An indifference approach to cost of capital constraints: KVA and beyond

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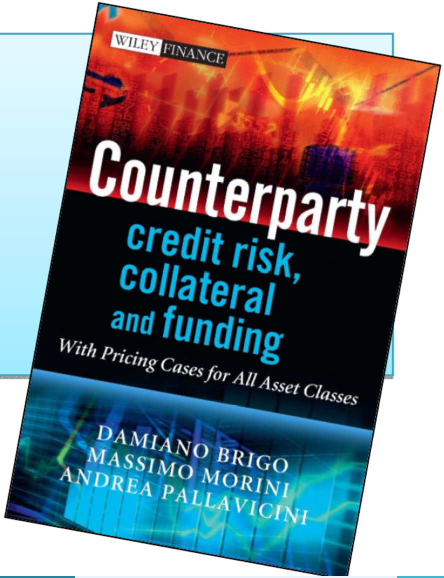
Full paper **https://arxiv.org/abs/1708.05319**

Presentation based on Book (working on 2nd Edition)



A Little History: CVA and DVA can be sizeable. Citigroup:

1Q 2009: “Revenues also included [...] a net 2.5$ billion positive Credit Valuation Adjustment (CVA) on derivative positions, excluding monolines, mainly due to the widening of Citi’s CDS spreads” (DVA)

CVA mark to market losses: BIS

”During the financial crisis, however, roughly two-thirds of losses attributed to counterparty credit risk were due to CVA losses and only about one-third were due to actual defaults.”

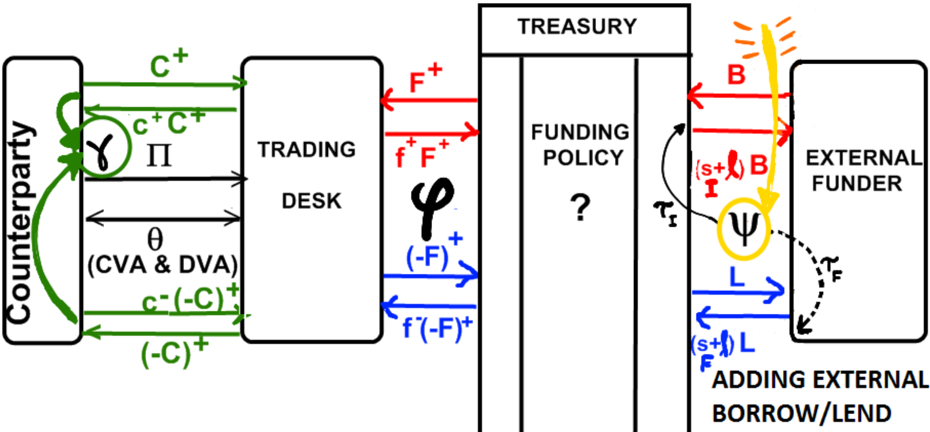
Collateral not always effective as a guarantee: B. et al [19]

For trades subject to strong contagion at default of the counterparty, like CDS, collateral can leave a sizeable CVA [Initial Margin?]

FVA can be sizeable too. JP Morgan:

Wall St Journal, Jan 14, 2014: ‘[...] So what is a funding valuation adjustment, (FVA) and why did it cost J.P. Morgan Chase $1.5 billion?’

Valuation adj.: default, collateral and funding flows



Nonlinear valuation

$$(*) \quad V_t = \mathbb{E}_t \left[\underbrace{\Pi(t, T \wedge \tau)}_{Basic\ cash\ flows} + \underbrace{\gamma(t)}_{collateral\ carry} + \underbrace{1_{\{\tau<T\}} D(t,\tau)\theta_\tau}_{closeout\ cashflows} + \underbrace{\varphi(t)}_{funding\ carry} \right]$$

