M2S1 PROBABILITY AND STATISTICS
SHORT REVISION QUESTIONS

Chapter 1. Definitions, Terminology, Notation

1. What is the **sample space** of an experiment ?
2. In probability theory, what is an **event** ?
3. What are the three basic set operations that are used in probability theory ?
4. If events E, F are **mutually exclusive**, what can you say about $E \cap F$?
5. Define a **partition** of an event F.
6. What are the three **axioms of probability** ?
7. Define the **conditional probability** of E given F.
8. If events E, F are **independent**, what can you say about $\mathrm{P}(E \cap F)$?
9. If events E, F are **not** independent, what can you say about $\mathrm{P}(E \cap F)$?
10. What is the **Theorem of Total Probability** ?
11. How does the Theorem of Total Probability assist in the calculation of probabilities ?
12. What is **Bayes Theorem** ?

Chapter 2. Probability Distributions and Random Variables

13. What is a **random variable** ?
14. What is the difference between a **discrete** and a **continuous** random variable ?
15. For a **discrete** random variable X, define the **probability mass function**, f_X, of X. Which two conditions must f_X satisfy ?
16. For a discrete random variable X, define the **cumulative distribution function**, or **cdf**, F_X, of X. What properties does F_X exhibit ?
17. For a discrete random variable X, how are f_X and F_X related ?
18. For a **continuous** random variable X, define the **cumulative distribution function**, or **cdf**, F_X, of X. What properties does F_X exhibit ?
19. For a **continuous** random variable X, define the **probability density function**, or **pdf**, f_X, of X. Which two conditions must f_X satisfy ?
20. For a continuous random variable X, how are f_X and F_X related ?
21. What is the **reliability function** of a discrete/continuous random variable ?
22. Define the **expectation** of a function $g(X)$ of random variable X, $\mathrm{E}_{f_X}[X]$. Hence define the **variance** of a random variable X, $\mathrm{Var}_{f_X}[X]$. ?
23. What are the expectation and variance of $2(X+2)$ in terms of the expectation and variance of X ?
24. What is the k**th moment** of a probability distribution ?
25. What is the k**th central moment** of a probability distribution ?
26. If X is a random variable with cdf F_X, and Y is a random variable defined by $Y = g(X)$ for some function g, how is the distribution of Y calculated ?
27. If discrete random variables X and Y are related by $Y = g(X)$, and g is 1-1, how is mass function f_Y calculated from f_X ?
28. If continuous random variables X and Y are related by $Y = g(X)$, and g is 1-1, how is density function f_Y calculated from f_X ?
29. What is the **Jacobian**, $J(y)$, of a univariate transformation g, a 1-1 transformation from $x \in \mathbb{X}$ to $y \in \mathbb{Y}$?

30. For discrete random variables $X_1, ..., X_k$, define the **joint mass function**, $f_{X_1,...,X_k}$, and the **joint cumulative distribution function**, $F_{X_1,...,X_k}$.

31. For continuous random variables $X_1, ..., X_k$, define the **joint cumulative distribution function**, $F_{X_1,...,X_k}$, and the **joint density function**, $f_{X_1,...,X_k}$.

32. For discrete random variables $X_1, ..., X_k$ with joint mass function $f_{X_1,...,X_k}$, how is the **marginal mass function** of X_i, f_{X_i}, calculated ?

33. For continuous random variables $X_1, ..., X_k$ with joint density function $f_{X_1,...,X_k}$, how is the **marginal density function** of X_i, f_{X_i}, calculated ?

34. For random variables $X_1, ..., X_k$ with joint mass/density function $f_{X_1,...,X_k}$, define the **conditional mass/density function** of X_i given the other variables $X_1, ..., X_{i-1}, X_{i+1}, ..., X_k$.

35. What are the necessary and sufficient conditions that must hold if random variables $X_1, ..., X_k$ are **independent** ?

36. What is the **Chain Rule** for random variables ?

37. What is the **Law of Iterated Expectation** ?

38. For a vector of k continuous random variables $\mathbf{X} = (X_1, ..., X_k)$ with joint density function $f_{X_1,...,X_k}$, consider the vector of continuous random variables $\mathbf{Y} = (Y_1, ..., Y_k)$, defined by $\mathbf{Y} = \mathbf{g}(\mathbf{X})$ for some multivariate 1-1 function $\mathbf{g}$. How is the joint density function of $\mathbf{Y}$, $f_{Y_1,...,Y_k}$, calculated ?

39. What are the **covariance** and **correlation** of two random variables ? What is the connection between the covariance and the **dependence** of two random variables ?

40. Why is the correlation a more useful measure of the association between two variables than the covariance ?

41. If random variable $X = X_1 + ... + X_k$, how can the expectation and variance of X be calculated in terms of the expectations, variances, and covariances of $X_1, ..., X_k$?

42. What is the **convolution formula**, and what is it used for ?

43. For random variables $X_1, ..., X_k$, define the **order statistics** $Y_1, ..., Y_k$ derived from $X_1, ..., X_k$. If random variables $X_1, ..., X_k$ are independent and identically distributed with mass/density function f_X, and $Y_1, ..., Y_k$ are the corresponding order statistics, calculate

(i) the distribution of the minimum order statistic Y_1
(ii) the distribution of the maximum order statistic Y_k

44. Find the forms for the **reliability function** for the minimum and maximum order statsistics ?

45. An organism contains a number, $N \geq 0$, of .potentially cancerous cells that will form tumours at random times $T_1, ...T_N$. that are identically distributed with pdf f_T What is the pdf of the random time at which the first tumour develops ?

