M2S1 - ASSESSED COURSEWORK 2

To be handed in no later than Friday, 17th November

1. Suppose that X is a continuous random variable with probability density function (pdf) given by

$$f_X(x) = \left(\frac{\lambda}{2\pi}\right)^{1/2} \exp\left\{-\frac{\lambda}{2}(x-\mu)^2\right\} \qquad -\infty < x < \infty$$

where $\lambda > 0$ and μ are real constants.

(a) Find the expectation of X.
(b) Find the expectation of the function $g(X)$ where $g(t) = e^t$.
(c) Suppose now that random variable Y is defined in terms of X by

$$Y = e^X.$$

Find the pdf of Y, and show that the expectation of Y is

$$\exp\left\{\mu + \frac{1}{2\lambda}\right\}$$

2. (a) Suppose that X and Y are continuous random variables with joint pdf given by

$$f_{X,Y}(x,y) = cx(1-y) \qquad 0 < x < 1,\ 0 < y < 1$$

and zero otherwise for some constant c.

(i) Are X and Y independent random variables ? Justify your answer.

(ii) Find the value of c.

(iii) Find the probability

$$\mathrm{P}[\ X < Y\ ] = \int_A\!\!\int f_{X,Y}(x,y)\ dxdy$$

where the set A is the set $\{(x,y)\ :\ 0 < x < y < 1\}$.

(b) Now suppose that the joint pdf of X and Y is given by

$$f_{X,Y}(x,y) = 24xy \qquad x > 0,\ y > 0,\ x + y < 1$$

and zero otherwise.

Find the marginal pdf of X, f_X given by

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\ dy$$

where, recall, the integrand is only non-zero for some part of the integral range.

Hint: before attempting to compute the integrals for 2(a) and (b), sketch the region on which the joint density is non-zero.