Nonlinear valuation under initial & variation margins, funding costs, gap default closeout and multiple curves

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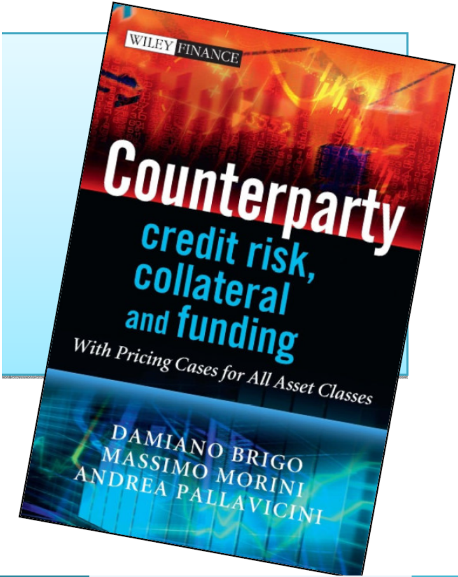
Content I

Presentation based on Book (working on 2nd Edition)



CVA and DVA can be sizeable. Citigroup:

1Q 2009: “Revenues also included [...] a net 2.5$ billion positive CVA on derivative positions, excluding monolines, mainly due to the widening of Citi’s CDS spreads” (DVA)

CVA mark to market losses: BIS

”During the financial crisis, however, roughly two-thirds of losses attributed to counterparty credit risk were due to CVA losses and only about one-third were due to actual defaults.”

Collateral not always effective as a guarantee: B. et al [19]

For trades subject to strong contagion at default of the counterparty, like CDS, collateral can leave a sizeable CVA.

FVA can be sizeable too. JP Morgan:

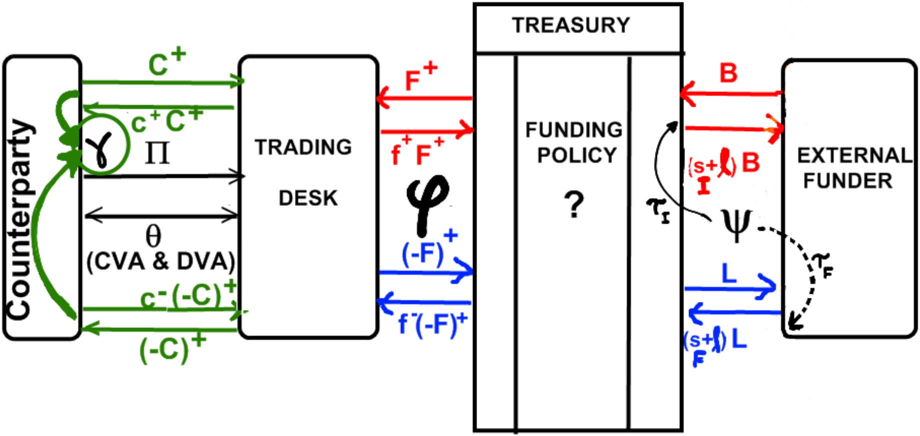
Wall St Journal, Jan 14, 2014: ‘[...] So what is a funding valuation adjustment, and why did it cost J.P. Morgan Chase $1.5 billion?’

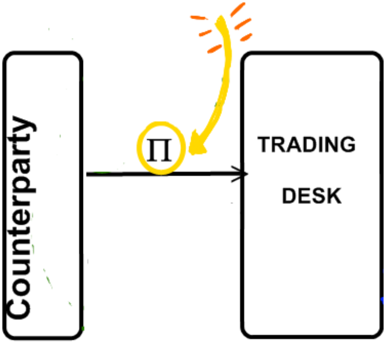
So all these effects are being modeled carefully...

... by the banking industry, in a consistent sound framework, right?

(with apologies to Blade Runner and The Matrix).
Let's try to do it properly (B. et al 2005-2015, 1st comprehensive & consistent analysis with default, collateral & funding in Pallavicini Perini & B. (2011)[79]).

Valuation with credit, collateral, funding & multi-curves





CLASSIC DISCOUNTED
CASH FLOWS

Basic Payout pre-credit and funding: Cash Flows

- Calculate prices by discounting cash-flows under the r-measure $\mathbb{Q}$. Collateral & funding are modeled as additional cashflows, as for Credit & Debit Valuation Adjustments (CVA & DVA)
- We start from derivative's basic cash flows without credit, collateral of funding risks

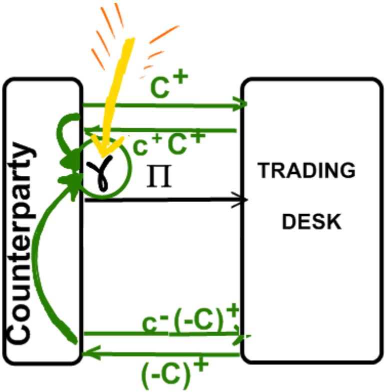
$$V_t \quad := \quad \mathbb{E}_t[\,\Pi(t, T \wedge \tau) + \dots]$$

where

$\longrightarrow$ $\tau := \tau_C \wedge \tau_I$ is the first default time (typically $\tau_{C,I}$ follow intensity models, and under conditional independence $\lambda = \lambda_I + \lambda_C$), and

$\longrightarrow$ $\Pi(t, u)$ is the sum of all payoff terms from t to u, discounted at t

Cash flows are stopped either at the first default or at portfolio's expiry if defaults happen later. We call $V_t^0 = \mathbb{E}_t \Pi(t, T)$



Adding pre-default Collateral Flows

Basic Payout with Collateral Costs & Benefits

- As second contribution we consider the collateralization procedure and we add its cash flows.

$$V_t := \mathbb{E}_t[\,\Pi(t, T \wedge \tau)\,] + \mathbb{E}_t[\,\gamma(t, T \wedge \tau; C) + \dots\,]$$

where

$\longrightarrow$ C_t is the collateral account defined by the CSA,

$\longrightarrow$ $\gamma(t, u; C)$ are the collateral margining costs up to time u.