46. How do you proceed in Q45 if N is unknown, but has discrete probability distribution $f_N(n)$ defined for $n = 0, 1, 2, ...$?

47. For random variable X with mass/density function f_X, define the **moment generating function**, or mgf, of X, M_X, and the **probability generating function**, or pgf, of X, G_X. How is the mgf M_X used to calculate the moments of f_X ?

48. How is the mgf M_Y related to the mgf M_X if random variables X and Y are related by $Y = aX + b$ (in a location/scale transformation) ?

49. If X is a discrete random variable, what is the connection between the pgf and the mgf of X. ?

50. If $X_1, ..., X_k$ are independent random variables, with mgfs M_{X_i} for $i = 1, ..., k$ respectively, and $Y = X_1 + ...X_k$, what is the mgf of Y, M_Y ? What is the form of M_Y if $X_1, ..., X_k$ are identically distributed ?

Chapter 3. Discrete probability distributions

51. What is the form of the mgf for the **discrete uniform distribution** ?

52. What is the form of the mgf for the **Bernoulli distribution** ?

53. What is the form of the mgf for the **binomial distribution** ?

54. How are the Bernoulli and binomial distributions related ?

55. What is the form of the mgf for the **Poisson distribution** ?

56. How are the binomial and Poisson distributions related ?

57. What is the distribution of $Y = X_1 + X_2$, where $X_1 \sim Poisson(\lambda_1), X_2 \sim Poisson(\lambda_2)$?

58. What assumptions underlie the **Poisson process** ?

59. How are the Poisson process and the Poisson distribution related ?

60. What is the form of the mgf for the **geometric distribution** ?

61. Derive the form of the mgf for the Geometric distribution.

62.If $X \sim Geometric(\theta)$, and $m, n > 0$, show that P$[X > n + m \mid X > m]$ =P$[X > n]$.

63. How are the Bernoulli and the geometric distributions related ?

64. What is the form of the mgf for the **negative binomial distribution** ?

65. How are the binomial and the negative binomial distributions related ?

66. How are the Bernoulli and the negative binomial distributions related ?

67. How are the geometric and the negative binomial distributions related ?

68. What is the alternative representation of the negative binomial distribution, and to what experimental situation does it correspond ?

69. How are the Poisson and the negative binomial distributions related ?

Chapter 4. Continuous probability distributions

70. What is the form of the mgf for the **continuous uniform distribution** ?

71. What is the form of the mgf for the **exponential distribution** ?

72. How are the Poisson process and the exponential distribution related ?

73. What is the **lack of memory property** ?

74. How are the continuous uniform and the exponential distributions related ?

75. What is the form of the mgf for the **gamma distribution** ?

76. What is the distribution of $Y = X_1 + X_2$, where $X_1 \sim Ga(\alpha_1, \beta), X_2 \sim Ga(\alpha_2, \beta)$?

77. How are the gamma and **chi-square** distributions related ?

78. If independent random variables $X_1, ..., X_n$ have chi-square distributions with degrees of freedom $m_1, ..., m_n$ respectively, what is the distribution of $Y = X_1 + ... + X_n$?

79. How are the gamma and **beta** distributions related ?

80. How are the beta and the continuous uniform distributions related ?

81. What is the form of the mgf for the **normal distribution** ?

82. What is the distribution of $Y = X_1 + X_2$, where $X_1 \sim N(\mu_1, \sigma_1^2), X_2 \sim N(\mu_2, \sigma_2^2)$?

83. What is the distribution of $Y = X^2$, where $X \sim N(0, 1)$?

84. What is the distribution of $Y = aX + b$ where $X \sim N(0, 1)$?

85. How are the normal, **Cauchy** and **Student-t** distributions related ?

86. What is the Weibull distrubution, and how is it connected to the Exponential distribution ?

87. If X and Y are continuous random variables, and $X|Y = y \sim Exponential(y)$, and $Y \sim Gamma(\alpha, \beta)$, find the (marginal) cdf of X

Chapter 5. Probability results and Limit Theorems

88. State, prove, and explain the implication of, the **Central Limit Theorem** ?

89. Find a Normal approximation to the distribution of $X \sim Poisson\,(\lambda)$?

90. What is meant by "the sequence of random variables **converges in distribution** to random variable X" ?

91. What is the **limiting distribution** of a sequence of random variables ?

92. What are three possible situations relating to the form and existence of a limiting distribution ?

93. What is meant by "the sequence of random variables **converges in probability** to c" ?

94. What are the **Chebychev Inequality** and the **Weak Law of Large Numbers** ?

95. What is meant by "the sequence of random variables **converges in probability** to random variable X" ?

Chapter 6. Introduction to Statistical Analysis

96. What is a **statistic** ?

97. What is the **sampling distribution** of a statistic ?

98 If $X_1, ..., X_n$ are a random sample from a Normal distribution with expectation μ and variance σ^2. What is the joint (sampling) distribution of the two estimators the sample mean $\bar{X}$ and sample variance s^2 ?

99. What is the difference between the **method of moments** and the **method of maximum likelihood** for estimating parameters in a probability model on the basis of an observed sample ?

100. What is the distinction between an **estimate** and an **estimator** ? Define what is meant by an **unbiased** estimator.

[The answers to all these questions can be found somewhere in the course notes, assessed work and exercises; most are quite straightforward, but a few are quite obscure, so you may have to search hard/try to deduce them for yourself ..]