M3/M4S3 STATISTICAL THEORY II

COURSE SUMMARY

1. ASYMPTOTIC THEORY

1.1 Basic Mathematical Tools and Notation

- limits, order notation, continuity, limits of functions
- supremum, infimum
- limit superior, limit inferior

1.2 Probability Spaces

- Sigma Algebras
- Measure Spaces
- Measure and its properties : continuity of measure
- Measurable functions
- Indicator Functions
- Representations using Simple Functions

1.3 Integration

- Integral with respect to measure: Lebesgue-Stieltjes Integration
- Simple Functions and their integrals
- Integrals of general functions: The Supremum Definition, existence and integrability
- Sets of measure zero and almost sure/almost everywhere existence
- Basic Properties of integrals with respect to measure

1.4 Convergence for Random Variables

- Convergence in Law
- Convergence in Probability
- Convergence in r^{th} mean
- Convergence almost surely
- Stochastic Order Notation
- Relating the Modes of Convergence
- *Theorem: Equivalence of almost sure convergence definitions*
- Relations between the modes of convergence and Converses
- *Theorem: The Borel-Cantelli Lemma*

1.5 Laws of Large Numbers

- Characteristic Functions and their properties
- Mean-Value Theorem and Taylor Theorem
- *Theorem: The Weak and Strong Laws of Large Numbers*
- Consistency
- The empirical distribution function
- *Theorem: The Glivenko Cantelli Lemma*

1.6 Convergence of Transformed Sequences

- *Theorem : Slutsky's Theorems*
- Continuous Mapping Theorem

1.7 Central Limit Theorems

- *Theorem: The basic (Lindeberg-Levy) CLT*
- Continuity Theorem
- *Theorem: The Lindeberg-Feller CLT for non-iid case (statement not proof)*
- Asymptotic Normality
- *Delta Method (Cramer's Theorem) for transformed variables*
- The Chi-square statistic and its distribution
- Hellinger Distance
- *Theorem: Asymptotic Distribution of Sample Quantiles*

2. Likelihood Theory and Extensions

2.1 Extending the Strong Law

- The Le Cam Result on Uniform Strong Consistency

2.2 Maximum Likelihood Estimation

- Basic MLE approach; solutions to the Likelihood Equations
- Kullback-Liebler divergence
- *Theorem: Properties of the KL divergence*
- *Theorem: Wald Theorem on the Consistency of the MLE (Statement not Proof)*

2.3 Asymptotic Behaviour of the MLE

- Efficient Estimation: Score Function and Fisher Information.
- *Theorem: The Existence and Asymptotic Normality of the MLE (Statement not Proof)*

2.4 Efficiency and The Cramer-Rao Bound

- *Theorem: The Information Inequality and the Cramer-Rao bound*
- Efficiency Improvements: Newton's Method and Method of Scoring.
- *Theorem: One Step Estimation*

2.5 Likelihood-Based Hypothesis Tests

- Likelihood-Ratio Statistic
- Wald Statistic
- Rao/Score Statistic
- Asymptotic Distributions under the null hypothesis
- Asymptotic Behaviour under the alternative
- Composite Null Hypotheses and Nuisance Parameters

2.7 Modified Likelihoods

- Partitioning the Information matrix in the presence of nuisance parameters
- Orthogonality
- Profile Likelihood
- Approximation formulae ($p^\star$-formula)
- Connections with likelihood-ratio tests
- Marginal and Conditional Likelihood
- Modified Profile Likelihood

2.7 Beyond Likelihood

- Estimating Functions

3. **Bayesian Theory**

3.1 **Simple Bayesian Computations**

- Reasons for and against the Bayesian Approach
- Conjugate Analysis
- Choosing a Prior

3.2 **The Bayes Linear Model**

- Conjugate Bayes Inference in Regression and the Linear Model

3.3 **Posterior Approximation**

- Quadratic Approximation

3.4 **Bayesian Estimation**

- Loss Functions
- Bayesian Credible Intervals

3.5 **Bayesian Testing**

- Posterior Probabilities on Models
- Bayes Factors
- Bayesian Information Criterion

3.6 **Representation Theorems**

- Exchangeability
- *Theorem: The 0-1 representation theorem*
- The General Representation Theorem