Assessing Euro exit probabilities

via market implied quantities and historical data

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Agenda I

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Based on 2001-15 Research and on Books
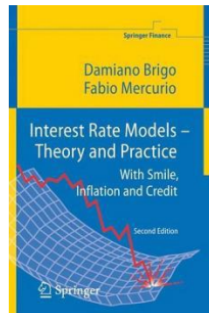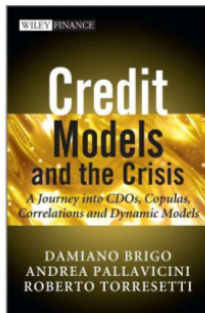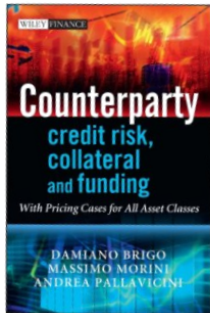
Many papers available for free in SSRN, arXiv, Repec,
See references at the end of the course

CDS and Defaultable Bonds: Intensity Models

Currently the two most important credit products, besides loans, are defaultable bonds and credit default swaps (CDS).

The risk in these products is driven primarily by default of the bond issuer or of the CDS reference entity, and secondarily by interest rate dynamics. We denote the short term interest rate at time t by r_t.

Default risk is typically included by modeling the default time, a random variable denoted by

$$\boxed{\tau}$$

and the recovery rate at default, R_{EC}, or the loss given default L_{GD} rate, $L_{GD} = 1 - R_{EC}$.

In intensity models the random default time τ is exponentially distributed, the distribution characterized by a given *intensity*.

CDS and Defaultable Bonds: Intensity Models

A strictly positive curve $t \mapsto \lambda_t$ called *default intensity* (or hazard rate) is given for the bond issuer or the CDS reference name.

The *cumulated intensity* (or hazard function) is the process $t \mapsto \int_0^t \lambda_s \, ds =: \Lambda_t$. Since λ is positive, Λ is increasing in time.

We will see in a minute λ is local default probability and credit spread.

More in detail, default time defined as inverse of cumulative intensity on an exponential random variable ξ with mean 1 and independent of λ

$$\tau = \Lambda^{-1}(\xi).$$

We now show how τ is simulated in an intensity model.
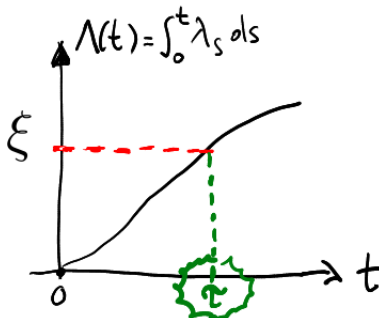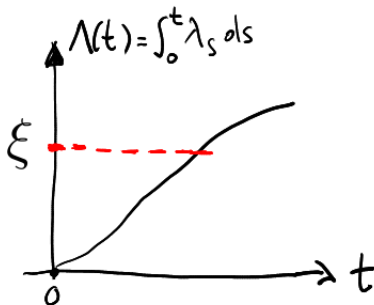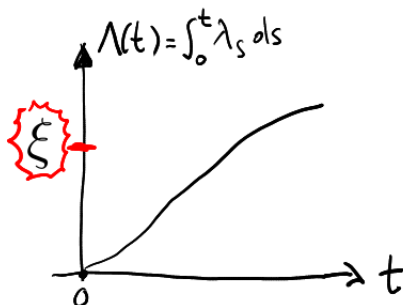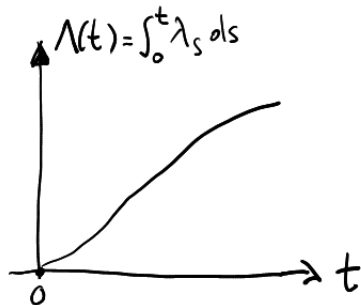
CDS and Defaultable Bonds: Intensity Models

Credit spreads are very volatile, with vols often above 50% (see eg B. (2005) [6]). Hence the intensity should be random (stochastic process) to allow for credit spread volatility.

Such models and a related discussion on CDS calibration and options are presented in B et al (2005, 2010) [2, 7], see also [3, 4]. Here we assume deterministic intensities $t \mapsto \lambda_t$, later even constant $t \mapsto \lambda$.