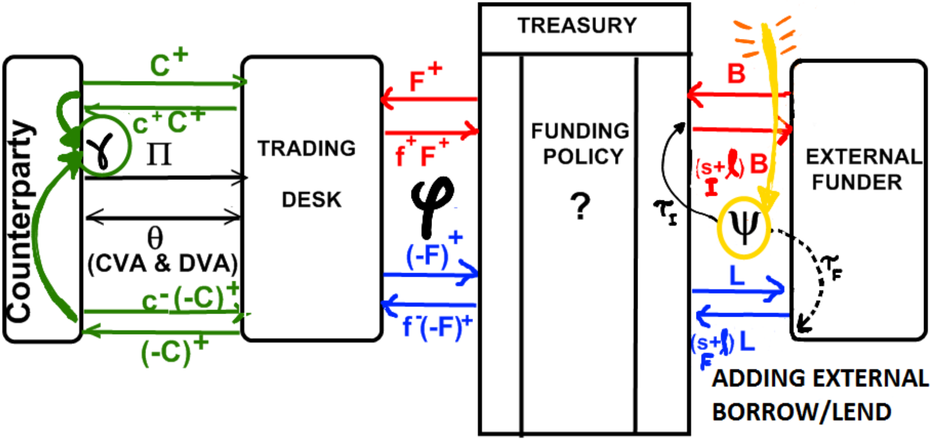
Can we interpret:

$$\mathbb{E}_t \left[\Pi + 1_{\{\tau<T\}} D(t,\tau)\theta \right] : \quad \text{RiskFree Price + DVA - CVA?}$$
$$\mathbb{E}_t[\gamma + \varphi] : \quad \text{Funding adjustment ColVA+FVA?}$$

Not really. $V_t = F_t + H_t + C_t$ (re–hypo) $\Rightarrow \varphi$ term depends on future $F = V - H - C$ to distinguish f^+ & f^-, & the closeout depends on future V too. All terms depend on V (all risks), no neat separation.

Nonlinear Equation! (FBSDE, SL-PDE)

$$V_t = \mathbb{E}_t \left[\Pi + \gamma(t, V) + 1_{\{\tau<T\}} D(t,\tau)\theta_\tau(V) + \varphi(t, V) \right]$$



FVA(FCA and FBA) eplained by the costs & benefits bank faces in borrowing/lending externally to service trade ($\mathbb{E}\psi$)

Replication approach stretched ($\mathbb{E}$ under $\mathbb{Q}$) beyond its limits? In the academic literature, see for example B. et al (2011-2017) [79, 80, 78, 36, 24, 25] & papers by Capponi, Crepey and others.

Kapital Valuation Adj? This talk in a nutshell

Strengthening of capital requirements induced banks to consider charging a so called capital valuation adjustment (KVA) to clients

Initial attempt to embed capital costs in "replication" framework as above, adding "capital account". Quite dubious (already for FVA...).

A more sensible approach might be relating costs of capital to ex-ante risk-adjusted target performance of the bank, for example ex ante RAROC = Expected P&L / VaR . How would this work?

When starting new trade with client, this generates new risk VaR.

This reduces RAROC. The improved P&L from the new trade may not be enough to compensate the new risk.

To keep target as before bank needs charge a profit margin.

(CVA is subject to capital: KVA on CVA? Additive? Nonlinear again?)

Capital costs: setting and previous literature I

Previous Literature: initial KVA in Green et al [112] relies on replication argument similar to our setting above for C/D/FVA. Unsatisfactory since it is not derived from first principles. Struggles with constraints and objectives defined under the real world measure $\mathbb{P}$.

Albanese et al [99] do not assume a replication argument but still postulate a form for KVA instead of deriving it from financial considerations. Their KVA doesn't depend explicitly on the regulatory constraints that motivated the KVA introduction in the first place.

Prampolini & Morini [116] assume definition of KVA & discuss how it interacts with CVA. KVA $\approx$ analogous of CVA for unhedgable trades.

We adopt an indifference pricing approach, which is similar in spirit to what has been done for example for FVA in Duffie et al [100].

A first example based on RAROC I

One period model. Bank is endowed with some cash C_0, and there is a liquid arbitrage-free market, with d securities with vector prices S_0 (deterministic) at $t = 0$ and S_1 (random) at $t = 1$.

A strategy on the liquid market is $\theta \in \mathbb{R}^d$.

Bank borrows & lends money at spread λ over the risk-free rate r. This is for computational convenience, easily generalized to different λ's.

Aside from liquid market, bank consider a OTC trade with payoff qY (q amount of product, Y product payoff) with a counterparty.

Counterparty interested in *buying* from the bank q units of payoff Y and they ask the bank for a quote on this product.

