M2S1 - ASSESSED COURSEWORK 2

To be handed in no later than Tuesday, 20th November

(a). Continuous random variables X and Y have joint pdf given by

$$f_{X,Y}(x,y) = c_1(x+y) \qquad 0 \leq x, y \leq 1$$

and zero otherwise, for constant c_1. Find

(i) the value of c_1

(ii) the marginal pdf of X, f_X

(iii) the probability

$$\mathrm{P}\left[Y < \frac{1}{2}\right]$$

(iv) the probability

$$\mathrm{P}\left[Y < X^2\right]$$

(v) Now suppose that the range (and hence the joint distribution) of (X, Y) is changed so that

$$0 \leq x, y \leq 1 \textbf{ and } 0 \leq x + y \leq 1$$

with joint pdf

$$f_{X,Y}(x,y) = c_2(x+y)$$

for constant c_2.

Re-evaluate $\mathrm{P}[Y < X^2]$ for this new specification - you may leave your answer in terms of

$$\alpha = \frac{-1+\sqrt{5}}{2}$$

[Hint: sketch the regions of interest in (iii), (iv) and (v).]

(b) Now suppose that the random variables X and Y have joint pdf specified as

$$f_{X,Y}(x,y) = \frac{1}{2\pi} \exp\left\{-\frac{1}{2}\left(x^2+y^2\right)\right\} \qquad -\infty \leq x, y \leq \infty$$

and suppose that X and Y correspond to the **Cartesian** x- and y-coordinates of a (random) point in the plane.

(i) Suppose that the **polar** coordinates of the point (radius, angle measured from the positive real axis) are random variables (R, T). Find the joint pdf of (R, T), $f_{R,T}$.

(ii) Consider random variables

$$U = \exp\left\{-\frac{R^2}{2}\right\} \qquad V = \frac{T}{2\pi}$$

Are random variables U and V independent ? Justify your answer.