- The second expected value originates what is occasionally called Liquidity Valuation Adjustment (LVA) in simplified versions of this analysis. We will show this in detail later.
- If $C > 0$ collateral has been overall posted by the counterparty to protect us, and we have to pay interest c^+.
- If $C < 0$ we posted collateral for the counterparty (and we are remunerated at interest c^-).

Basic Payout with Collateral Costs & Benefits

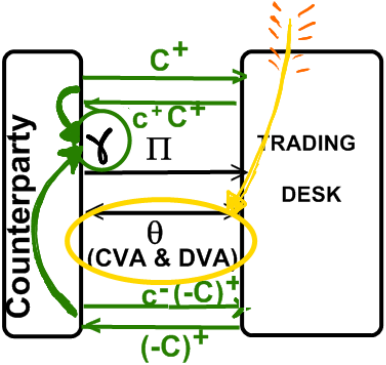
- The cash flows due to the margining procedure on the time grid $\{t_k\}$ are equal to (Linearization of exponential bond formulas in the continuously compounded rates)

$$\gamma(t,u;C) \approx -\sum_{k=1}^{n-1} 1_{\{t \le t_k < u\}} D(t,t_k) C_{t_k} \alpha_k (\tilde{c}_{t_k}(t_{k+1}) - r_{t_k}(t_{k+1}))$$

where $\alpha_k = t_{k+1} - t_k$ and the collateral accrual rates are given by

$$\tilde{c}_t := c_t^+ 1_{\{C_t > 0\}} + c_t^- 1_{\{C_t < 0\}}$$

Note that if the collateral rates in $\tilde{c}$ are both equal to the risk free rate, then this term is zero.



Adding default closeout Cash Flows (trading CVA and DVA after collateralization)

Close-Out: Trading-CVA/DVA after Collateralization

- As third contribution we consider the cash flow happening at 1st default, and we have

$$\begin{aligned} V_t &:= \mathbb{E}_t[\,\Pi(t, T \wedge \tau)\,] + \mathbb{E}_t[\,\gamma(t, T \wedge \tau; C)\,] \\ &+ \mathbb{E}_t\Big[\,1_{\{\tau < T\}} D(t,\tau)\theta_\tau(C,\varepsilon) + \dots\Big] \end{aligned}$$

  where ε_τ is the close-out amount, or residual value of the deal at default, "exposure at default"
- Replacement closeout, $\varepsilon_\tau = V_\tau$ (nonlinearity/recursion!). Under risk-free closeout, $\varepsilon_\tau = \mathbb{E}_\tau[\,\Pi(\tau, T)\,]$ (easier)
- We define the on-default cash flow θ_τ by including the pre-default value of the collateral account used by the close-out netting rule to reduce exposure. We can include liquidation delays (see [36]).

Close-Out: Trading-CVA/DVA after Collateralization

- The on-default cash flow $\theta_\tau(C, \varepsilon)$ can be calculated by following ISDA documentation. We obtain (LGD′ rehypothecation recovery)

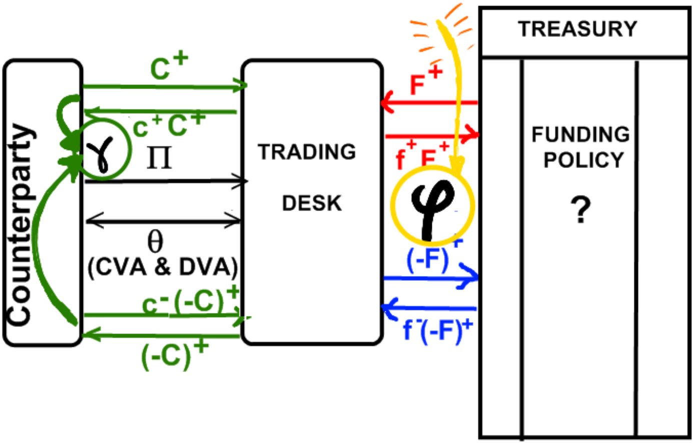
$$\begin{aligned}
\theta_\tau(C, \varepsilon) &:= \varepsilon_\tau - 1_{\{\tau=\tau_C<\tau_I\}}\Pi_{\text{CVAcoll}} + 1_{\{\tau=\tau_I<\tau_C\}}\Pi_{\text{DVAcoll}} \\
\Pi_{\text{CVAcoll}} &= \text{LGD}_C(\varepsilon_\tau^+ - C_{\tau^-}^+)^+ + \text{LGD}'_C(-(-\varepsilon_\tau)^+ + (-C_{\tau^-})^+)^+ \\
\Pi_{\text{DVAcoll}} &= \text{LGD}_I((-\varepsilon_\tau)^+ - (-C_{\tau^-})^+)^+ + \text{LGD}'_I(C_{\tau^-}^+ - \varepsilon_\tau^+)^+
\end{aligned}$$

- In case of re-hypothecation, when $\text{LGD}_C = \text{LGD}'_C$ and $\text{LGD}_I = \text{LGD}'_I$,

$$\Pi_{\text{DVAcoll}} = \text{LGD}_I(-(\varepsilon_\tau - C_{\tau^-}))^+, \quad \Pi_{\text{CVAcoll}} = \text{LGD}_C(\varepsilon_\tau - C_{\tau^-})^+.$$

