MISCELLANEOUS USEFUL GENERAL RESULTS FOR M2S1

- **SERIES SUMMATIONS**

GEOMETRIC $$\frac{1}{1-z} = 1 + z + z^2 + ... = \sum_{k=0}^{\infty} z^k \qquad (|z| < 1)$$

EXPONENTIAL $$e^z = 1 + z + \frac{z^2}{2!} + ... = \sum_{k=0}^{\infty} \frac{z^k}{k!} \qquad (z \in \mathbb{R})$$

BINOMIAL $$(1+z)^n = 1 + nz + \frac{n(n-1)}{2!} z^2 + ... + nz^{n-1} + z^n = \sum_{k=0}^{n} \binom{n}{k} z^k$$

$$(n > 0)$$

NEG. BINOMIAL $$\frac{1}{(1-z)^{n+1}} = 1 + (n+1)\, z + \frac{(n+1)\,(n+2)}{2!} z^2 + ... = \sum_{k=0}^{\infty} \binom{n+k}{k} z^k$$

$$(n > 0,\ |z| < 1)$$

LOGARITHMIC $$-\log(1-z) = z + \frac{z^2}{2} + \frac{z^3}{3} + ... = \sum_{k=1}^{\infty} \frac{z^k}{k} \qquad (|z| < 1)$$

$$\log(1+z) = z - \frac{z^2}{2} + \frac{z^3}{3} - ... = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{z^k}{k} \qquad (|z| < 1)$$

- **EXPONENTIAL FUNCTION** For real $x > 0$

$$\lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n = \lim_{n\to\infty} \left(1 - \frac{x}{n}\right)^{-n} = e^x$$

$$\lim_{n\to\infty} \left(1 - \frac{x}{n}\right)^n = \lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^{-n} = e^{-x}$$

- **TAYLOR SERIES** For real function f and real number c

$$f(x) = \sum_{k=0}^{\infty} \frac{(x-c)^k}{k!} f^k(c)$$

where

$$f^k(c) = \frac{d^k}{dx^k} \{f(x)\}_{x=c}$$

(under the usual regularity assumptions)

- **INTEGRATION METHODS:** When faced with an integral, try
  - direct integration, including the "integration chain rule"

    $$\frac{d}{dx}\{f(g(x))\} = g'(x)\, f'(g(x)) \qquad \therefore \qquad \int g'(x)\, f'(g(x))\, dx = f(g(x)) + \text{constant}$$

  - by parts
  - substitution
  - the special probability integration trick: if you want to compute

    $$\int g(x)\, dx$$

    but notice that g is proportional to a pdf f_X, that is, say

    $$g(x) = \frac{1}{c} f_X(x) \qquad \text{or} \qquad f_X(x) = c g(x)$$

    then you can immediately write down that

    $$\int g(x)\, dx = \int \frac{1}{c} f_X(x) dx = \frac{1}{c} \int f_X(x) dx = \frac{1}{c} \qquad \text{as} \int f_X(x) dx = 1$$