MISCELLANEOUS USEFUL GENERAL RESULTS FOR M2S1

- **SERIES SUMMATIONS**

GEOMETRIC
$$\frac{1}{1-z} = 1 + z + z^2 + ... = \sum_{k=0}^{\infty} z^k \qquad (|z| < 1)$$

EXPONENTIAL
$$e^z = 1 + z + \frac{z^2}{2!} + ... = \sum_{k=0}^{\infty} \frac{z^k}{k!} \qquad (z \in \mathbb{R})$$

BINOMIAL
$$(1+z)^n = 1 + nz + \frac{n(n-1)}{2!}z^2 + ... + nz^{n-1} + z^n = \sum_{k=0}^{n} \binom{n}{k} z^k$$
$$(n > 0)$$

NEG. BINOMIAL
$$\frac{1}{(1-z)^{n+1}} = 1 + (n+1)\,z + \frac{(n+1)\,(n+2)}{2!}z^2 + ... = \sum_{k=0}^{\infty} \binom{n+k}{k} z^k$$
$$(n > 0,\ |z| < 1)$$

LOGARITHMIC
$$-\log(1-z) = z + \frac{z^2}{2} + \frac{z^3}{3} + ... = \sum_{k=1}^{\infty} \frac{z^k}{k} \qquad (|z| < 1)$$

$$\log(1+z) = z - \frac{z^2}{2} + \frac{z^3}{3} - ... = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{z^k}{k} \qquad (|z| < 1)$$

- **EXPONENTIAL FUNCTION** For real $x > 0$

$$\lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n = \lim_{n\to\infty} \left(1 - \frac{x}{n}\right)^{-n} = e^x$$

$$\lim_{n\to\infty} \left(1 - \frac{x}{n}\right)^n = \lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^{-n} = e^{-x}$$

- **TAYLOR SERIES** For real function f and real number c

$$f(x) = \sum_{k=0}^{\infty} \frac{(x-c)^k}{k!} f^k(c)$$

where

$$f^k(c) = \frac{d^k}{dx^k} \{f(x)\}_{x=c}$$

(under the usual regularity assumptions)

- **INTEGRATION METHODS:** When faced with an integral, try
  - direct integration, including the "integration chain rule"

$$\frac{d}{dx}\left\{f\left(g\left(x\right)\right)\right\} = g'\left(x\right) f'\left(g\left(x\right)\right) \qquad \therefore \qquad \int g'\left(x\right) f'\left(g\left(x\right)\right) dx = f\left(g\left(x\right)\right) + \text{constant}$$

  - by parts
  - substitution
  - the special probability integration trick: if you want to compute

$$\int g\left(x\right) dx$$

    but notice that g is proportional to a pdf f_X, that is, say

$$g\left(x\right) = \frac{1}{c} f_X(x) \qquad \text{or} \qquad f_X\left(x\right) = cg\left(x\right)$$

    then you can immediately write down that

$$\int g\left(x\right) dx = \int \frac{1}{c} f_X(x) dx = \frac{1}{c} \int f_X(x) dx = \frac{1}{c} \qquad \text{as } \int f_X(x) dx = 1$$