- In case of no collateral re-hypothecation

$$\Pi_{\text{CVAcoll}} = \text{LGD}_C(\varepsilon_\tau^+ - C_{\tau^-}^+)^+, \ \Pi_{\text{DVAcoll}} = \text{LGD}_I((-\varepsilon_\tau)^+ - (-C_{\tau^-})^+)^+$$



Adding costs of funding the trade through the treasury (and offsetting Repo market)

Funding Costs of the Replication Strategy

- As fourth contribution we consider the cost of funding for the hedging procedures and we add the relevant cash flows (Pallavicini et al (2011)[79]).

$$\begin{aligned} V_t &:= \mathbb{E}_t[\,\Pi(t, T\wedge\tau)\,] + \mathbb{E}_t\left[\,\gamma(t, T\wedge\tau; C) + 1_{\{\tau<T\}} D(t,\tau)\theta_\tau(C,\varepsilon)\,\right] \\ &+ \mathbb{E}_t[\,\varphi(t, T\wedge\tau; F, H)\,] \end{aligned}$$

The last term, especially in simplified versions, is related to what is called FVA in the industry. We will point this out once we get rid of the rate r.

$\longrightarrow$ F_t is the cash account for the replication of the trade,
$\longrightarrow$ H_t is the risky-asset account in the replication,
$\longrightarrow$ $\varphi(t, u; F, H)$ are the cash F and hedging H funding costs up to u.

- In classical Black Scholes on Equity, for a call option (no credit risk, no collateral, no funding costs),

$$V_t^{\text{Call}} = \Delta_t S_t + \eta_t B_t =: H_t + F_t, \quad \tau = +\infty, \quad C = \gamma = \varphi = 0.$$

Funding Costs of the Replication Strategy

- Continuously compounding format and linearizing exponentials:

$$
\begin{aligned}
\varphi(t,u) &\approx -\sum_{j=1}^{m-1} 1_{\{t\le t_j<u\}} D(t,t_j)(F_{t_j}+H_{t_j})\alpha_k \left(\tilde{f}_{t_j}(t_{j+1}) - r_{t_j}(t_{j+1})\right) \\
&+ \sum_{j=1}^{m-1} 1_{\{t\le t_j<u\}} D(t,t_j) H_{t_j}\alpha_k \left(\tilde{h}_{t_j}(t_{j+1}) - r_{t_j}(t_{j+1})\right) \\
\tilde{f}_t &:= f_t^{+} 1_{\{F_t+H_t>0\}} + f_t^{-} 1_{\{F_t+H_t<0\}} \quad \tilde{h}_t := h_t^{+} 1_{\{H_t>0\}} + h_t^{-} 1_{\{H_t<0\}}
\end{aligned}
$$

- $\mathbb{E}$ of φ is related to the so called FVA. If treasury funding rates $\tilde{f}$ are same as asset lending/borrowing $\tilde{h}$ OR if H is perfectly collateralized with collateral included via re-hypothecation ($H=0$)

$$
\varphi(t,u) \approx \sum_{j=1}^{m-1} 1_{\{t\le t_j<u\}} D(t,t_j) F_{t_j}\alpha_k \left(r_{t_j}(t_{j+1}) - \tilde{f}_{t_j}(t_{j+1})\right)
$$

Funding Costs of the Replication Strategy

- If we distinguish borrowing and lending explicitly $\varphi(t,u) \approx$

$$\sum_{j=1}^{m-1} 1_{t \le t_j < u} D(t,t_j)\alpha_k \left[\underbrace{-(F_{t_j})^+ \left(f^+_{t_j}(t_{j+1}) - r_{t_j}(t_{j+1}) \right)}_{\mathbb{E}:\text{Funding Cost Adj: FCA}} + \underbrace{(-F_{t_j})^+ \left(f^-_{t_j}(t_{j+1}) - r_{t_j}(t_{j+1}) \right)}_{\mathbb{E}:\text{Funding Benefit Adj: FBA}} \right]$$

- If further treasury borrows/lends at risk free $\tilde{f} = r \Rightarrow \varphi = FVA = 0$.

Funding rates (more on this later)

f^+ & f^- are policy driven. EG, $f^+ = r + s^I + \ell^+$, $f^- = r + s^F + \ell^-$ ($s^{I/F}$ is our bank/the funder spread, typically $s^{I/F} = \lambda^{I/F}\, \mathrm{L}_{\mathrm{GD}\,I/F}$), ℓ are liquidity bases driven by treasury policy and market (CDS-Bond basis).

A Trader's explanation of the funding cash flows φ I

1. **Time** t: I wish to buy a call option with maturity T whose current price is $V_t = V(t, S_t)$. I need V_t cash to do that. So I borrow V_t cash from my bank treasury and buy the call.
2. I receive the collateral C_t for the call, that I give to the treasury.
3. Now I wish to hedge the call option I bought. To do this, I plan to repo-borrow $\Delta_t = \partial_S V_t$ stock on the repo-market.
4. To do this, I borrow $H_t = \Delta_t S_t$ cash at time t from the treasury.
5. I repo-borrow an amount Δ_t of stock, posting cash H_t guarantee.
6. I sell the stock I just obtained from the repo to the market, getting back the price H_t in cash.
7. I give H_t back to treasury.
8. Outstanding: I hold the Call; My debt to the treasury is $V_t - C_t$; I am Repo borrowing Δ_t stock.

