CHAPTER 1

1. DEFINITIONS, TERMINOLOGY, NOTATION

1.1. EVENTS AND THE SAMPLE SPACE

DEFINITION

An **experiment** is a one-off or repeatable process or procedure for which

(a) there is a well-defined set of *possible* outcomes

(b) the *actual* outcome is not known with certainty

DEFINITION

A **sample outcome**, ω, is precisely one of the possible outcomes of an experiment.

DEFINITION

The **sample space**, Ω, of an experiment is the set of all possible outcomes.

NOTE : Ω is a set in the mathematical sense, so set theory notation can be used. For example, if the sample outcomes are denoted $\omega_1, ..., \omega_k$, say, then

$$\Omega = \{\omega_1, ..., \omega_k\} = \{\omega_i : i = 1, ..., k\},$$

and $\omega_i \in \Omega$ for $i = 1, ..., k$.

The sample space of an experiment can be

- a FINITE list of sample outcomes, $\{\omega_1, ..., \omega_k\}$
- an INFINITE list of sample outcomes, $\{\omega_1, \omega_2, ...\}$
- an INTERVAL or REGION of a real space, $\left\{\omega \ : \ \omega \in A \subseteq \mathbb{R}^d\right\}$

DEFINITION

An **event**, E, is a designated collection of sample outcomes. Event E **occurs** if the actual outcome of the experiment is one of this collection.

Special Cases of Events

The event corresponding to collection of *all* sample outcomes is Ω.

The event corresponding to a collection of *none* of the sample outcomes is denoted $\emptyset$.

i.e. The sets $\emptyset$ and Ω are also events, termed the **impossible** and the **certain** event respectively, and for any event E, $E \subseteq \Omega$.

1.2. OPERATIONS IN SET THEORY

Set theory operations can be used to manipulate events in probability theory. Consider events $E, F \subseteq \Omega$. Then the three basic operations are

UNION	$E \cup F$	"E or F or both occur"
INTERSECTION	$E \cap F$	"both E and F occur"
COMPLEMENT	$E^{'}$	"E does not occur"

Properties of Union/Intersection operators

Consider events $E, F, G \subseteq \Omega$.

COMMUTATIVITY	$E \cup F = F \cup E$ $E \cap F = F \cap E$
ASSOCIATIVITY	$E \cup (F \cup G) = (E \cup F) \cup G$ $E \cap (F \cap G) = (E \cap F) \cap G$
DISTRIBUTIVITY	$E \cup (F \cap G) = (E \cup F) \cap (E \cup G)$ $E \cap (F \cup G) = (E \cap F) \cup (E \cap G)$
DE MORGAN'S LAWS	$(E \cup F)^{'} = E^{'} \cap F^{'}$ $(E \cap F)^{'} = E^{'} \cup F^{'}$

NOTE: Union and intersection are *binary* operators, that is, they take only two arguments, and thus the bracketing in the above equations is necessary. For more than two events, $E_1, E_2, ..., E_k$, say, write

$$\bigcup_{i=1}^{k} E_i = E_1 \cup ... \cup E_k \qquad \text{and} \qquad \bigcap_{i=1}^{k} E_i = E_1 \cap ... \cap E_k$$

for the union and intersection of $E_1, E_2, ..., E_k$. The extension to an *infinite* collection of events is immediate.

DEFINITION

Events E and F are **mutually exclusive** if $E \cap F = \emptyset$, that is, if events E and F cannot both occur.

If the sets of sample outcomes represented by E and F are **disjoint** (have no common element), then E and F are mutually exclusive.

DEFINITION

Events $E_1, ..., E_k \subseteq \Omega$ form a **partition** of event $F \subseteq \Omega$ if

(a) $E_i \cap E_j = \emptyset$ for $i \neq j$, $i, j = 1, ..., k$ (b) $\bigcup_{i=1}^{k} E_i = F$.

so that each element of the collection of sample outcomes corresponding to event F is in *one and only one* of the collections corresponding to events $E_1, ...E_k$.