M2S1 - EXERCISES 2

1. For which values of the constant c do the following functions define valid probability mass functions for a discrete random variable X, taking values on range $\mathbb{X} = \{1, 2, 3, ...\}$:

$$\text{(a)} \quad f_X(x) = c/2^x \qquad \text{(b)} \quad f_X(x) = c/(x2^x)$$

$$\text{(c)} \quad f_X(x) = c/(x^2) \qquad \text{(d)} \quad f_X(x) = c2^x/x!$$

In each case, calculate (where possible) $\text{P}[X > 1]$ and $\text{P}[X \text{ is even}]$

2. n identical fair coins are tossed. Those that show Heads are tossed again, and the number of Heads obtained on the second set of tosses defines a discrete random variable X. Assuming that all tosses are independent, find the range and probability mass function of X.

3. A point is to be selected from an integer lattice restricted to the triangle with corners at $(1,1)$, $(n,1)$ and (n,n) for positive integer n. If all points are equally likely to be selected, find the probability mass functions for the two discrete random variables X and Y corresponding to the $x-$ and $y-$ coordinates of the selected points respectively.

4. A continuous random variable X has pdf given by

$$f_X(x) = c(1-x)x^2 \qquad 0 < x < 1$$

and zero otherwise. Find the value of c, the cdf of X, F_X, and $\text{P}[X > 1/2]$.

5. A function f is defined by

$$f(x) = k/x^{k+1} \qquad x > 1$$

and zero otherwise. For what values of k is f a valid pdf ? Find the cdf of X.

6. A continuous random variable X has pdf given by

$$f_X(x) = \begin{cases} x & 0 < x < 1 \\ 2-x & 1 \leq x < 2 \end{cases}$$

and zero otherwise. Sketch f_X, and find the cdf F_X.

7. A continuous random variable X has cdf given by

$$F_X(x) = c(\alpha x^\beta - \beta x^\alpha) \qquad 0 \leq x \leq 1$$

for constants $1 \leq \beta < \alpha$, and zero otherwise.

Find the value of constant c, and evaluate the rth moment of X.

8. A continuous random variable X has cdf given by

$$F_X(x) = \frac{2\beta x}{\beta^2 + x^2} \qquad 0 \leq x \leq \beta$$

for constant $\beta > 0$. Find the pdf of X, and show that the expectation of X is

$$\beta(1 - \log 2)$$