Denote by $\mathbb{Q}$ the pricing probability measure, or "Risk Neutral Measure", characterizing no-arbitrage. Prices of financial products are expected values $\mathbb{E}^{\mathbb{Q}}$ under this measure of cash flows discounted at the risk free rate r. We write $\mathbb{E} = \mathbb{E}^{\mathbb{Q}}$.

We will denote by $\mathbb{P}$ the Physical or Historical probability measure instead, under which risk is measured (VaR, ES are statistics of the loss distribution under $\mathbb{P}$).

CDS and Defaultable Bonds: Intensity Models

Look at a defaultable zero coupon bond with zero recovery $\bar{P}$ and compare it with the default-free bond P. A quick calculation yields

$$P(0, T) = \mathbb{E}\left[e^{-\int_0^T r_s \, ds}\right], \qquad \bar{P}(0, T) = \mathbb{E}\left[e^{-\int_0^T (r_s + \boxed{\lambda_s}) \, ds}\right]$$

So the price of a defaultable bond is like the price of a default-free bond *where the risk free discount short rate r has been replaced by r plus a spread λ*. Additional yield compensates for default risk.

Terms similar to $\bar{P}$ are also involved in CDS calculations

We further have (exponential ξ): $\mathbb{Q}(\tau \in [t, t + dt)|\tau > t, "\lambda[0, t]") = \lambda_t \, dt$. "probability of defaulting in $[t, t + dt)$ given no default before t is $\lambda_t \, dt$".

λ **is instantaneous credit spread &** $\lambda \, dt$ **is local default probability**

ξ **is pure jump to default risk**

The case with constant intensity $\lambda_t = \lambda$: CDS

If the intensity λ_t is constant, and S^{MID} is the quoted CDS premium spread at which the two legs of the CDS have the same value, then (see [3] or [4])

$$\lambda = \frac{S^{MID}}{1 - R_{EC}}$$

from which we see that also **the CDS premium rate S is indeed a sort of CREDIT SPREAD, or INTENSITY.**

Intuition: $S = \lambda\,(1 - R_{EC}) = \lambda\,L_{GD}$,   meaning
CDS Spread $\approx$ Default Probability $\times$ (1 - Recovery),
or annual compensation for the risk of losing 1-R_{EC}.

$t \mapsto \lambda_t$ can be taken as time dependent too, in which case CDS market rates S^{MID} for different maturities can be used to imply the intensity. We give the Lehman example now.

- **August 23, 2007**: Lehman announces that it is going to shut one of its home lending units (*BNC Mortgage*) and lay off 1,200 employees. The bank says it would take a \$52 million charge to third-quarter earnings.
- **March 18, 2008**: Lehman announces better than expected first-quarter results (but profits have more than halved).
- **June 9, 2008**: Lehman confirms the booking of a \$2.8 billion loss and announces plans to raise \$6 billion in fresh capital by selling stock. Lehman shares lose more than 9% in afternoon trade.
- **June 12, 2008**: Lehman shakes up its management; its chief operating officer and president, and its chief financial officer are removed from their posts.
- **August 28, 2008**: Lehman prepares to lay off 1,500 people. The Lehman executives have been knocking on doors all over the world seeking a capital infusion.
- **September 9, 2008**: Lehman shares fall 45%.
- **September 14, 2008**: Lehman files for bankruptcy protection and hurtles toward liquidation after it failed to find a buyer.

Lehman Brothers CDS Calibration: July 10th, 2007

On the left part of this Table we report the values of the quoted CDS spreads before the beginning of the crisis. We see that the spreads are very low. In the middle of Table 1 we have the results of the exact calibration obtained using a *piecewise constant* intensity model.
For the time being ignore the last two columns
Recovery at 40%! Actual one will be $\approx 8\%$.

T_i	S_i (bps)	λ_i	Surv (Int)	σ_i	Surv (AT1P)
10 Jul 2007			100.0%		100.0%
1y	16	0.267%	99.7%	29.2%	99.7%
3y	29	0.601%	98.5%	14.0%	98.5%
5y	45	1.217%	96.2%	14.5%	96.1%
7y	50	1.096%	94.1%	12.0%	94.1%
10y	58	1.407%	90.2%	12.7%	90.2%

Table: Results of calibration for July 10th, 2007.