Problem: *what is the price $P(q)$ that the bank is willing to make to the counterparty for the contract with pay-off Y.*

A first example based on RAROC II

In this initial example we use RAROC (Sironi et al [118]) to assess the value of a contract for a bank.

Ex-ante RAROC $= \mathbb{E}^{\mathbb{P}}[$ Portfolio P&L$]/$VaR[Portfolio]

Notation: **From now on $\mathbb{E} = \mathbb{E}^{\mathbb{P}}$.**

Evaluate RAROC of portfolio without the product qY and RAROC of our portfolio including the product, and choose price $P(q)$ so that the RAROC stays the same.

In other words our bank will be *indifferent* wrt entering the deal.

A first example based on RAROC III

To proceed more in detail, for our chosen strategy θ we have:

$$Assets = (\theta^T S_1)^+ + (qY)^+ + (C_0 - \theta^T S_0 + P(q))^+(1 + r + \lambda^B);$$

$$Liabilities = (\theta^T S_1)^- + (qY)^- + (C_0 - \theta^T S_0 + P(q))^-(1 + r + \lambda^B).$$

For convenience define the equity at time 1 of the bank as:

$$\begin{aligned} X(\theta, q) &= Assets - Liabilities \\ &= qY + \theta^T(S_1 - S_0(1 + r + \lambda)) + (C_0 + P(q))(1 + r + \lambda^B). \end{aligned}$$

Assume VaR is proportional to the standard deviation:

$$RAROC(\theta, q) = \frac{\mathbb{E}[X(\theta, q)] - C_0}{m\sqrt{Var[X(\theta, q)]}}. \quad (1)$$

A first example based on RAROC IV

Find $P(q)$ s.t. $RAROC(\theta, 0) = RAROC(\theta', q)$, where θ' is strategy that the bank chooses in the case when it also trades the new product.

θ' in principle different from θ, possibly includes hedge of qY and will need to change from θ to take into account capital constraints.

In this first example we assume θ is not changed. We assume the new product contains only unhedgeable risks, independent from other products, and that *there are no capital constraints* impacting θ. Then we have to solve

$$RAROC(\theta, 0) = RAROC(\theta, q).$$

Under our assumption of independence between Y and S_1 , with straightforward calculations and expanding for small q we obtain

$$P(q) \simeq \frac{1}{1+r+\lambda}\left(-q\mathbb{E}[Y] + RAROC(\theta,0)\left(\frac{1}{2}\frac{mq^2 Var[Y]}{\sqrt{Var[X(\theta,0)]}}\right)\right)$$

A first example based on RAROC V

$$P(q) \simeq \frac{1}{1+r+\lambda}\left(-q\mathbb{E}[Y] + RAROC(\theta,0)\left(\frac{1}{2}\frac{mq^2 Var[Y]}{\sqrt{Var[X(\theta,0)]}}\right)\right)$$

where we see that acceptable price is the discounted expectation of the cash flows due to our new product plus an adjustment (KVA?) proportional to the variance of the new product and the $RAROC(\theta, 0)$.

In the next part we are going to address the issues of the choice of θ, the hedging and also the presence of capital constraints impacting θ.

The general linear case (whole bank view) I

We now look again at an indifference price but

i) we address how to choose the bank's strategy on the liquid market (optimisation approach);
ii) inlcude regulatory capital constraints

On i) bank chooses θ to maximise value. Admissible set $\Theta(q) \subseteq \mathbb{R}^d$

$$\widetilde{v}(q) = \sup_{\theta \in \Theta(q)} \mathbb{E}\left[X(\theta, q)\right]. \tag{2}$$

We focus on regulatory capital constraints that can be expressed as constraints on the Expected Shortfall (ES) or Value at Risk (V@R).

The general linear case (whole bank view) II

For our purposes we approximate capital constraints with:

$$\Theta(q) = \left\{ \theta \in \mathbb{R}^2 \mid C_0 = \nu \sqrt{\theta^T A \theta + 2q\theta^T b + \sigma_Y^2 q^2} \right\}, \qquad (3)$$

where σ_Y^2 is the variance of the payoff Y, and

$$Cov \left[\begin{pmatrix} S_1 \\ Y \end{pmatrix} \right] = \begin{pmatrix} A & b \\ b^T & \sigma_Y^2 \end{pmatrix}.$$

In practice we are computing the capital constraint as an ES or V@R limit while approximating ES or V@R with a multiple ν of the standard deviation of the distribution of $X(\theta, q)$. Equivalently we are approximating the distribution of $X(\theta, q)$ with a distribution such that its ES or V@R is a multiple ν of its standard deviation.

