M2S1 - ASSESSED COURSEWORK 1

To be handed in no later than Wednesday, 3rd November, 12.00pm.

Please hand in to the Mathematics General Office
as dictated by Departmental regulations.

1. The location of a particle in a two dimensional lattice extending over the set

$$\mathbb{X} \equiv \{(x,y) : x = 0,1,2,3,...,y = 0,1,2,3,...\}$$

is a discrete random variable. The probability that the particle is at location (x,y) is given by

$$\frac{c(\gamma,\phi)\gamma^x}{x!\phi^y} \qquad x = 0,1,2,3,...,y = 0,1,2,3,...$$

where $c(\gamma,\phi)$ is a constant that does not depend on x or y, for parameters $\gamma > 0$ and $\phi > 1$. The units of the lattice are taken to be 10^{-9} metres.

(a) Find the value of constant $c(\gamma,\phi)$, as a function of the two parameters.

[3 MARKS]

(b) Find the probability that the particle is at $(1,1)$, if $\gamma = 1$, and $\phi = 3$.

[2 MARKS]

(c) Find the probability that the particle lies on the line $y = 1$, if $\gamma = 1$ and $\phi = 3$.

[3 MARKS]

(d) Find the probability that the particle is further than 2 units (i.e. $2 \times 10^{-9}m$) away from $(0,0)$ in any direction, if $\gamma = 1$ and $\phi = 3$.

[3 MARKS]

2. Suppose that X is a continuous random variable with range $\mathbb{R}$ and cdf given by

$$F_X(x) = \frac{c(\mu,\sigma,\alpha)}{\left\{1+\exp\left\{-\left(\frac{x-\mu}{\sigma}\right)\right\}\right\}^{\alpha}} \qquad -\infty < x < \infty$$

and zero otherwise, where c is a normalizing constant that may depend on the three parameters μ, σ, α, where $-\infty < \mu < \infty$, $\sigma > 0$, and $\alpha > 0$.

(a) Find $c(\mu,\sigma,\alpha)$.

[2 MARKS]

(b) Find the probability density function, f_X,of X.

[2 MARKS]

(c) Find the value of x, x_M, for which $F_X(x_M) = 0.5$. Is f_X symmetric about x_M ? Justify your answer.

[2 MARKS]

(d) Find the range $\mathbb{Y}$, cdf F_Y, and pdf f_Y of transformed random variable

$$Y = F_X(X).$$

[4 MARKS]

PLEASE SHOW ALL WORKING. YOU MAY NOT USE MAPLE.