Lehman Brothers CDS Calibration: June 12th, 2008

Middle of the crisis. CDS spreads S_i have increased with respect to the previous case, but are not very high, indicating the fact that the market is aware of the difficulties suffered by Lehman but thinks that it can come out of the crisis. Notice that now the term structure of both R and *intensities* is inverted. This is typical of names in crisis (buyers worry more about short term default than long term one, locally)

T_i	S_i (bps)	λ_i	Surv (Int)	σ_i	Surv (AT1P)
12 Jun 2008			100.0%		100.0%
1y	397	6.563%	93.6%	45.0%	93.5%
3y	315	4.440%	85.7%	21.9%	85.6%
5y	277	3.411%	80.0%	18.6%	79.9%
7y	258	3.207%	75.1%	18.1%	75.0%
10y	240	2.907%	68.8%	17.5%	68.7%

Table: Results of calibration for June 12th, 2008.

Lehman Brothers CDS Calibration: Sept 12th, 2008

In this Table we report the results of the calibration on September 12th, 2008, just before Lehman's default. We see that the spreads are now very high, corresponding to lower survival probability and higher intensities than before.

T_i	S_i (bps)	λ_i	Surv (Int)	σ_i	Surv (AT1P)
12 Sep 2008			100.0%		100.0%
1y	1437	23.260%	79.2%	62.2%	78.4%
3y	902	9.248%	65.9%	30.8%	65.5%
5y	710	5.245%	59.3%	24.3%	59.1%
7y	636	5.947%	52.7%	26.9%	52.5%
10y	588	6.422%	43.4%	29.5%	43.4%

Table: Results of calibration for September 12th, 2008.

Until the end, rating agencies maintained a good rating for Lehman[1]

In our [S&P] view, Lehman [...] had adequate liquidity relative to reasonably severe and foreseeable temporary stresses. [...] We believe the downfall of Lehman reflected escalating fears that led to a loss of confidence – ultimately becoming a real threat to Lehmans viability in a way that fundamental credit analysis could not have anticipated with greater levels of certainty

If we check from published transition matrices what has been the probability that a S&P "A" rated name defaulted we have

$\mathbb{P}$(A rated name defaults within 1y) $= 0$ in $2005/6/7$, and 0.38% in 2008

Compare with CDS market: $\mathbb{Q}$ default prob in 1y: 21%.
0.38% vs 21%. **Huge risk premium between $\mathbb{P}$ and $\mathbb{Q}$ for Lehman.**
This is a regular feature: market implied $\mathbb{Q}$ default probabilities are typically larger than fundamental history-based ones under $\mathbb{P}$.

[1] http://ww2.cfo.com/banking-capital-markets/2008/09/
/rating-itself-sp-defends-lehmans-a/ accessed Sept 9 2014

Euro exit probability index : Preliminaries

Although in the sector of financials, Lehman example highlights:

1. Often, traditional credit ratings are not very responsive, but they are relatively stable.
2. Market implied default probabilities/intensities (CDS & bonds) are more responsive but also volatile, & they contain risk premia.

This is confirmed by sovereign data.

	Cyprus	Italy	Greece
PD	51%	48%	37%
PD Vol	25%	41%	35% [61%]

Table: PDs in the period 17/04/2013 to 2/12/2014 [14/07/2014 to 13/03/2015] and their historical volatilities

Key idea: a mixed approach. Can we combine the two approaches? Retain responsive probabilities without excessive volatilities?

Cepix: Definition. Market implied PD

Define the intensity from a liquid sovereign bond of country i:

$$\lambda_i = \frac{BondMarketCreditSpread_i}{1 - Recovery_i} = \frac{\text{BondYield}_i - \text{RiskFreeRate}}{1 - \text{Recovery}_i}.$$

The risk free rate can be taken as the lowest government bond yield in the Euro area at the relevant time for the rating class Aaa (Moodys).

From the hazard rate one immediately computes the survival probability over a given time horizon, and from this the default probability. In continuous compounding, the default probability (PD) over one year reads (MI=market implied)

$$\text{PD}_i^{MI} = \mathbb{Q}(\text{Default of country } i \text{ before 1y}) = 1 - e^{-\lambda_i \, 1y} \approx \lambda_i$$

for small λ_i.

Cepix: Definition. Rating implied PD

$$PD_i^{RA} = \mathbb{P}(\text{Default of rating class of country } i \text{ before 1y})$$

(from rating transition matrices). Notice this is a default probability under $\mathbb{P}$.