The general linear case (whole bank view) III

We are supposing that the capital constraint is binding, or that is optimal for the bank to have the minimum amount of capital required.

Note: here endowment C_0 plays also role of capital of the bank: we are optimising the whole portfolio of the bank over one period.

Now, similarly to what we did in the first example, we look for $P(q)$ such that the bank is *indifferent* with respect to enter the contract.

$$\widetilde{v}(0) = \widetilde{v}(q).$$

The treatment of the maximisation problem (2) $\widetilde{v}(q) = \sup_{\theta \in \Theta(q)} \mathbb{E}\left[X(\theta, q)\right]$ is classic (see for example [103, 108]).

The general linear case (whole bank view) IV

Main result for whole-bank view. Consider, for a fixed q, the problem of finding P such that

$$\widetilde{v}(q) = \widetilde{v}(0),$$

$$\widetilde{v}(q) = \sup_{\theta \in \Theta(q)} \mathbb{E}\left[qY + \theta^T(S_1 - S_0(1 + r + \lambda)) + (C_0 + P(q))(1 + r + \lambda)\right]$$

$$\Theta(q) = \left\{\theta \in \mathbb{R}^2 \mid C_0 = \nu\sqrt{\theta^T A\theta + 2q\theta^T b + \sigma_Y^2 q^2}\right\}. \tag{4}$$

Set $\hat{\mu} = (S_1 - S_0(1 + r + \lambda))$, $\mu = \mathbb{E}[(S_1 - S_0(1 + r + \lambda))]$.

The general linear case (whole bank view) V

The indifference Equation has a solution and is given by

$$P(q) = \frac{1}{1+r+\lambda}\left(-q\mathbb{E}[Y] + \frac{1}{2}\left(\frac{1}{\chi(0)} - \frac{1}{\chi(q)}\right)\mu^T A^{-1}\mu + qb^T A^{-1}\mu\right),$$

where $\chi(q)$ is the Lagrange multiplier associated with the problem (4) whose explicit expression is known (see full paper). Moreover if the deal is small compared to the bank's portfolio (i.e. q is small) we have the following ($D^\lambda = 1/(1+r+\lambda)$):

$$P(q) \simeq D^\lambda\left(-q\mathbb{E}[Y] + \left(\frac{(\sigma_Y^2 - b^T A^{-1} b)q^2\nu}{C_0}\right)\frac{\sqrt{\mu^T A^{-1}\mu}}{2} + qb^T A^{-1}\mu\right)$$

The general linear case (whole bank view) VI

$$P(q) \simeq D^{\lambda}\left(-q\mathbb{E}[Y] + \left(\frac{(\sigma_Y^2 - b^T A^{-1} b)q^2\nu}{C_0}\right)\frac{\sqrt{\mu^T A^{-1}\mu}}{2} + q b^T A^{-1}\mu\right)$$

The price that would make the bank break even is approximated by the sum of three terms:

- a term which is minus the expectation of the payoff under $\mathbb{P}$;
- a correction due to the presence of the capital constraint (KVA?);
- and a term that represents the expected excess return of the hedging portfolio, i.e. the difference between the expected value of the hedging portfolio at time 1 and its price at time 0 accrued at $1 + r + \lambda$.

The general linear case (whole bank view) VII

Lastly, in the spirit of the previous example, introduce RAROC. Our expected P&L, associated with the best strategy, is:

$$\widetilde{v}(0) - C_0 = \left(\frac{\mu^T A^{-1}}{2\chi(0)}\right)\mu + C_0(r+\lambda) = \sqrt{\mu A^{-1}\mu}\,\frac{C_0}{\nu} + C_0(r+\lambda)$$

To simplify our formulae we choose the same ES metric we used as capital constraint as denominator for our RAROC. Hence we have that:

$$RAROC(\bar{\theta}, 0) = \frac{\sqrt{\mu A^{-1}\mu}\frac{C_0}{\nu} + C_0(r+\lambda)}{C_0} = \frac{\sqrt{\mu A^{-1}\mu}}{\nu} + r + \lambda.$$