A Trader's explanation of the funding cash flows φ II

9. **Time $t + dt$:** I need to close the repo. To do that I need to give back Δ_t stock. I need to buy this stock from the market. To do that I need $\Delta_t S_{t+dt}$ cash.
10. I thus borrow $\Delta_t S_{t+dt}$ cash from the bank treasury.
11. I buy Δ_t stock and I give it back to close the repo and I get back the cash H_t deposited at time t plus interest $h_t H_t$.
12. I give back to the treasury the cash H_t I just obtained, so that the net value of the repo operation has been

$$H_t(1 + h_t\, dt) - \Delta_t S_{t+dt} = -\Delta_t\, dS_t + h_t H_t\, dt$$

    Notice that this $-\Delta_t dS_t$ is the right amount I needed to hedge V in a classic delta hedging setting.
13. I close the derivative position, the call option, and get V_{t+dt} cash.

A Trader's explanation of the funding cash flows φ III

14. I have to pay back the collateral plus interest, so I ask the treasury the amount $C_t(1 + c_t\, dt)$ that I give back to the counterparty.
15. My outstanding debt plus interest (at rate f) to the treasury is $V_t - C_t + C_t(1 + c_t\, dt) + (V_t - C_t) f_t\, dt = V_t(1 + f_t\, dt) + C_t(c_t - f_t\, dt)$. I then give to the treasury the cash V_{t+dt} I just obtained, the net effect being

$$V_{t+dt} - V_t(1 + f_t\, dt) - C_t(c_t - f_t)\, dt = dV_t - f_t V_t\, dt - C_t(c_t - f_t)\, dt$$

16. I now have that the total amount of flows is :

$$-\Delta_t\, dS_t + h_t H_t\, dt + dV_t - f_t V_t\, dt - C_t(c_t - f_t)\, dt$$

A Trader's explanation of the funding cash flows φ IV

17. Now I present–value the above flows in t in a risk neutral setting.

$$\mathbb{E}_t[-\Delta_t\, dS_t + h_t H_t\, dt + dV_t - f_t V_t\, dt - C_t(c_t - f_t)\, dt] =$$

$$= -\Delta_t(r_t - h_t)S_t\, dt + (r_t - f_t)V_t\, dt - C_t(c_t - f_t)\, dt - d\varphi(t)$$

$$= -H_t(r_t - h_t)\, dt + (r_t - f_t)(H_t + F_t + C_t)\, dt - C_t(c_t - f_t)\, dt - d\varphi(t)$$

$$= (h_t - f_t)H_t\, dt + (r_t - f_t)F_t\, dt + (r_t - c_t)C_t\, dt - d\varphi(t)$$

This derivation holds assuming that $\mathbb{E}_t[dS_t] = r_t S_t\, dt$ and $\mathbb{E}_t[dV_t] = r_t V_t\, dt - d\varphi(t)$, where $d\varphi$ is a dividend of V in $[t, t + dt)$ expressing the funding costs. Setting the above expression to zero we obtain

$$d\varphi(t) = (h_t - f_t)H_t\, dt + (r_t - f_t)F_t\, dt + (r_t - c_t)C_t\, dt$$

which coincides with the definition given earlier.

The nonlinear nature of adjusted prices

$$(*) \quad V_t = \mathbb{E}_t\left[\,\Pi(t, T\wedge\tau) + \gamma(t) + 1_{\{\tau<T\}}D(t,\tau)\theta_\tau + \varphi(t)\,\right]$$

Can we interpret:

$\mathbb{E}_t\left[\,\Pi(t, T\wedge\tau) + 1_{\{\tau<T\}}D(t,\tau)\theta_\tau(C,\varepsilon)\,\right]$: RiskFree Price + DVA - CVA?

$\mathbb{E}_t[\,\gamma(t, T\wedge\tau) + \varphi(t, T\wedge\tau; F, H)\,]$: Funding adjustment LVA+FVA?

Not really. This is not a decomposition. It is an equation. In fact since

$$V_t = F_t + H_t + C_t \quad \text{(re–hypo)}$$

we see that the φ present value term depends on future $F_t = V_t - H_t - C_t$ and generally the closeouts depend on future V too. All terms feed each other and there is no neat separation of risks.

Under assumptions on market information, cash flows & dynamics of dependence one can fully specify the equation as a FBSDE or, adding a Markovian assumption for market risk, as a semi-linear PDE.

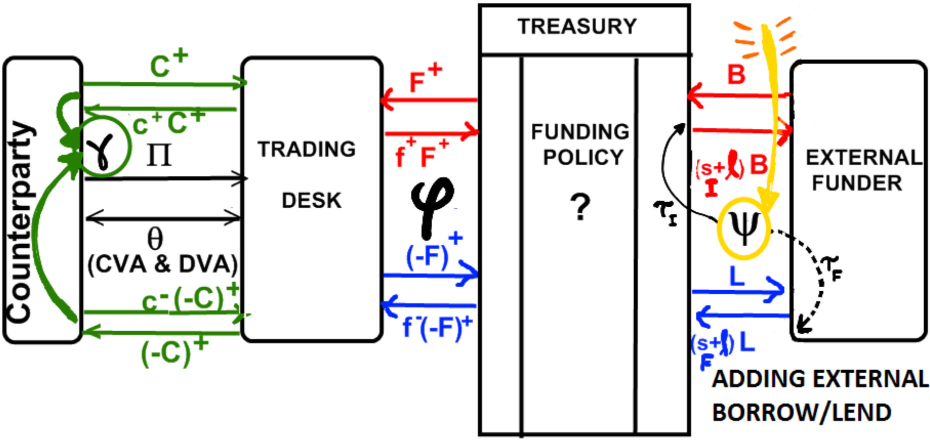
Interpretation: pricing measure & discounting