However, as we observed in the Lehman case, this probability is not always updated in a responsive fashion, so on its own it would not suffice for most practical purposes.

Cepix: Definition. Comining PD^{MI} and PD^{RA}

We define the CEPIX PD, the euro exit probability for country *i*, as a weighted average of market implied and rating agencies PDs:

$$PD_i = w_i PD_i^{MI} + (1 - w_i) PD_i^{RA}.$$

Weights are defined as

$$w_i = \frac{Debt_i / GDP_i}{\max_k (Debt_k / GDP_k)}.$$

We weight market implied data more for countries whose debt is larger compared to GDP. We reason that the market has more incentive to hedge credit risk for those countries, so that market implied information should be weighted more.

Cepix: Definition. Comining PD^{MI} and PD^{RA}

Finally, the obtained combined probabilities PD$_i$ s are smoothed over an historical window by using an exponentially weighted moving average (EWMA) with period of 20, i.e.

$$PD_t^{EWMA} = \alpha PD_t + (1 - \alpha)PD_{t-1}^{EWMA}$$

where $\alpha = 2/(period + 1)$ with *period* $= 20$.

Cepix: Definition

Sovereign bonds from which we extract intensities should be chosen primarily according to two features:

1. Maturity close to the desired horizon;
2. High liquidity.

In case one decided to use sovereign CDS contracts instead, the same criteria should be applied. Moreover, CDS offering protection on Euro countries often pay protection denominated in US dollars and are subject to FX effects, see for example [5].

For liquidity, in general an input price average between bid/ask should be considered. If bid/ask is too large a different more liquid maturity may be considered.

Cepix: Definition

As a source to determine PD_i^{RA}, i.e. to determine probabilities of default suggested by the relevant rating agency, we chose Moodys. Since Moodys assigns only 4 different PDs to all 21 ratings, we use cubic interpolation to derive the PD for each rating separately. For all ratings Caa1 to C we take Moodys proposed single PD value and we do not extrapolate PDs as this might be misleading. Moreover, only few countries are rated with these grades.
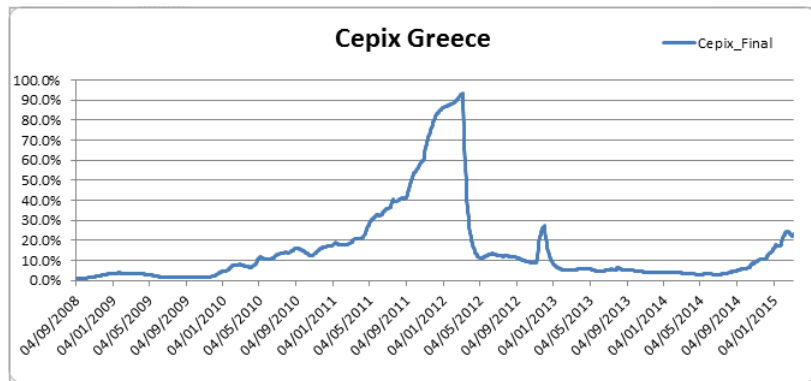
Cepix: history and highlights. Greece I
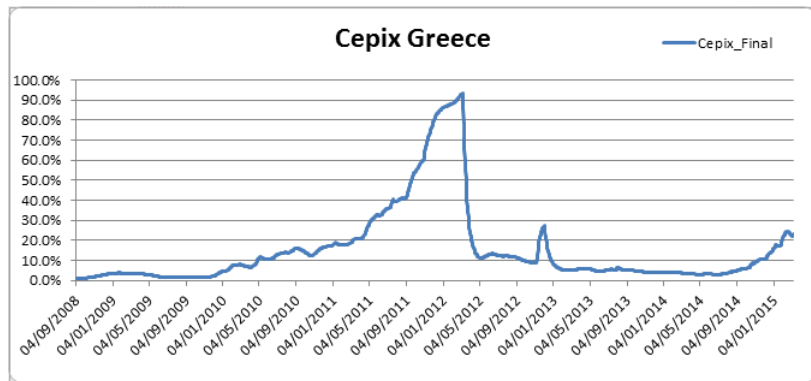
High debt/gdp, so mostly market implied.
In March 2012 Greece announced sovereign default, largest in history
by a government. Recognized as a "credit event" and triggered the
relevant payments in the related CDS.