Notation: define RAROC $= h := \sqrt{\mu A^{-1}\mu}/\nu + r + \lambda$ and obtain

$$P(q) \simeq D^{\lambda}\left(-q\mathbb{E}[Y] + \frac{1}{2}\left(\frac{(\sigma_Y^2 - b^T A^{-1} b)q^2\nu^2}{C_0}\right)(h - r - \lambda) + qb^T A^{-1}\mu\right.$$

The general linear case (whole bank view) VIII

$P(q) \simeq$

$$D^\lambda \left(-q\mathbb{E}[Y] + \boxed{\frac{1}{2}\left(\frac{(\sigma_Y^2 - b^T A^{-1} b) q^2 \nu^2}{C_0}\right)(h - r - \lambda)} + q b^T A^{-1} \mu \right).$$

with $h =$RAROC$(\tilde{\theta}, 0)$. This formula clearly highlights the adjustment due to capital constraint that needs to be made so that the bank can break even.

The full nonlinear case: Shareholders I

While in the previous section we took the whole bank perspective, now we analyse how capital constraints play a role in the shareholder's valuation of a deal.

Shareholder's objective function is different from the whole-bank one, since in case of default (negative equity at time 1 in our model) they have limited liability. This means that the best strategy for a shareholder solves the following optimisation problem:

$$v(q) = \sup_{\theta \in \Theta(q)} \mathbb{E}\left[(X(\theta, q))^+ \right].$$

Note the positive part, that was missing in the whole-bank case.

Notation: bank's default is $D(\theta, q) = \{\omega \in \Omega \,|\, X(q, \theta) \leq 0\}$ and $D^c(\theta, q)$ will denote the survival event.

The full nonlinear case: Shareholders II

Again, we wish to determine the indifference price $P(q)$ such that

$$v(q) = v(0).$$

In this part we focus on an approximated solution. We consider the following expansion for $v(q)$ near 0

$$v(q) = v(0) + q\frac{dv}{dq}|_{q=0} + q^2\frac{1}{2}\frac{d^2v}{d^2q}|_{q=0} + o(q^2),$$

and, inspired from our linrarization based result for whole-bank, we look for price P which is a second degree polynomial in q such that

$$q\frac{dv}{dq}|_{q=0} + q^2\frac{1}{2}\frac{d^2v}{d^2q}|_{q=0} = 0.$$

The full nonlinear case: Shareholders III

This is a second order extension of what in the literature is usually called *marginal price*.

Here we focus on second order expansion since we want to understand the impact of capital constraints that in our model are represented by variance constraints & hence are second order in q.

The full nonlinear case: Shareholders IV

Main result nonlinear case - shareholders. Consider, for a fixed q, the problem of finding P such that

$$v(q) = v(0),$$

$$\begin{aligned} v(q) &= \sup_{\theta \in \Theta(q)} \mathbb{E}\left[(qY + \theta^T(S_1 - S_0/D^\lambda) + (C_0 + P(q))/D^\lambda)^+ \right], \\ \Theta(q) &= \left\{ \theta \in \mathbb{R}^2 \mid C_0 = \nu \sqrt{\theta^T A \theta + 2q\theta^T b + \sigma_Y^2 q^2} \right\}. \end{aligned} \tag{5}$$

The marginal (second order) indifference price is $P(q) \simeq$

$$\frac{\mathbb{E}\left[-1_{\{D^c\}} Y\right] + \chi(0) 2\theta^T b}{\mathbb{P}[D^c](1 + r + \lambda)} q - \frac{\psi(0) - \chi(0) 2\sigma_Y^2 - 2\chi'(0)(2\theta^T b) + \mathfrak{C}(0)}{2(1 + r + \lambda)} q^2,$$