Using nonlinear Feynman Kac (coming from $\exists!$ of the FBSDE/SLPDE sol in B. & al 2015 [24]) we write the formula ($\pi_u\, du = \Pi(u, u + du)$)

$$V_t = \int_t^T \mathbb{E}^h\{D(t,u;f)[\pi_u + (\tilde{\theta_u} - \lambda_u V_u) + (f_u - c_u)C_u]|\mathcal{F}_t\}du$$

- This eq depends only on market rates, no theoretical r_t. invariance
- $\mathbb{Q}^h$ is the probability measure where the drift of the risky assets is h, the repo rate, that in turn depends on H and hence on V itself. Nonlinearity. Deal dependent measure.
- We discount at funding. Note that f depends on V, non-linearity. Deal dependent discount curve.
- θ_u are trading CVA and DVA after collateralization
- $(f_u - c_u)C_u$ is the cost of funding collateral with the treasury
- NO Explicit funding term for the replica as this has been absorbed in the discount curve and in the collateral cost

Treasury CVA & DVA



Treasury CVA & DVA

Include default risk of funder and funded ψ, leading to CVA$_F$ & DVA$_F$.

$$V_t = \mathbb{E}_t\Big[\,\Pi(t, T\wedge\tau) + \gamma(t) + 1_{\{\tau<T\}} D(t,\tau)\theta_\tau + \varphi(t) + \psi(t,\tau_F,\tau)\,\Big]$$

FVA $= -$FCA$+$FBA from f^+ & f^- largely offset by $\mathbb{E}\psi$ after immersion, approx & linearization.

Assume $H = 0$ (perfectly collateralized hedge with re–hypothecation), once f^+ & f^- are decided by policy, under *immersion*

- Underlying $\Pi(t, T)$ is not credit sensitive, technically $\mathcal{F}_t$-measurable; $\mathcal{F}$ pre-default filtration, $\mathcal{G}$ full filtration.
- τ_I and τ_C and τ_F are $\mathcal{F}$ conditionally independent (credit spreads can be correlated, jumps to default are independent);

we obtain a practical decomposition of price into

V = RiskFreePrice - CVA + DVA + LVA - FCA + FBA -CVA$_F$+ DVA$_F$

$$RiskFreePr = \int_t^T \mathbb{E}\left\{\pi_u \,|\mathcal{F}_t\right\}du; \quad LVA = \int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)(r_u - \tilde{c}_u)C_u|\mathcal{F}_t\right\}du$$

$$-CVA = \int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)\left[-\mathrm{L}_{\mathrm{GD}_C}\lambda_C(u)(V_u - C_{u-})^+\right]|\mathcal{F}_t\right\}du$$

$$DVA = \int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)\left[\mathrm{L}_{\mathrm{GD}_I}\lambda_I(u)(-(V_u - C_{u-}))^+\right]|\mathcal{F}_t\right\}du$$

$$-FCA = -\int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)\left[({f^+}_u - r_u)(V_u - C_u)^+\right]|\mathcal{F}_t\right\}du$$

$$FBA = \int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)\left[({f^-}_u - r_u)(-(V_u - C_u))^+\right]|\mathcal{F}_t\right\}du$$

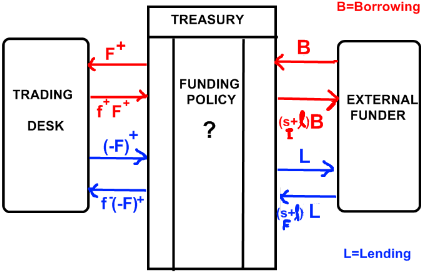
$$-CVA_F = \int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)\left[\mathrm{L}_{\mathrm{GD}_F}\lambda_F(u)(-(V_u - C_u))^+\right]|\mathcal{F}_t\right\}du$$

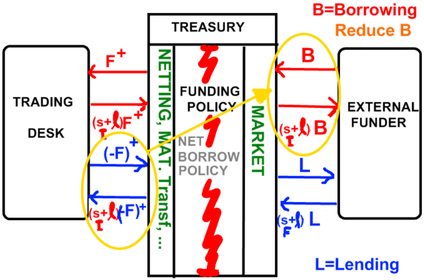
$$DVA_F = \int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)\left[\mathrm{L}_{\mathrm{GD}_I}\lambda_I(u)(V_u - C_u)^+\right]|\mathcal{F}_t\right\}du$$

Treasury CVA & DVA

To further specify the split, we need to assign f^+ (borrow) & f^- (lend)
There are two possible simple treasury models to assign f.

EXTERNAL FUNDING BENEFIT (EFB) VS **REDUCED BORROWING BENEFIT (RBB)**





$$f^+ = \mathrm{LGD}_I \lambda_I + \ell^+ =: s_I + \ell^+ \qquad\qquad f^+ = s_I + \ell^+$$

$$f^- = \mathrm{LGD}_F \lambda_F + \ell^- =: s_F + \ell^- \qquad\qquad f^- = s_I + \ell^-$$