# Cepix: history and highlights. Greece II

The debt write-off had a size of €107 billion, and lowered the Greek debt level to roughly €240bn in March 2012 from the previous level of €350bn. Cepix anticipated these events by starting the steady increase from 2% in December 2009 to 93.0% on 05 March 2012.

Cepix: history and highlights. Greece III
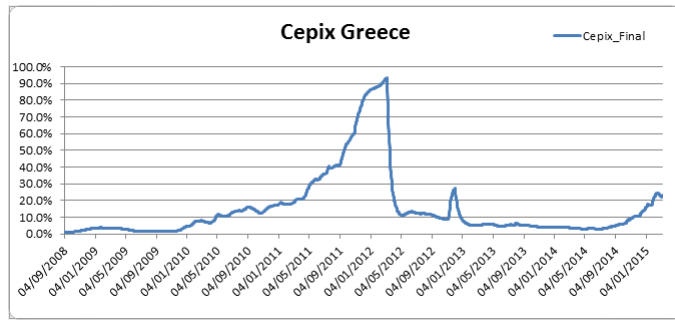
Afterwards, the index lowered to rise again abruptly, although on a smaller scale, starting on 6 November 2012 from a value of 8.6%. The index rose up to 27.1% in 5 December 2012 (sharp spike on the right hand side). This 2nd jump reflected the uncertainty on Greece restoring the budget into a self-financing situation (delayed reform schedule and worsened recession).

Cepix: history and highlights. Greece IV

Cepix Greece
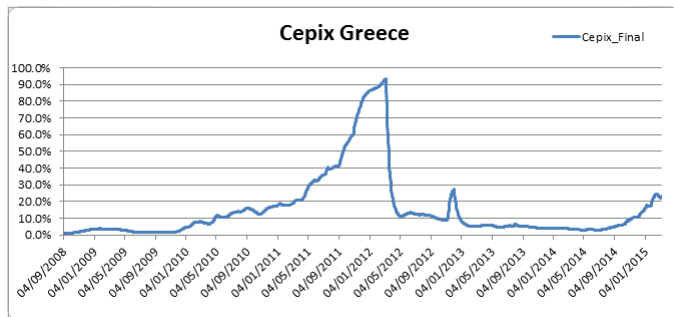
On 12 November 2012 international institutions decided on the next bailout program for Greece by lowering interest rates and prolonging debt maturities, thus providing Greece with €10bn for debt-buy-back. Towards the end of 2014, Greece Cepix has shown a rise in Greece exit likelihood (last part of plot), moving from 11% at the beginning of December to approximately 18% in mid-January (final part of plot).

Cepix: history and highlights. Greece V

This last near-double jump correlates directly with the announcement (Dec 9 2014) to bring the Greek presidential elections forward.

Whilst 18% looked relatively low, it suggested the market was less worried about a default in Greece than the rating agencies.

Varoufakis, the Greek economist who supported the favorite candidate Tsipras (Syriza party) and who would become finance minister later, had argued that "Exit from the euro is not an idea that a Syriza government will ever entertain or use as a negotiating strategy".

The market was worrying more about a possible Grexit after elections had been announced than before, but in absolute terms probabilities were still lower than one would have expected given Greece rating.

Cepix: history and highlights. Greece VI

Still, the subsequent initial failure of negotiations between Greece and
a group of Eurozone countries on the conditions of the extension of the
bailout that was to expire on February 28, revived fears on debt
refinancing, sparking turmoil and, potentially, a bank run.

This event raised Cepix to high levels for the third time, 25% on
January 18, 2015.

On the following day the agreement between Greece and creditors
was struck to extend a bailout program by four months. Cepix started
to move slightly downwards but continued to remain on the high level
of 23% until March 13 , as short- and long-term risks still loomed over
the following period.

