M2S1 - Probability and Statistics II : Examination news

This is a broad summary of the material we covered in the course; it is not exhaustive, but does list the main topics. Essentially, no general topics are excluded, and this covers lecture material and handouts, plus exercises and assessed coursework.

Chapter 1. Definitions, Terminology, Notation

All the basic definitions, terminology and notation of events-based probability theory are examinable. You should understand (things like) sample spaces, events, set-theory binary operations (i.e. union and intersection), the definitions of mutual exclusive events and partitions. Partitions and disjoint unions are examinable.

The basic axioms, definitions and results in probablity theory are examinable. You should be familiar with the probability axioms, conditional probability, independence, and the two basic theorems in probability, the Theorem of Total Probability, and Bayes Theorem, and you should be able to apply them in relatively complex problems. The basic results about counting techniques are examinable in the context of other problems.

Chapter 2. Probability Distributions and Random Variables

Random variables and their probability distributions, and related concepts and definitions formed the bulk of the course, and therefore the majority of this Chapter is examinable. You should be familiar with discrete and continuous random variables, and the basic properties of probability mass, probability density, and cumulative distribution functions. You also need to know about expectations of simple functions of random variables, like variances and other moments etc, and general properties of expectations.

The extensions of the basic ideas to the multivariate setting are examinable; you need to know how to relate and compute joint, marginal and conditional distributions (even when the ranges of the variables are of the form $0 < X < Y < Z$.....). You also need to know about independence of random variables, and how to diagnose it in a general multivariate setting. The Law of Iterated Expectation and its proof are also examinable

The general technique for single variable transformations - the formula, it's proof in the case where the transformation function is 1-1, and how it is applied - is examinable, so you need to know and understand about changes of variable and Jacobians etc. In the multivariate case, the proof of the result in the 1-1 case is not examinable, but you need to know how to apply the general transformation result, and how to calculate Jacobians etc.

Multivariate expectations, such as covariances, correlations and related results and concepts are examinable, as are general expectation results for sums of random variables.

The convolution formula (for the sum of two random variables), and it's proof are examinable; you should be able to apply the formula for specific distributions.

Order statistics; the basic definitions and interpretations, and the results of the three theorems given on the handouts are all examinable, but you don't have to remember the general formulae; they will be given to you if needed. The general proofs are not examinable. However, you should know how to derive the distributions for extreme order statistics (i.e. Min and Max). Also, you should be familiar with applying the formulae given random variables with a particular distribution.

Finally in this Chapter, the definitions and simple results relating to moment generation functions are examinable. You should be able to prove and apply the simple properties such as those relating mgfs to moments, linear (location-scale) transformations of random variables, and sums of independent random variables etc. Factorial/probability generating functions are also examinable (as, of course, they are directly related to mgfs).

Chapters 3 and 4. Special Discrete and Continuous probability distributions
These two chapters contain definitions, information and connections between the most commonly used distributions in probability and statistics. All of the material we studied is examinable, with the exception of the mathematical assumptions and proof relating to the Poisson process; you should however be familiar with the experimental context of the Poisson process, and it's connection with standard distributions.

You will be given a formula sheet in the exam (identical to the one given out in lectures) containing the forms of the mass/density functions that we studied in detail in these chapters, plus expectations, variances, and mgfs; you do not have to memorize the forms of the pdfs of, say, the Student-t, Cauchy, F, Weibull distributions, but you should be familiar with how to derive them using standard transformation techniques.

Chapter 5. Probability results and Limit Theorems
You need to understand the general concept of limiting behaviour of random variables, in terms of, say, limiting cases of parameter values for connections between distributions, or behaviour for "large" collections of random variables.

The Central Limit Theorem (section 5.1), its statement and proof, and implications for approximation results are all examinable. For sections 5.2 and 5.3, all material is examinable, including the Chebychev Inequality and the Weak Law of Large Numbers from Assessed 3 . You need to be familiar with convergence in distribution in terms of cdfs/mgfs, limiting distributions, degenerate limiting distributions, and convergence in probability, and you should be able to apply these concepts to specific problems (i.e. given a specific sequence of random variables, you should be able to find the limiting distribution, assess whether it converges in probability to a given constant, does not converge in distribution etc.)

Chapter 6. Introduction to Statistical Inference
All topics examinable; elementary definifitions (statistic, sampling distribution etc), the result (but not the proof) of the theorem for Normal variables in section 6.1, its implications; terminology for, and methods of Estimation (section 6.2)

Finally, some general hints. In the exam, if you are asked explictly to show, prove, or derive a particular result, you must do this in full detail. If you are asked to state a result, you only have to state it. If you use a result in any calculation or derivation that has been proved or stated in lectures, you may state and use the result without proof unless explictly told otherwise.

Each of the five questions on the exam will contain a significant proportion of **bookwork** based on knowledge of the basic definitions, results and theorems contained in the course. The remainder of the question will usually require an **application of a fundamental technique** to a particular problem of a similar nature the material contained on the Exercise and Assessed worksheets. Usually the two parts of the question will be related.

PLEASE ASK IF YOU NEED ANY FURTHER CLARIFICATION