RiskFreePrice - CVA + DVA + LVA - FCA + FBA -CVA$_F$+ DVA$_F$

$$RiskFreePr = \int_t^T \mathbb{E}\left\{\pi_u \,|\mathcal{F}_t\right\}du; \quad LVA = \int_t^T \mathbb{E}\left\{D(t,u;r{+}\lambda)(r_u{-}\tilde{c}_u)C_u|\mathcal{F}_t\right\}du$$

$$-CVA = \int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)\left[-\mathrm{LGD}_C\lambda_C(u)(V_u - C_{u-})^+\right]|\mathcal{F}_t\right\}du$$

$$DVA = \int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)\left[\mathrm{LGD}_I\lambda_I(u)(-(V_u - C_{u-}))^+\right]|\mathcal{F}_t\right\}du$$

$$-FCA = -\int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)\left[({f^+}_u - r_u)(V_u - C_u)^+\right]|\mathcal{F}_t\right\}du$$

$$FBA = \int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)\left[({f^-}_u - r_u)(-(V_u - C_u))^+\right]|\mathcal{F}_t\right\}du$$

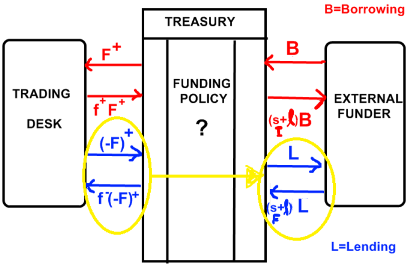
$$-CVA_F = \int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)\left[\mathrm{LGD}_F\lambda_F(u)(-(V_u - C_u))^+\right]|\mathcal{F}_t\right\}du$$

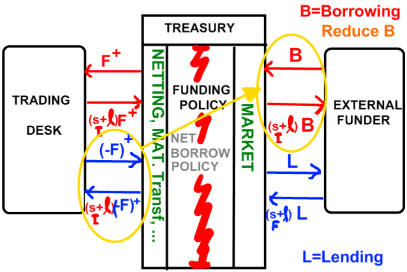
$$DVA_F = \int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)\left[\mathrm{LGD}_I\lambda_I(u)(V_u - C_u)^+\right]|\mathcal{F}_t\right\}du$$

Treasury CVA & DVA

To further specify the split, we need to assign f^+ (borrow) & f^- (lend)
There are two possible simple treasury models to assign f.

EXTERNAL FUNDING BENEFIT (EFB) VS **REDUCED BORROWING BENEFIT (RBB)**





$$f^+ = \mathrm{LGD}_I \lambda_I + \ell^+ =: s_I + \ell^+ \qquad f^+ = s_I + \ell^+$$

$$f^- = \mathrm{LGD}_F \lambda_F + \ell^- =: s_F + \ell^- \qquad f^- = s_I + \ell^-$$

Treasury CVA & DVA

$$-CVA = \int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)\left[-s_C(u)(V_u - C_{u-})^+\right]|\mathcal{F}_t\right\}du$$

$$DVA = \int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)\left[s_I(u)(-(V_u - C_{u-}))^+\right]|\mathcal{F}_t\right\}du$$

$$-FCA = -\int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)\left[(s_I(u)+\ell_u^+)(V_u - C_u)^+\right]|\mathcal{F}_t\right\}du$$

$$FBA = \int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)\left[(\underbrace{f_u^- - r_u}_{\text{EFB: } s_F + \ell^-;\ \text{RBB: } s_I + \ell^-})(-(V_u - C_u))^+\right]|\mathcal{F}_t\right\}du$$

$$-CVA_F = \int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)\left[s_F(u)(-(V_u - C_u))^+\right]|\mathcal{F}_t\right\}du$$

$$DVA_F = \int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)\left[s_I(u)(V_u - C_u)^+\right]|\mathcal{F}_t\right\}du$$

Treasury CVA & DVA

The benefit of lending back to the treasury, two different models:

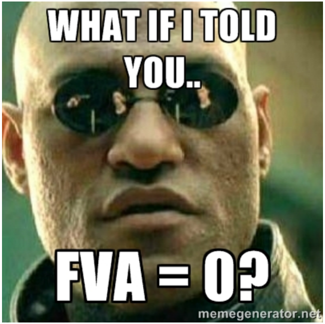
1. External funding benefit (EFB) policy: when desk lends back to treasury, treasury lends to F for interest $f^- = r + s^F + \ell^-$. Hence

$$V_{\text{EFB}} = V^0 - CVA + DVA + ColVA \underbrace{-FCA}_{-DVA_F - FCA_\ell} + \underbrace{FBA}_{CVA_F + FBA_\ell} + DVA_F - CVA_F$$

2. Reduced borrowing benefit (RBB) policy: whenever trading desk lends back to the treasury, the latter reduces the desk loan outstanding and the desk saves at interest $f^- = r + s^l + \ell^-$. Hence

$$V_{\text{RBB}} = V^0 - CVA + DVA + ColVA \underbrace{-FCA}_{-DVA_F - FCA_\ell} + \underbrace{FBA}_{DVA + FBA_\ell} + DVA_F$$

A lot of work & discussion on FVA. What if I told you...