Treasury CVA & DVA

$$-CVA = \int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)\left[-s_C(u)(V_u - C_{u-})^+\right]|\mathcal{F}_t\right\}du$$

$$DVA = \int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)\left[s_I(u)(-(V_u - C_{u-}))^+\right]|\mathcal{F}_t\right\}du$$

$$-FCA = -\int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)\left[(s_I(u)+\ell_u^+)(V_u - C_u)^+\right]|\mathcal{F}_t\right\}du$$

$$FBA = \int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)\left[(\underbrace{f_u^- - r_u}_{\text{EFB: } s_F + \ell^-;\ \text{RBB: } s_I + \ell^-})(-(V_u - C_u))^+\right]|\mathcal{F}_t\right\}du$$

$$-CVA_F = \int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)\left[s_F(u)(-(V_u - C_u))^+\right]|\mathcal{F}_t\right\}du$$

$$DVA_F = \int_t^T \mathbb{E}\left\{D(t,u;r+\lambda)\left[s_I(u)(V_u - C_u)^+\right]|\mathcal{F}_t\right\}du$$

Treasury CVA & DVA

The benefit of lending back to the treasury, two different models:

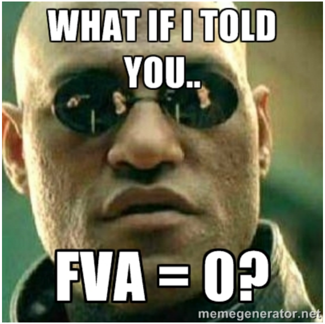
1. External funding benefit (EFB) policy: when desk lends back to treasury, treasury lends to F for interest $f^- = r + s^F + \ell^-$. Hence

$$V_{\text{EFB}} = V^0 - CVA + DVA + LVA \underbrace{-FCA}_{-DVA_F - FCA_\ell} + \underbrace{FBA}_{CVA_F + FBA_\ell} + DVA_F - CVA_F$$

2. Reduced borrowing benefit (RBB) policy: whenever trading desk lends back to the treasury, the latter reduces the desk loan outstanding and the desk saves at interest $f^- = r + s^l + \ell^-$. Hence

$$V_{\text{RBB}} = V^0 - CVA + DVA + LVA \underbrace{-FCA}_{-DVA_F - FCA_\ell} + \underbrace{FBA}_{DVA + FBA_\ell} + DVA_F$$

The "corporate finance" moment: What if I told you...



Hull White argued FVA =0.
Modigliani Miller? (MM)

- Mkt prices follow rnd walks,
- No taxes, No costs for
- bankruptcy or agency,
- No asymmetric information,
- & market is efficient

then value of firm does not depend on how firm is financed.
Without MM or corporate finance:

$$V_{\text{EFB}} = V^0 - CVA + DVA + LVA \underbrace{-FCA}_{-DVA_F - FCA_\ell} + \underbrace{FBA}_{CVA_F + FBA_\ell} + DVA_F - CVA_F$$

If bases $\ell = 0$, & if $r = \tilde{c}$, & if... $V_{\text{EFB}} = V^0 - CVA + DVA$ (no funding)
Too many if's? Even then, internal fund transfers happening.

Nonlinearities due to funding

Aggregation–dependent and asymmetric valuation

Valuation of a portfolio is aggregation dependent. Value of portfolio is not sum of values of assets. More: Without funding, price to one entity is minus price to the other one. No more with funding.

Price of Value? Charging clients?

Funding adjusted "price"? Not a price in conventional sense. Use it for cost/profitability analysis or internal fund transfers, but can we charge it to a client? How can client check our price is fair if she has no access to our funding policy & parameters and vice versa if client charges us?

Consistent global modeling across asset classes and risks

Once aggregation is set, funding valuation is non–separable. Holistic consistent modeling across trading desks & asset classes needed

Nonlinearities due to funding

Nonlinearity Valuation Adjustment (NVA) (B. et al (2014)[25])

NVA analyzes the double counting involved in forcing linearization. Our numerical examples for simple call options show that NVA can easily reach 2 or 3% of the deal value even in relatively standard settings.

Multiple interest rate curves (Pallavicini & B. (2013) [78])

.. can be embedded in the credit-funding theory above, explaining multiple curves as an effect of collateralization & funding policy

CCPs and initial margins (B. & Pallavicini (2014)[36])

These too can be included by adding the initial margin account cash flows and customizing it to the relevant initial margin rule, depending on the CCP or specific Standard CSA is trading over the counter

NVA: numerical example I

Equity call option (long or short), $r = 0.01$, $\sigma = 0.25$, $S_0 = 100$, $K = 80$, $T = 3y$, $V_0 = 28.9$ (no credit risk or funding/collateral costs). Precise credit curves are given in the paper.

$$NVA = V_0(nonlinear) - V_0(\text{linearized})$$

Table: NVA with default risk and collateralization

Funding Rates bps			Default risk, low[a]		Default risk, high[b]	
			Long	Short	Long	Short
f^+	f^-	$\hat{f}$				
300	100	200	-3.27 (11.9%)	-3.60 (10.5%)	-3.16 (11.4%)	-3.50 (10.1%)
100	300	200	3.63 (10.6%)	3.25 (11.8%)	3.52 (10.2%)	3.13 (11.3%)

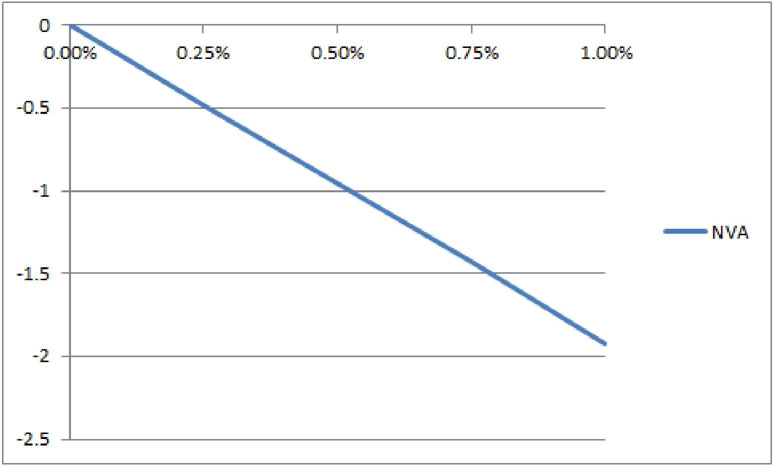
The percentage of the total call price corresponding to NVA is reported in parentheses.
[a] Based on the joint default distribution D_{low} with low dependence.
[b] Based on the joint default distribution D_{high} with high dependence.