The full nonlinear case: Shareholders V

$$\frac{\mathbb{E}\left[-1_{\{D^c\}}Y\right]+\chi(0)2\theta^T b}{\mathbb{P}[D^c](1+r+\lambda)}q-\frac{\psi(0)-\chi(0)2\sigma_Y^2-2\chi'(0)(2\theta^T b)+\mathfrak{C}(0)}{2(1+r+\lambda)}q^2,$$

where if we indicate $D^c(h)=D^c(\theta^*,q+h)$ and $D^c=D^c(0)$, we have

$$\psi(0)=\lim_{h\to 0}\mathbb{E}\left[(Y+P(0)(1+r+\lambda))\frac{1_{\{D^c(h)\}}-1_{\{D^c\}}}{h}\right]$$

$$\mathfrak{C}(0)=\sum_{i=1}^{d}\left(\sum_{j=1}^{d}\left(\chi(0)\frac{\partial^2 g}{\partial\theta_i\partial\theta_j}|_{q=0}-\frac{\partial^2 f}{\partial\theta_i\partial\theta_j}|_{q=0}\right)\frac{\partial\theta_j^*}{\partial q}|_{q=0}\right)\frac{\partial\theta_i^*}{\partial q}|_{q=0},$$

Also in this case we can identify the different contributions to the price: the expected value term $\mathbb{E}\left[1_{\{D^c\}}Y\right]$, the two covariance bits $-q\chi(0)2\theta^T b$ and $-2q^2\chi'(0)(2\theta^T b)$, a convexity adjustment $(\psi(0)-\mathfrak{C}(0))q^2$ term and a capital part $-\chi(0)2\sigma_Y^2$.

Optimizing the median: shareholder = whole bank I

Now we formulate our problem, including limited liability (shareholder), using the median instead of the expected value. On the use of quantiles function as objective functions for decision making and asset pricing see for example [110, 117].

We define a *median* $\mathcal{M}[X]$ of a real valued random variable X as

$$\mathcal{M}[X] := \inf_m \left\{ m \,|\, \mathbb{P}[X \leq m] \geq \frac{1}{2} \right\}$$

We make the reasonable assumption that there exist at least a $\bar{\theta} \in \Theta(q) = \left\{ \theta \in \mathbb{R}^2 \mid C_0 = \nu\sqrt{\theta^T A\theta + 2q\theta^T b + \sigma_Y^2 q^2} \right\}$ s.t.

$$q m_Y + \bar{\theta}^T(m_1 - S_0(1 + r + \lambda)) + (C_0 + P(q))(1 + r + \lambda) > 0,$$

Optimizing the median: shareholder = whole bank II

where $m_1 = \mathbb{E}[S_1]$ and $m_Y = \mathbb{E}[Y]$. This is equivalent to say that there exist an admissible portfolio choice such that on average the bank is solvent at time 1. Using the median as objective function we can then obtain the following.

Optimizing the median: shareholder = whole bank III

Median result. Consider, for a fixed q, the shareholder problem of finding P such that

$$w(q) = w(0),$$

where

$$\begin{aligned} w(q) &= \sup_{\theta \in \Theta(q)} \mathcal{M}\left[(qY + \theta^T(S_1 - S_0/D^\lambda) + (C_0 + P(q))/D^\lambda)^+\right], \\ \Theta(q) &= \left\{\theta \in \mathbb{R}^2 \mid C_0 = \nu\sqrt{\theta^T A\theta + 2q\theta^T b + \sigma_Y^2 q^2}\right\}. \end{aligned} \tag{6}$$

This indifference problem has the same solution as the indifference problem in the whole-bank case with the mean (and the median)

$$P(q) = \frac{1}{1+r+\lambda}\left(q\mathbb{E}[-Y] + \frac{1}{2}\left(\frac{1}{\chi(0)} - \frac{1}{\chi(q)}\right)\mu^T A^{-1}\mu + 2qb^T A^{-1}\mu\right).$$

Optimizing the median: shareholder = whole bank IV

$$P(q) = \frac{1}{1+r+\lambda}\left(q\mathbb{E}[-Y] + \frac{1}{2}\left(\frac{1}{\chi(0)} - \frac{1}{\chi(q)}\right)\mu^T A^{-1}\mu + 2qb^T A^{-1}\mu\right).$$

or small deal approximation:

$$P(q) \simeq D^\lambda \left(-q\mathbb{E}[Y] + \left(\frac{(\sigma_Y^2 - b^T A^{-1} b)q^2\nu}{C_0}\right)\frac{\sqrt{\mu^T A^{-1}\mu}}{2} + qb^T A^{-1}\mu\right)$$

Hence we see that the median approach gives the same result as the linear one.

In practice this means that the use of the median as a performance indicator puts shareholders on the same ground as the whole bank, aligning their interests with the bond holders, also for what concerns the impact of capital requirements on the valuation process.

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