Hull White argued FVA =0.
Modigliani Miller? (MM)

- Mkt prices follow rnd walks,
- No taxes, No costs for
- bankruptcy or agency,
- No asymmetric information,
- & market is efficient

then value of firm does not depend on how firm is financed.
Without MM or corporate finance:

$$V_{\text{EFB}} = V^0 - CVA + DVA + ColVA \underbrace{-FCA}_{-DVA_F - FCA_\ell} + \underbrace{FBA}_{CVA_F + FBA_\ell} + DVA_F - CVA_F$$

If bases $\ell = 0$, & if $r = \tilde{c}$, & if... $V_{\text{EFB}} = V^0 - CVA + DVA$ (no funding)
Too many if's? Even then, internal fund transfers happening.

NVA: numerical example I

Equity call option (long or short), $r = 0.01$, $\sigma = 0.25$, $S_0 = 100$, $K = 80$, $T = 3y$, $V_0 = 28.9$ (no credit risk or funding/collateral costs). Precise credit curves are given in the paper.

$$NVA = V_0(nonlinear) - V_0(\text{linearized})$$

Table: NVA with default risk and collateralization

Funding Rates bps			Default risk, low[a]		Default risk, high[b]	
			Long	Short	Long	Short
f^+	f^-	$\hat{f}$				
300	100	200	-3.27 (11.9%)	-3.60 (10.5%)	-3.16 (11.4%)	-3.50 (10.1%)
100	300	200	3.63 (10.6%)	3.25 (11.8%)	3.52 (10.2%)	3.13 (11.3%)

The percentage of the total call price corresponding to NVA is reported in parentheses.
[a] Based on the joint default distribution D_{low} with low dependence.
[b] Based on the joint default distribution D_{high} with high dependence.

NVA: numerical example II

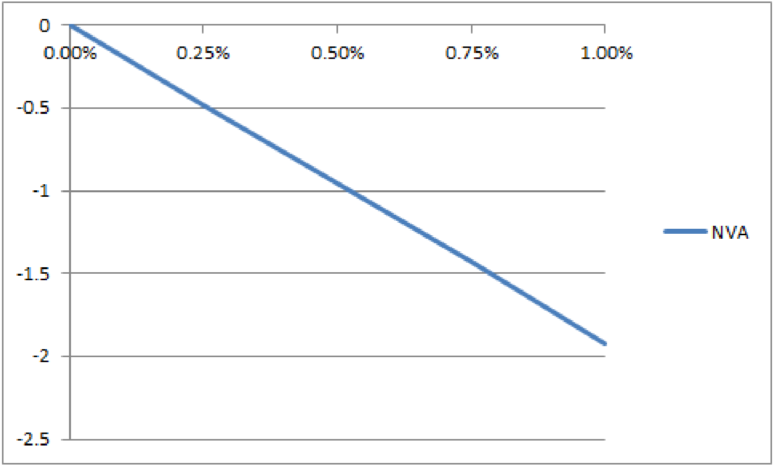
Table: NVA with default risk, collateralization and rehypothecation

Funding Rates bps			Default risk, low[a]		Default risk, high[b]	
			Long	Short	Long	Short
f^+	f^-	$\hat{f}$				
300	100	200	-4.02 (14.7%)	-4.45 (12.4%)	-3.91 (14.0%)	-4.35 (12.0%)
100	300	200	4.50 (12.5%)	4.03 (14.7%)	4.40 (12.2%)	3.92 (14.0%)

