M2S1 - ASSESSED COURSEWORK 2

To be handed in no later than Friday, 26th November, 12.00pm.

Please hand in to the Mathematics General Office
as dictated by Departmental regulations.

1. Let the joint density for random variables X and Y be defined by

$$f_{X,Y}(x,y) = c(1-y) \qquad 0 < x < y < 1$$

and zero otherwise, for some constant c.

(a) Find c.

[2 MARKS]

(b) Evaluate $E_{f_X}[X]$ and $E_{f_X}[X^2]$, and hence evaluate $Var_{f_X}[X]$.

[3 MARKS]

(c) Derive the conditional density, $f_{Y|X}(y|x)$, and the conditional expectation

$$E[(1-Y)|X=x].$$

for general $x, 0 < x < 1$. Hence or otherwise, evaluate $E_{f_Y}[Y]$ and $Cov_{f_{X,Y}}[X,Y]$

[3 MARKS]

(d) Evaluate $P[Y < 2X]$.

[2 MARKS]

2. (a) Let the joint density for random variables U and V be defined by

$$f_{U,V}(u,v) = \frac{3}{2}u^2(1-|v|) \qquad -1 < u < 1, -1 < v < 1$$

and zero otherwise.

(i) Are U and V independent ? Justify your answer.

[2 MARKS]

(ii) Let

$$A \equiv \{(u,v) : 0 < u < 1, 0 < v < u\}.$$

Compute $P[(U,V) \in A]$

[3 MARKS]

(b) Let the joint density for random variables R and S be defined by

$$f_{R,S}(r,s) = 2r \qquad 0 < r < 1,\ 0 < s < 1$$

and zero otherwise. Find the probability

$$P\left[R^2 < S < R\right].$$

[5 MARKS]

PLEASE SHOW ALL WORKING. YOU MAY NOT USE MAPLE.