Cepix: history and highlights. Greece VII
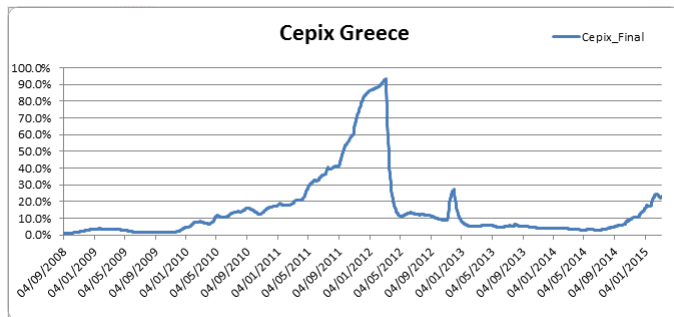
First, on February 24 the reform list delivered by Greece was admittedly accepted by EU finance ministers, but the four-month extension still required an approval of national parliaments in Greece and Germany. Second, the bailout amount would not be paid out to Greece until Greece fulfilled all conditions. As we wrapped up the index project, the situation of Greece was still evolving and uncertain.

# Cepix: history and highlights. Slovenia & Cyprus I
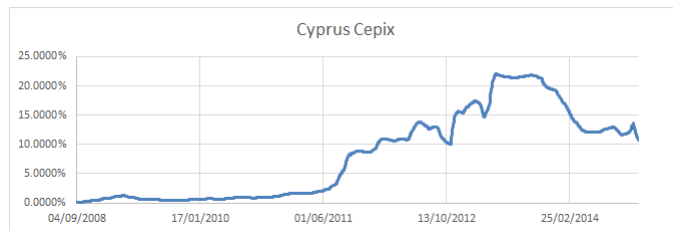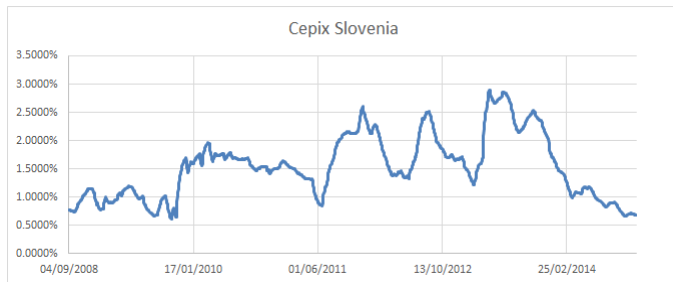
Cepix: history and highlights. Slovenia & Cyprus II

The index jumps showed signs of increasing sovereign default risk for these countries ahead of the bailout actions that were taken to avoid their default. Likelihood of the sovereign default did not originally derive from public debt and was of different origin in the two states.

- Slovenia, the trigger was the indebtedness of the citizens and local companies;
- Cyprus: one of the key factors was the large pile of Greek bonds among the banks assets.

The Cepix weight of market implied default probability for Slovenia amounted to about 40% and was much lower than for Cyprus, the latter having a weight of about 65% due to higher public debt in relation to GDP.

Cepix: history and highlights. Slovenia & Cyprus III

Cepix Slovenia February-April 2013
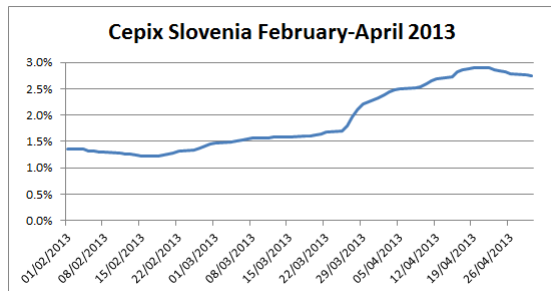
The increase in the Cepix value for Slovenia has gone from 1.22% to 2.89% in the period from 18 February to 19 April 2013. This properly reflected the Slovenian deteriorating creditworthiness due to banks mounting losses on their loans. In 2012 and 2013 the ratio of non-performing to total loans rose from 13.2% to 17.4%. That was the highest level in the Euro zone after Greece and Ireland.

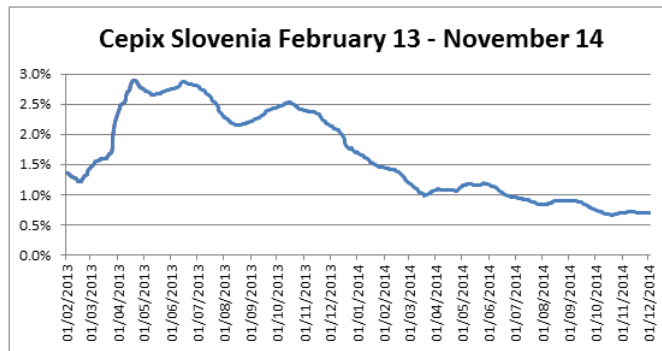# Cepix: history and highlights. Slovenia & Cyprus IV

The bad loans mainly occurred in lending to businesses burdened by debt, which is particularly high in relation to equity.