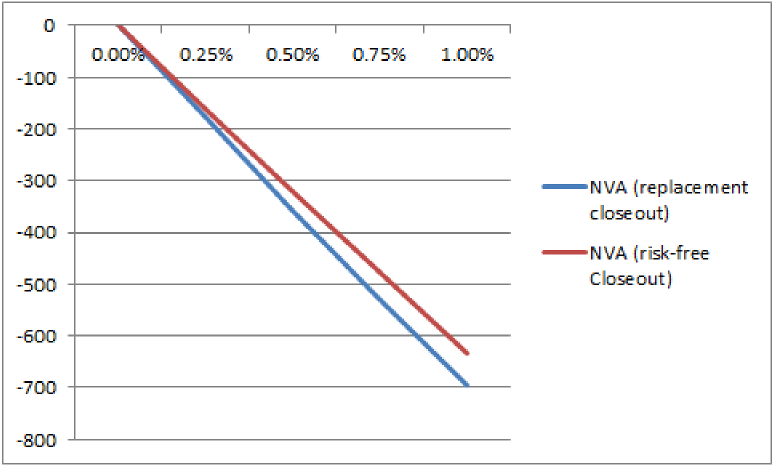
The percentage of the total call price corresponding to NVA is reported in parentheses.

[a] Based on the joint default distribution D_{low} with low dependence.

[b] Based on the joint default distribution D_{high} with high dependence.



NVA for long call as a function of $f^{+} - f^{-}$, with $f^{-} = 1\%$, $f+$ increasing over 1% and $\hat{f}$ increasing accordingly. NVA expressed as an additive price component on a notional of 100, risk free option price 29. Risk free closeout. For example, $f + -f- = 25bps$ results in NVA=-0.5 circa, 50 bps $\Rightarrow$ NVA = -1



NVA for long call as a function of $f^+ - f^-$, with $f^- = 1\%$, $f+$ increasing over 1% and $\hat{f}$ increasing accordingly. NVA expressed as a percentage (in bps) of the linearized $\hat{f}$ price. For example, $f^+ - f^- = 25$bps results in NVA=-100bps = -1% circa, replacement closeout relevant (red/blue) for large $f^+ - f^-$

Multiple Interest Rate Curves

Derive interest rate dynamics consistently with credit, collateral and funding costs as per the above master valuation equations.

- We use our maket based (no r_t) master equation to price OIS & find OIS equilibrium rates. Collateral fees will be relevant here, driving forward OIS rates.
- Use master equation to price also one period swaps based on LIBOR market rates. LIBORs are market given and not modeled from first principles from bonds etc. Forward LIBOR rates obtained by zeroing one period swap and driven both from primitive market LIBOR rates and by collateral fees.
- We'll model OIS rates and forward LIBOR/SWAP jointly, using a mixed HJM/LMM setup
- In the paper we look at non-perfectly collateralized deals too, where we need to model treasury funding rates.
- See `http://ssrn.com/abstract=2244580`

Pricing under Initial Margins: SCSA and CCPs

CCPs: Default of Clearing Members, Delays, Initial Margins...

Our general theory can be adapted to price under Initial Margins, both under CCPs and SCSA.

The type of equations is slightly different but quantitative problems are quite similar.

See B. and Pallavicini (2014) for details. See also "Brigo, D. and A. Pallavicini (2014). Nonlinear consistent valuation of CCP cleared or CSA bilateral trades with initial margins under credit, funding and wrong-way risks. Journal of Financial Engineering 1 (1), 1-60." Here we give a summary.

Pricing under Initial Margins: SCSA and CCPs

So far all the accounts that need funding have been included within the funding netting set defining F_t.

If additional accounts needed, for example segregated initial margins, as with CCP or SCSA, their funding costs must be added.

Initial margins kept into a segregated account, one posted by the investor ($N_t^I \leq 0$) and one by the counterparty ($N_t^C \geq 0$):

$$\begin{aligned}\varphi(t,u) \;&:=\; \int_t^u dv\,(r_v - f_v)F_v D(t,v) - \int_t^u dv\,(f_v - h_v)H_v D(t,v) \quad (1)\\ &+\; \int_t^u dv(f_v^{N^C} - r_v)N_v^C + \int_t^u dv(f_v^{N^I} - r_v)N_v^I\,,\end{aligned}$$

with $f_t^{N^C}$ & $f_t^{N^I}$ assigned by the Treasury to the initial margin accounts. $f^N \neq f$ as initial margins not in funding netting set of the derivative.

Pricing under Initial Margins: SCSA and CCPs

$$\ldots + \int_t^u dv(f_v^{N^C} - r_v)N_v^C + \int_t^u dv(f_v^{N^I} - r_v)N_v^I$$

Assume for example $f > r$. The party that is posting the initial margin has a penalty given by the cost of funding this extra collateral, while the party which is receiving it reports a funding benefit, but only if the contractual rules allow to invest the collateral in low-risk activity, otherwise $f = r$ and there are no price adjustments.