NVA: numerical example II

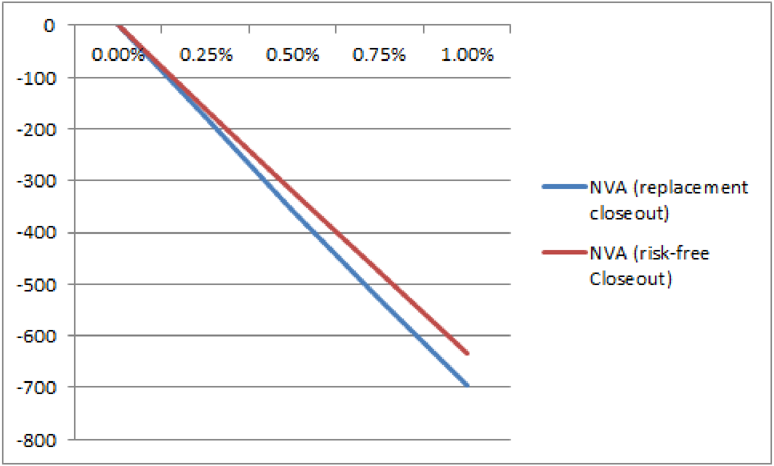
Table: NVA with default risk, collateralization and rehypothecation

Funding Rates bps			Default risk, low[a]		Default risk, high[b]	
			Long	Short	Long	Short
f^+	f^-	$\hat{f}$				
300	100	200	-4.02 (14.7%)	-4.45 (12.4%)	-3.91 (14.0%)	-4.35 (12.0%)
100	300	200	4.50 (12.5%)	4.03 (14.7%)	4.40 (12.2%)	3.92 (14.0%)

The percentage of the total call price corresponding to NVA is reported in parentheses.
[a] Based on the joint default distribution D_{low} with low dependence.
[b] Based on the joint default distribution D_{high} with high dependence.



NVA for long call as a function of $f^+ - f^-$, with $f^- = 1\%$, $f+$ increasing over 1% and $\hat{f}$ increasing accordingly. NVA expressed as an additive price component on a notional of 100, risk free option price 29. Risk free closeout. For example, $f + -f- = 25bps$ results in NVA=-0.5 circa, 50 bps $\Rightarrow$ NVA = -1



NVA for long call as a function of $f^+ - f^-$, with $f^- = 1\%$, $f+$ increasing over 1% and $\hat{f}$ increasing accordingly. NVA expressed as a percentage (in bps) of the linearized $\hat{f}$ price. For example, $f^+ - f^- = 25$bps results in NVA=-100bps = -1% circa, replacement closeout relevant (red/blue) for large $f^+ - f^-$

Multiple Interest Rate Curves

Derive interest rate dynamics consistently with credit, collateral and funding costs as per the above master valuation equations.

- We use our maket based (no r_t) master equation to price OIS & find OIS equilibrium rates. Collateral fees will be relevant here, driving forward OIS rates.
- Use master equation to price also one period swaps based on LIBOR market rates. LIBORs are market given and not modeled from first principles from bonds etc. Forward LIBOR rates obtained by zeroing one period swap and driven both from primitive market LIBOR rates and by collateral fees.
- We'll model OIS rates and forward LIBOR/SWAP jointly, using a mixed HJM/LMM setup
- In the paper we look at non-perfectly collateralized deals too, where we need to model treasury funding rates.
- See `http://ssrn.com/abstract=2244580`

Pricing under Initial Margins: SCSA and CCPs I

CCPs: Default of Clearing Members, Delays, Initial Margins...

Our general theory can be adapted to price under Initial Margins, both under CCPs and SCSA.

The type of equations is slightly different but quantitative problems are quite similar.

See B. and Pallavicini (2014) [36] for details (60 pages published paper). Here we give a summary.

Pricing under Initial Margins: SCSA and CCPs II

So far all the accounts that need funding have been included within the funding netting set defining F_t.

If additional accounts needed, for example segregated initial margins, as with CCP or SCSA, their funding costs must be added.

Initial margins kept into a segregated account, one posted by the investor ($N_t^I \leq 0$) and one by the counterparty ($N_t^C \geq 0$):

$$\begin{aligned} \varphi(t,u) \;&:=\; \int_t^u dv\,(r_v - f_v)F_v D(t,v) - \int_t^u dv\,(f_v - h_v)H_v D(t,v) \quad (1) \\ &+\; \int_t^u dv(f_v^{N^C} - r_v)N_v^C + \int_t^u dv(f_v^{N^I} - r_v)N_v^I\,, \end{aligned}$$

with $f_t^{N^C}$ & $f_t^{N^I}$ assigned by the Treasury to the initial margin accounts. $f^N \neq f$ as initial margins not in funding netting set of the derivative.

Pricing under Initial Margins: SCSA and CCPs III

$$\ldots + \int_t^u dv (f_v^{N^C} - r_v) N_v^C + \int_t^u dv (f_v^{N^I} - r_v) N_v^I$$

Assume for example $f > r$. The party that is posting the initial margin has a penalty given by the cost of funding this extra collateral, while the party which is receiving it reports a funding benefit, but only if the contractual rules allow to invest the collatera in low-risk activity, otherwise $f = r$ and there are no price adjustments.