The Slovenian banking industry possessed assets worth around 130% of GDP at this time and the three largest Slovenian state-controlled banks sought imminent recapitalization.

On 12 December 2013, Slovenia announced a plan to inject €3bn into the country's three largest banks to help cover a €4.8bn, or 15% of GDP, capital shortfall in the financial sector. This resulted in the rise of the countrys sovereign debt to 76.5% of GDP, from only 22% of GDP in 2008. In 2013 and 2014 reforms continued to be implemented across the banking and corporate sectors and the economy could soon show signs of improvement.

Cepix: history and highlights. Slovenia & Cyprus V

The clean-up of banks balance sheets, the privatization reducing the role of the state in the economy and a significant fiscal consolidation created the conditions needed for regaining investors trust. This was reflected in Cepix gradually falling to 0.7% in December 2014.

Cepix Slovenia February 13 - November 14

Cepix: history and highlights. Slovenia & Cyprus VI
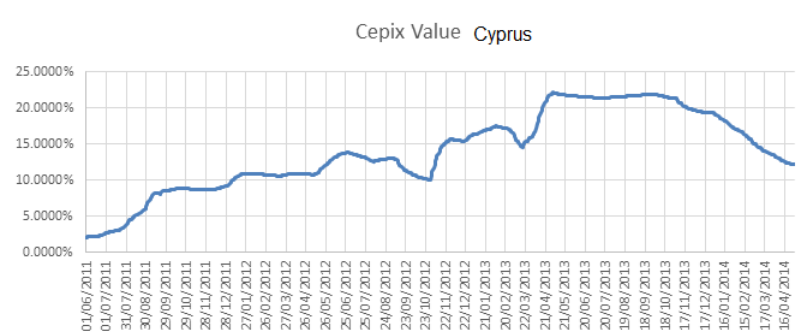
Cepix for Cyprus increased heavily in the period from June 2011 to May 2013, from the level of 2.1% to 22.0%. On 25 June 2012, Cyprus requested a bailout from the European Financial Stability Facility / *European Stability Mechanism* (hello!).

Cepix: history and highlights. Slovenia & Cyprus VII

What caused this call for help was the exposure of the country banking sector to the Greek debt haircut. Cypriot banks lost €22bn due to the Greek debt haircut.

Rating agencies downgraded Cyprus to junk status and the government was unable to refund its state expenses. On 25 March 2013, it was agreed to close the most troubled Bank, Laiki Bank, which reduced the needed loan amount for the overall bailout package. The bailout amount of €10bn was granted without need for imposing a general levy on bank deposits. The €10bn bailout raised Cypriot debt from almost 90% to 143% of GDP.

However, this step in combination with banking sector reform, cutting budget spending & privatization started to yield results & dramatically lowered risk of insolvency. By April 2014 Cepix halved to 11%.

Cepix: history and highlights. Slovenia & Cyprus VIII

Cepix Slovenia
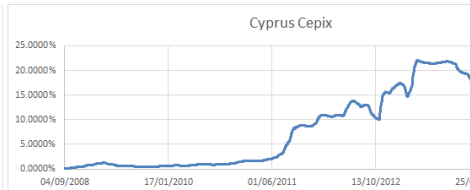
Cyprus Cepix

The observed large nominal Cepix differences between Slovenia and Cyprus reveal the important difference in credit severity.

Slovenia was able to cope with the failing banks on its own without bailout from the international institutions, contrary to Cyprus.

Conclusions

We designed an index based on a mixture of market implied and rating implied default probabilities, the weights in the mixture being determined by debt over gdp ratios.

The index performed well and we highlighted its performance in relation with the credit crises of Greece, Slovenia and Cyprus.

The index would allow us to run a similar analysis for other countries with interesting credit features, such as Italy, Portugal, Spain, France and Ireland, among others.

It may be useful as a tool to monitor the credit situation of sovereign entities in the Euro area.

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      Probability Index (CEPIX): Methodology and History of an Index
      Expressing Euro Exit Likelihoods. Journal of Financial
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      http://ssrn.com/abstract=2622631

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