We can describe the default procedure with initial margins and delay by assuming that at 1st default τ the surviving party enters a deal with a cash flow ϑ, at maturity $\tau + \delta$ (DELAY!).

δ 5d (CCP) or 10d (SCSA).

Pricing under Initial Margins: SCSA and CCPs

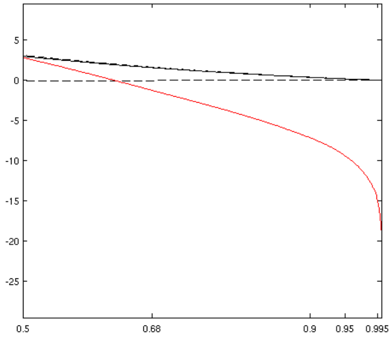
For a CCP cleared contract priced by the clearing member we have $N^I_{\tau^-} = 0$, whatever the default time, since the clearing member does not post the initial margin.

We assume that each margining account accrues continuously at collateral rate c_t.
We may further

- include funding default cloeseout and also
- define the Initial Margin as a percentile of the mark to market at time $\tau + \delta$.

This is done explicitly in the paper.

Now a few numerical examples:



Ten-year receiver IRS traded with a CCP.

Prices are calculated from the point of view of the CCP client.

Mid-credit-risk for CCP clearing member, high for CCP client.

Initial margin posted at various confidence levels q.

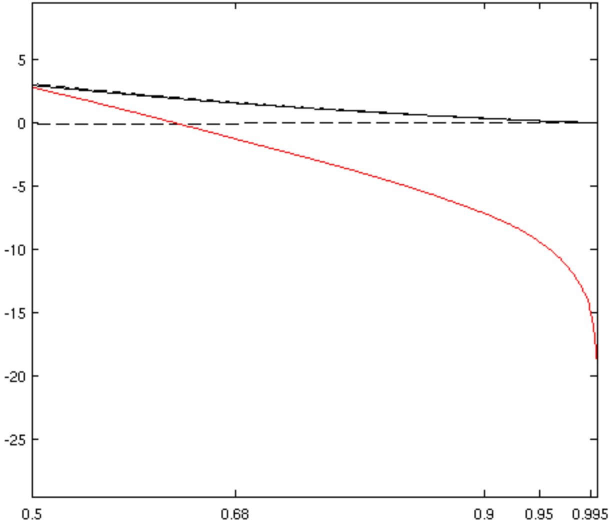
Prices in basis points with a notional of one Euro

Black continuous line: price inclusive of residual CVA and DVA after margining but not funding costs

Dashed black lines represent CVA and the DVA contributions.

Red line is the price inclusive both of credit & funding costs.

Symmetric funding policy. No wrong way correlation overnight/credit.


5
0
-5
-10
-15
-20
-25
0.5
0.68
0.9
0.95
0.995

CCP Pricing: Tables (see paper for WWR etc)

Table: Prices of a ten-year receiver IRS traded with a CCP (or bilaterally) with a mid-risk parameter set for the clearing member (investor) and a high-risk parameter set for the client (counterparty) for initial margin posted at various confidence levels q. Prices are calculated from the point of view of the client (counterparty). Symmetric funding policy. WWR correlation $\bar{\rho}$ is zero. Prices in basis points with a notional of one Euro.

	Receiver, CCP, $\beta^- = \beta^+ = 1$				Receiver, Bilateral, $\beta^- = \beta^+ = 1$			
q	**CVA**	**DVA**	**MVA**	**FVA**	**CVA**	**DVA**	**MVA**	**FVA**
50.0	-0.126	3.080	0.000	-0.1574	-2.1317	4.3477	0.0000	-0.0842
68.0	-0.066	1.605	-2.933	0.1251	-1.1176	2.2613	-4.1389	0.2491
90.0	-0.015	0.357	-8.037	0.5492	-0.2578	0.4997	-11.3410	0.7924
95.0	-0.007	0.154	-10.316	0.7205	-0.1149	0.2151	-14.5561	1.0250
99.0	-0.001	0.025	-14.590	1.0290	-0.0204	0.0346	-20.5869	1.4544
99.5	-0.001	0.013	-16.154	1.1402	-0.0107	0.0176	-22.7947	1.6107
99.7	-0.000	0.008	-17.233	1.2165	-0.0070	0.0114	-24.3164	1.7184
99.9	-0.000	0.004	-19.381	1.3684	-0.0035	0.0056	-27.3469	1.9326