We can describe the default procedure with initial margins and delay by assuming that at 1st default τ the surviving party enters a deal with a cash flow ϑ, at maturity $\tau + \delta$ (DELAY!).

δ 5d (CCP) or 10d (SCSA).

Pricing under Initial Margins: SCSA and CCPs IV

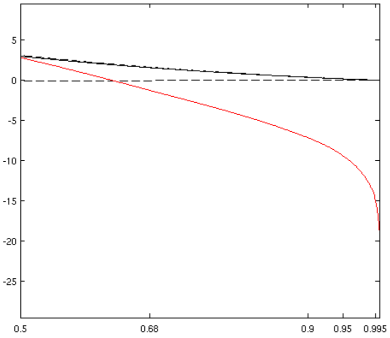
For a CCP cleared contract priced by the clearing member we have $N^I_{\tau^-} = 0$, whatever the default time, since the clearing member does not post the initial margin.

We assume that each margining account accrues continuously at collateral rate c_t.
We may further

- include funding default cloeseout and also
- define the Initial Margin as a percentile of the mark to market at time $\tau + \delta$.

This is done explicitly in the paper.

Now a few numerical examples:



Ten-year receiver IRS traded with a CCP.

Prices are calculated from the point of view of the CCP client.

Mid-credit-risk for CCP clearing member, high for CCP client.

Initial margin posted at various confidence levels q.

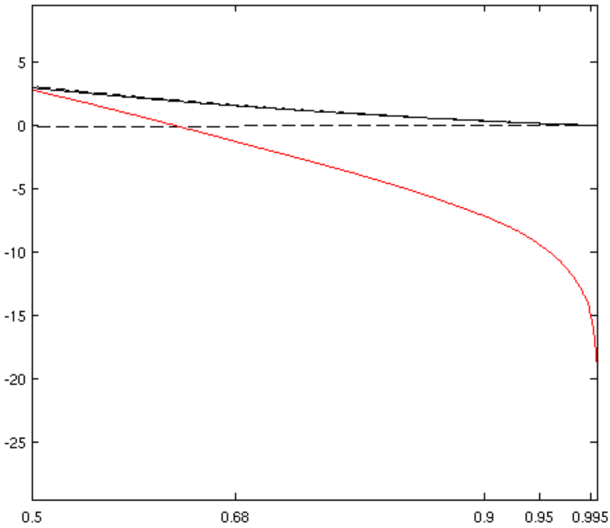
Prices in basis points with a notional of one Euro

Black continuous line: price inclusive of residual CVA and DVA after margining but not funding costs

Dashed black lines represent CVA and the DVA contributions.

Red line is the price inclusive both of credit & funding costs.

Symmetric funding policy. No wrong way correlation overnight/credit.


5
0
-5
-10
-15
-20
-25
0.5
0.68
0.9
0.95
0.995

CCP Pricing: Tables (see paper for WWR etc)

Table: Prices of a ten-year receiver IRS traded with a CCP (or bilaterally) with a mid-risk parameter set for the clearing member (investor) and a high-risk parameter set for the client (counterparty) for initial margin posted at various confidence levels q. Prices are calculated from the point of view of the client (counterparty). Symmetric funding policy. WWR correlation $\bar{\rho}$ is zero. Prices in basis points with a notional of one Euro.

	Receiver, CCP, $\beta^- = \beta^+ = 1$				Receiver, Bilateral, $\beta^- = \beta^+ = 1$			
q	**CVA**	**DVA**	**MVA**	**FVA**	**CVA**	**DVA**	**MVA**	**FVA**
50.0	-0.126	3.080	0.000	-0.1574	-2.1317	4.3477	0.0000	-0.0842
68.0	-0.066	1.605	-2.933	0.1251	-1.1176	2.2613	-4.1389	0.2491
90.0	-0.015	0.357	-8.037	0.5492	-0.2578	0.4997	-11.3410	0.7924
95.0	-0.007	0.154	-10.316	0.7205	-0.1149	0.2151	-14.5561	1.0250
99.0	-0.001	0.025	-14.590	1.0290	-0.0204	0.0346	-20.5869	1.4544
99.5	-0.001	0.013	-16.154	1.1402	-0.0107	0.0176	-22.7947	1.6107
99.7	-0.000	0.008	-17.233	1.2165	-0.0070	0.0114	-24.3164	1.7184
99.9	-0.000	0.004	-19.381	1.3684	-0.0035	0.0056	-27.3469	1.9326

- Default Closeout and Credit Risk cash flows,
- Collateral Flows, Funding flows,
- & Treasury flows interact nonlinearly and should be included in valuation in a consistent way. No easy split in CVA, DVA, FVA etc
- Elementary financial facts like closeout clauses and asymmetric borrowing and lending rates lead to dramatic consequences
- Mathematical consequences: nonlinear valuation operators, FBSDEs or nonlinear PDEs
- Financially, this leads to deal dependent valuation measures, aggregation dependent prices that do not add up on portfolios, and a debate on price vs value and on whether it is right to charge the client for these adjustments.
- Adding more and more additive adjustments for new or neglected risks is a dangerous practice that can get out of control.
- It may be necessary to linearize, operationally, but the error should be kept under control (Nonlinearity Valuation Adjustment, NVA?)
- Latest adj K(apital)VA is a strange animal and does not fit well...

Coming soon: EVA

(Electricity-bill Valuation Adjustment)

Thank you for your attention!

Questions?

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