M2S1 - ASSESSED COURSEWORK 2

To be handed in no later than Friday, 25th November, 2.00pm.
Please hand in to the Mathematics General Office

(a) Continuous random variables R, X and Y have joint density specified in the following way: the marginal pdf for R, f_R, is defined by

$$f_R(r) = 4r^3 \qquad 0 < r < 1$$

and zero otherwise, and, for $0 < r < 1$, the joint conditional pdf for X and Y **given that** $R = r$, denoted $f_{X,Y|R}$, is given by

$$f_{X,Y|R}(x, y|r) = k(r)x^2y^2 \qquad -r < x < r,\ -r < y < r,\ 0 < x^2 + y^2 < r^2.$$

and zero otherwise, where normalizing constant $k(r)$ **depends on** r.

(i) Find the form of $k(r)$, for $0 < r < 1$.

[6 MARKS]

(ii) Find the joint marginal pdf for X and Y, denoted $f_{X,Y}$.

First find the joint pdf $f_{R,X,Y}$, then marginalize out over R.

[4 MARKS]

(b) Suppose that continuous random variables X_1 and X_2 have joint pdf given by

$$f_{X_1,X_2}(x_1, x_2) = \lambda_1\lambda_2 \exp\{-(\lambda_1 x_1 + \lambda_2 x_2)\} \qquad x_1 > 0,\ x_2 > 0.$$

and zero otherwise, for parameters $\lambda_1, \lambda_2 > 0$. Define random variables Y_1 and Y_2 by

$$Y_1 = \frac{X_1}{X_2} \qquad Y_2 = X_1X_2.$$

(i) Find the marginal cdf of Y_1, denoted F_{Y_1}.

[3 MARKS]

(ii) Find an expression for $P[Y_1 < Y_2]$.

[2 MARKS]

(iii) Find the covariance between X_2 and Y_2.

[5 MARKS]

In all cases, remember to state the support of joint/marginal/conditional pdf, or the ranges of the random variables. Show all working.